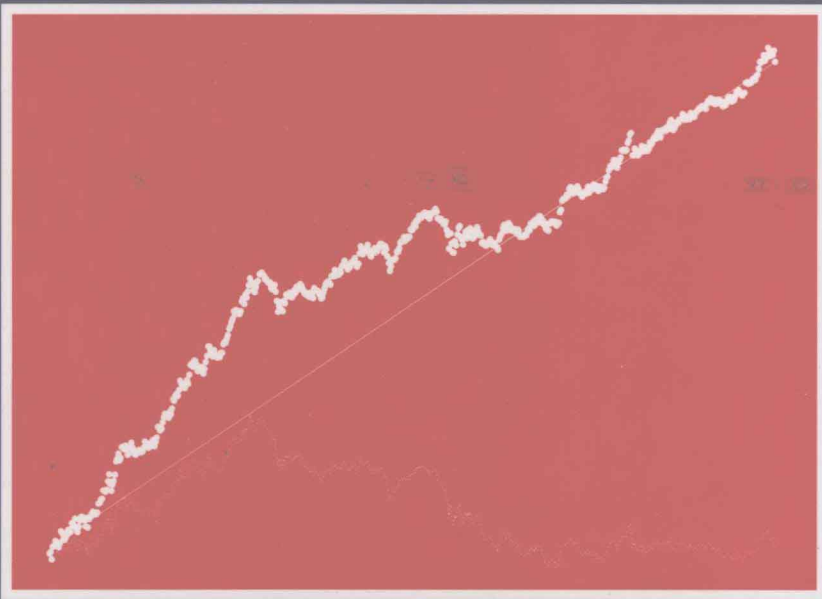


# Analysis of Financial Time Series

Second Edition



Ruey S. Tsay



WILEY SERIES IN PROBABILITY AND STATISTICS

# Analysis of Financial Time Series

Second Edition

RUEY S. TSAY

University of Chicago  
Graduate School of Business



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## Analysis of Financial Time Series

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To my parents and Teresa

# Preface

The subject of *financial time series analysis* has attracted substantial attention in recent years, especially with the 2003 Nobel awards to Professors Robert Engle and Clive Granger. At the same time, the field of financial econometrics has undergone various new developments, especially in high-frequency finance, stochastic volatility, and software availability. There is a need to make the material more complete and accessible for advanced undergraduate and graduate students, practitioners, and researchers. The main goals in preparing this second edition have been to bring the book up to date both in new developments and empirical analysis, and to enlarge the core material of the book by including consistent covariance estimation under heteroscedasticity and serial correlation, alternative approaches to volatility modeling, financial factor models, state-space models, Kalman filtering, and estimation of stochastic diffusion models.

The book therefore has been extended to 10 chapters and substantially revised to include S-Plus commands and illustrations. Many empirical demonstrations and exercises are updated so that they include the most recent data.

The two new chapters are Chapter 9, Principal Component Analysis and Factor Models, and Chapter 11, State-Space Models and Kalman Filter. The factor models discussed include macroeconomic, fundamental, and statistical factor models. They are simple and powerful tools for analyzing high-dimensional financial data such as portfolio returns. Empirical examples are used to demonstrate the applications. The state-space model and Kalman filter are added to demonstrate their applicability in finance and ease in computation. They are used in Chapter 12 to estimate stochastic volatility models under the general Markov chain Monte Carlo (MCMC) framework. The estimation also uses the technique of forward filtering and backward sampling to gain computational efficiency.

A brief summary of the added material in the second edition is:

1. To update the data used throughout the book.
2. To provide S-Plus commands and demonstrations.
3. To consider unit-root tests and methods for consistent estimation of the covariance matrix in the presence of conditional heteroscedasticity and serial correlation in Chapter 2.

4. To describe alternative approaches to volatility modeling, including use of high-frequency transactions data and daily high and low prices of an asset in Chapter 3.
5. To give more applications of nonlinear models and methods in Chapter 4.
6. To introduce additional concepts and applications of value at risk in Chapter 7.
7. To discuss cointegrated vector AR models in Chapter 8.
8. To cover various multivariate volatility models in Chapter 10.
9. To add an effective MCMC method for estimating stochastic volatility models in Chapter 12.

