

Reinforced Concrete Structures

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Preface

We hope that the content and the treatment of the subject of reinforced concrete structures in this book will appeal to students, teachers, and practicing members of the structural engineering profession.

The book has grown from two editions of seminar notes entitled *Ultimate Strength Design of Reinforced Concrete Structures* (Vol. 1), printed by the University of Canterbury for extension study seminars conducted for practicing structural engineers in New Zealand. Those early editions of seminar notes have been considerably extended and updated. Many years of experience in teaching theory and design, and in design and research, have helped to form ideas and to provide background material for the book.

The text emphasizes the basic behavior of reinforced concrete elements and of structures—in particular, their strength and deformation characteristics up to ultimate load. It endeavors to give the reader a thorough knowledge of the fundamentals of reinforced concrete. Such a background is essential to a complete and proper understanding of building codes and design procedures. The design engineer may be disappointed that the text does not extend into a range of design charts, tables, and examples. Such information is available elsewhere. The main purpose of the text is to bring about a basic understanding of the background to such applied material.

The current building code of the American Concrete Institute (ACI 318-71) is one of the most widely accepted reinforced concrete codes. It has been adopted by some countries and has strongly influenced the codes of many others. For this reason extensive reference is made to the ACI provisions, but comparison with other building codes appears where necessary. The book is not heavily code oriented, however. The emphasis is on why certain engineering decisions should be made, rather than how they should be executed. It is our belief that engineers should be capable of rationally assessing design procedures and should not be blind followers of code provisions.

The strength and serviceability approach to design is emphasized throughout the book because we believe that it is the most realistic method.

The book commences with a discussion of basic design criteria and the

properties of concrete and steel. The strength and deformation of reinforced concrete structural members with flexure, flexure and axial load, shear, and torsion are then presented in some depth, followed by a discussion of bond and anchorage. The service load behavior of reinforced concrete members is then examined, with emphasis on deflection and crack control. This material is followed by a treatment of frames and shear walls. Because we believe that correct proportioning of components is insufficient to ensure a successful design, the book ends with a discussion on the detailing of structural components and joints.

We have not attempted to treat the design of specific types of structures. Thorough understanding of the behavior of reinforced concrete components and of structural analysis should enable a designer to undertake the design of the common range of structures and to find solutions to special problems.

An aspect of the book that distinguishes it from most other texts on reinforced concrete is the treatment of the effects of earthquake loading and means of achieving design procedures for seismic-resistant structures. Seismic design is assuming more importance with the realization that seismic zones may be more extensive than has heretofore been assumed. Seismic design involves more than a consideration of additional static lateral loads on the structure. Proper attention to details, and an understanding of possible failure mechanisms, are essential if structures capable of surviving major earthquakes are to be designed. Considerations of behavior under intense seismic loading involve an understanding of the deformation characteristics of members and structures in the inelastic range, as well as the development of strength, and these areas are given due regard in the text.

A detailed discussion of slabs has been omitted because a book-length treatment is in preparation.

We hope that the book will serve as a useful text to teachers preparing a syllabus for undergraduate courses in reinforced concrete. Each major topic has been treated in enough depth to permit the book to be used by graduate students in advanced courses in reinforced concrete. It is hoped that many practicing engineers, particularly those facing the formidable task of having to design earthquake-resistant structures, will also find this book a useful reference.

We would be grateful for any constructive comments or criticisms that readers may have and for notification of any errors that they will inevitably detect.

The authors have received a great deal of assistance, encouragement, and inspiration from numerous sources. Thanks are due to our many colleagues at the University of Canterbury, particularly to Prof. H. J. Hopkins, who initiated a strong interest in concrete at this University, to Dr. A. J. Carr, who read part of the manuscript, and to Mrs. Alice Watt, whose patience when

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R. PARK
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I

The Design Approach

1.1 DEVELOPMENT OF WORKING STRESS AND ULTIMATE STRENGTH DESIGN PROCEDURES

Several of the early studies of reinforced concrete members were based on ultimate strength theories, for example, Thullie's flexural theory of 1897 and the parabolic stress distribution theory of Ritter in 1899. However at about 1900 the straight-line (elastic) theory of Coignet and Tedesco became generally accepted, mainly because elastic theory was the conventional method of design for other materials and also because it was thought that the straight-line distribution of stress led to mathematical simplification. In addition, tests had shown that the use of elastic theory with carefully chosen values for the allowable working stresses led to a structure displaying satisfactory behavior at the service loads and having an adequate margin of safety against collapse. Thus elastic theory has been the basis of reinforced concrete design for many years.

Recently there has been renewed interest in ultimate strength theory as a basis of design. After more than half a century of practical experience and laboratory tests, the knowledge of the behavior of structural concrete has vastly increased and the deficiencies of the elastic theory (working stress) design method have become evident. This has resulted in periodic adjustment to the working stress design method, but it has become increasingly apparent that a design method should be based on the actual inelastic properties of the concrete and steel. Thus ultimate strength design became accepted as an alternative to working stress design in the building codes for reinforced concrete of the American Concrete Institute (ACI) in 1956 and of the United Kingdom in 1957. These two design approaches may be summarized as follows.

Working Stress Design (Elastic Theory)

The sections of the members of the structure are designed assuming straight-line stress-strain relationships ensuring that at service loads the stresses in

the steel and the concrete do not exceed the allowable working stresses. The allowable stresses are taken as fixed proportions of the ultimate or yield strength of the materials; for example, for compression in bending 0.45 