The revision benefits greatly from constructive comments of colleagues, friends, and many readers on the first edition. I am indebted to them all. In particular, I thank J. C. Artigas, Spencer Graves, Chung-Ming Kuan, Henry Lin, Daniel Peña, Jeff Russell, Michael Steele, George Tiao, Mark Wohar, Eric Zivot, and students of my MBA classes on financial time series for their comments and discussions, and Rosalyn Farkas, production editor, at John Wiley. I also thank my wife and children for their unconditional support and encouragement. Part of my research in financial econometrics is supported by the National Science Foundation, the High-Frequency Finance Project of the Institute of Economics, Academia Sinica, and the Graduate School of Business, University of Chicago.

Finally, the website for the book is:

[gsbwww.uchicago.edu/fac/ruey.tsay/teaching/fts2](http://gsbwww.uchicago.edu/fac/ruey.tsay/teaching/fts2).

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# Preface for the First Edition

This book grew out of an MBA course in analysis of financial time series that I have been teaching at the University of Chicago since 1999. It also covers materials of Ph.D. courses in time series analysis that I taught over the years. It is an introductory book intended to provide a comprehensive and systematic account of financial econometric models and their application to modeling and prediction of financial time series data. The goals are to learn basic characteristics of financial data, understand the application of financial econometric models, and gain experience in analyzing financial time series.

The book will be useful as a text of time series analysis for MBA students with finance concentration or senior undergraduate and graduate students in business, economics, mathematics, and statistics who are interested in financial econometrics. The book is also a useful reference for researchers and practitioners in business, finance, and insurance facing value at risk calculation, volatility modeling, and analysis of serially correlated data.

The distinctive features of this book include the combination of recent developments in financial econometrics in the econometric and statistical literature. The developments discussed include the timely topics of value at risk (VaR), high-frequency data analysis, and Markov chain Monte Carlo (MCMC) methods. In particular, the book covers some recent results that are yet to appear in academic journals; see Chapter 6 on derivative pricing using jump diffusion with closed-form formulas, Chapter 7 on value at risk calculation using extreme value theory based on a nonhomogeneous two-dimensional Poisson process, and Chapter 9 on multivariate volatility models with time-varying correlations. MCMC methods are introduced because they are powerful and widely applicable in financial econometrics. These methods will be used extensively in the future.

Another distinctive feature of this book is the emphasis on real examples and data analysis. Real financial data are used throughout the book to demonstrate applications of the models and methods discussed. The analysis is carried out by using several computer packages; the SCA (the Scientific Computing Associates)

for building linear time series models, the RATS (regression analysis for time series) for estimating volatility models, and the S-Plus for implementing neural networks and obtaining postscript plots. Some commands required to run these packages are given in appendixes of appropriate chapters. In particular, complicated RATS programs used to estimate multivariate volatility models are shown in Appendix A of Chapter 9. Some Fortran programs written by myself and others are used to price simple options, estimate extreme value models, calculate VaR, and carry out Bayesian analysis. Some data sets and programs are accessible from the World Wide Web at <http://www.gsb.uchicago.edu/fac/ruey.tsay/teaching/fts>.

The book begins with some basic characteristics of financial time series data in Chapter 1. The other chapters are divided into three parts. The first part, consisting of Chapters 2 to 7, focuses on analysis and application of univariate financial time series. The second part of the book covers Chapters 8 and 9 and is concerned with the return series of multiple assets. The final part of the book is Chapter 10, which introduces Bayesian inference in finance via MCMC methods.

A knowledge of basic statistical concepts is needed to fully understand the book. Throughout the chapters, I have provided a brief review of the necessary statistical concepts when they first appear. Even so, a prerequisite in statistics or business statistics that includes probability distributions and linear regression analysis is highly recommended. A knowledge of finance will be helpful in understanding the applications discussed throughout the book. However, readers with advanced background in econometrics and statistics can find interesting and challenging topics in many areas of the book.

An MBA course may consist of Chapters 2 and 3 as a core component, followed by some nonlinear methods (e.g., the neural network of Chapter 4 and the applications discussed in Chapters 5–7 and 10). Readers who are interested in Bayesian inference may start with the first five sections of Chapter 10.

Research in financial time series evolves rapidly and new results continue to appear regularly. Although I have attempted to provide broad coverage, there are many subjects that I do not cover or can only mention in passing.

I sincerely thank my teacher and dear friend, George C. Tiao, for his guidance, encouragement, and deep conviction regarding statistical applications over the years. I am grateful to Steve Quigley, Heather Haselkorn, Leslie Galen, Danielle LaCouriere, and Amy Hendrickson for making the publication of this book possible, to Richard Smith for sending me the estimation program of extreme value theory, to Bonnie K. Ray for helpful comments on several chapters, to Steve Kou for sending me his preprint on jump diffusion models, to Robert E. McCulloch for many years of collaboration on MCMC methods, to many students in my courses on analysis of financial time series for their feedback and inputs, and to Jeffrey Russell and Michael Zhang for insightful discussions concerning analysis of high-frequency financial data. To all these wonderful people I owe a deep sense of gratitude. I am also grateful for the support of the Graduate School of Business, University of Chicago and the National Science Foundation. Finally, my heartfelt thanks to my wife, Teresa, for her continuous support, encouragement, and

understanding; to Julie, Richard, and Vicki for bringing me joy and inspirations; and to my parents for their love and care.

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# Contents

<b>Preface</b>	<b>xvii</b>
<b>Preface to First Edition</b>	<b>xix</b>
<b>1. Financial Time Series and Their Characteristics</b>	<b>1</b>
1.1 Asset Returns, 2	
1.2 Distributional Properties of Returns, 7	
1.2.1 Review of Statistical Distributions and Their Moments, 7	
1.2.2 Distributions of Returns, 13	
1.2.3 Multivariate Returns, 16	
1.2.4 Likelihood Function of Returns, 17	
1.2.5 Empirical Properties of Returns, 17	
1.3 Processes Considered, 20	
Exercises, 22	
References, 23	
<b>2. Linear Time Series Analysis and Its Applications</b>	<b>24</b>
2.1 Stationarity, 25	
2.2 Correlation and Autocorrelation Function, 25	
2.3 White Noise and Linear Time Series, 31	
2.4 Simple Autoregressive Models, 32	
2.4.1 Properties of AR Models, 33	
2.4.2 Identifying AR Models in Practice, 40	
2.4.3 Goodness of Fit, 46	
2.4.4 Forecasting, 47	

- 2.5 Simple Moving-Average Models, 50
  - 2.5.1 Properties of MA Models, 51
  - 2.5.2 Identifying MA Order, 52
  - 2.5.3 Estimation, 53
  - 2.5.4 Forecasting Using MA Models, 54
- 2.6 Simple ARMA Models, 56
  - 2.6.1 Properties of ARMA(1,1) Models, 57
  - 2.6.2 General ARMA Models, 58
  - 2.6.3 Identifying ARMA Models, 59
  - 2.6.4 Forecasting Using an ARMA Model, 61
  - 2.6.5 Three Model Representations for an ARMA Model, 62
- 2.7 Unit-Root Nonstationarity, 64
  - 2.7.1 Random Walk, 64
  - 2.7.2 Random Walk with Drift, 65
  - 2.7.3 Trend-Stationary Time Series, 67
  - 2.7.4 General Unit-Root Nonstationary Models, 67
  - 2.7.5 Unit-Root Test, 68
- 2.8 Seasonal Models, 72
  - 2.8.1 Seasonal Differencing, 73
  - 2.8.2 Multiplicative Seasonal Models, 75
- 2.9 Regression Models with Time Series Errors, 80
- 2.10 Consistent Covariance Matrix Estimation, 86
- 2.11 Long-Memory Models, 89
- Appendix: Some SCA Commands, 91
- Exercises, 93
- References, 96

### 3. Conditional Heteroscedastic Models

97

- 3.1 Characteristics of Volatility, 98
- 3.2 Structure of a Model, 99
- 3.3 Model Building, 101
  - 3.3.1 Testing for ARCH Effect, 101
- 3.4 The ARCH Model, 102
  - 3.4.1 Properties of ARCH Models, 104
  - 3.4.2 Weaknesses of ARCH Models, 106
  - 3.4.3 Building an ARCH Model, 106
  - 3.4.4 Some Examples, 109
- 3.5 The GARCH Model, 113
  - 3.5.1 An Illustrative Example, 116

3.5.2	Forecasting Evaluation, 121
3.5.3	A Two-Pass Estimation Method, 121
3.6	The Integrated GARCH Model, 122
3.7	The GARCH-M Model, 123
3.8	The Exponential GARCH Model, 124
3.8.1	An Alternative Model Form, 125
3.8.2	An Illustrative Example, 126
3.8.3	Second Example, 126
3.8.4	Forecasting Using an EGARCH Model, 128
3.9	The Threshold GARCH Model, 130
3.10	The CHARMA Model, 131
3.10.1	Effects of Explanatory Variables, 133
3.11	Random Coefficient Autoregressive Models, 133
3.12	The Stochastic Volatility Model, 134
3.13	The Long-Memory Stochastic Volatility Model, 134
3.14	Application, 136
3.15	Alternative Approaches, 140
3.15.1	Use of High-Frequency Data, 140
3.15.2	Use of Daily Open, High, Low, and Close Prices, 143
3.16	Kurtosis of GARCH Models, 145
	Appendix: Some RATS Programs for Estimating Volatility Models, 147
	Exercises, 148
	References, 151

#### **4. Nonlinear Models and Their Applications**

**154**

4.1	Nonlinear Models, 156
4.1.1	Bilinear Model, 156
4.1.2	Threshold Autoregressive (TAR) Model, 157
4.1.3	Smooth Transition AR (STAR) Model, 163
4.1.4	Markov Switching Model, 164
4.1.5	Nonparametric Methods, 167
4.1.6	Functional Coefficient AR Model, 175
4.1.7	Nonlinear Additive AR Model, 176
4.1.8	Nonlinear State-Space Model, 176
4.1.9	Neural Networks, 177
4.2	Nonlinearity Tests, 183
4.2.1	Nonparametric Tests, 183
4.2.2	Parametric Tests, 186
4.2.3	Applications, 190

- 4.3 Modeling, 191
- 4.4 Forecasting, 192
  - 4.4.1 Parametric Bootstrap, 192
  - 4.4.2 Forecasting Evaluation, 192
- 4.5 Application, 194
- Appendix A: Some RATS Programs for Nonlinear Volatility Models, 199
- Appendix B: S-Plus Commands for Neural Network, 200
- Exercises, 200
- References, 202

## **5. High-Frequency Data Analysis and Market Microstructure 206**

- 5.1 Nonsynchronous Trading, 207
- 5.2 Bid–Ask Spread, 210
- 5.3 Empirical Characteristics of Transactions Data, 212
- 5.4 Models for Price Changes, 218
  - 5.4.1 Ordered Probit Model, 218
  - 5.4.2 A Decomposition Model, 221
- 5.5 Duration Models, 225
  - 5.5.1 The ACD Model, 227
  - 5.5.2 Simulation, 229
  - 5.5.3 Estimation, 232
- 5.6 Nonlinear Duration Models, 236
- 5.7 Bivariate Models for Price Change and Duration, 237
- Appendix A: Review of Some Probability Distributions, 242
- Appendix B: Hazard Function, 245
- Appendix C: Some RATS Programs for Duration Models, 246
- Exercises, 248
- References, 250

## **6. Continuous-Time Models and Their Applications 251**

- 6.1 Options, 252
- 6.2 Some Continuous-Time Stochastic Processes, 252
  - 6.2.1 The Wiener Process, 253
  - 6.2.2 Generalized Wiener Processes, 255
  - 6.2.3 Ito Processes, 256
- 6.3 Ito's Lemma, 256
  - 6.3.1 Review of Differentiation, 256
  - 6.3.2 Stochastic Differentiation, 257

6.3.3	An Application, 258
6.3.4	Estimation of $\mu$ and $\sigma$ , 259
6.4	Distributions of Stock Prices and Log Returns, 261
6.5	Derivation of Black–Scholes Differential Equation, 262
6.6	Black–Scholes Pricing Formulas, 264
6.6.1	Risk-Neutral World, 264
6.6.2	Formulas, 264
6.6.3	Lower Bounds of European Options, 267
6.6.4	Discussion, 268
6.7	An Extension of Ito’s Lemma, 272
6.8	Stochastic Integral, 273
6.9	Jump Diffusion Models, 274
6.9.1	Option Pricing Under Jump Diffusion, 279
6.10	Estimation of Continuous-Time Models, 282
	Appendix A: Integration of Black–Scholes Formula, 282
	Appendix B: Approximation to Standard Normal Probability, 284
	Exercises, 284
	References, 285

## **7. Extreme Values, Quantile Estimation, and Value at Risk**

287

7.1	Value at Risk, 287
7.2	RiskMetrics, 290
7.2.1	Discussion, 293
7.2.2	Multiple Positions, 293
7.3	An Econometric Approach to VaR Calculation, 294
7.3.1	Multiple Periods, 296
7.4	Quantile Estimation, 298
7.4.1	Quantile and Order Statistics, 299
7.4.2	Quantile Regression, 300
7.5	Extreme Value Theory, 301
7.5.1	Review of Extreme Value Theory, 301
7.5.2	Empirical Estimation, 304
7.5.3	Application to Stock Returns, 307
7.6	Extreme Value Approach to VaR, 311
7.6.1	Discussion, 314
7.6.2	Multiperiod VaR, 316
7.6.3	VaR for a Short Position, 316
7.6.4	Return Level, 317



- 7.7 A New Approach Based on the Extreme Value Theory, 318
  - 7.7.1 Statistical Theory, 318
  - 7.7.2 Mean Excess Function, 320
  - 7.7.3 A New Approach to Modeling Extreme Values, 322
  - 7.7.4 VaR Calculation Based on the New Approach, 324
  - 7.7.5 An Alternative Parameterization, 325
  - 7.7.6 Use of Explanatory Variables, 328
  - 7.7.7 Model Checking, 329
  - 7.7.8 An Illustration, 330
- Exercises, 335
- References, 337

## 8. Multivariate Time Series Analysis and Its Applications 339

- 8.1 Weak Stationarity and Cross-Correlation Matrices, 340
  - 8.1.1 Cross-Correlation Matrices, 340
  - 8.1.2 Linear Dependence, 341
  - 8.1.3 Sample Cross-Correlation Matrices, 342
  - 8.1.4 Multivariate Portmanteau Tests, 346
- 8.2 Vector Autoregressive Models, 349
  - 8.2.1 Reduced and Structural Forms, 349
  - 8.2.2 Stationarity Condition and Moments of a VAR(1) Model, 351
  - 8.2.3 Vector AR( $p$ ) Models, 353
  - 8.2.4 Building a VAR( $p$ ) Model, 354
  - 8.2.5 Impulse Response Function, 362
- 8.3 Vector Moving-Average Models, 365
- 8.4 Vector ARMA Models, 371
  - 8.4.1 Marginal Models of Components, 375
- 8.5 Unit-Root Nonstationarity and Cointegration, 376
  - 8.5.1 An Error-Correction Form, 379
- 8.6 Cointegrated VAR Models, 380
  - 8.6.1 Specification of the Deterministic Function, 382
  - 8.6.2 Maximum Likelihood Estimation, 383
  - 8.6.3 A Cointegration Test, 384
  - 8.6.4 Forecasting of Cointegrated VAR Models, 385
  - 8.6.5 An Example, 385
- 8.7 Threshold Cointegration and Arbitrage, 390
  - 8.7.1 Multivariate Threshold Model, 391
  - 8.7.2 The Data, 392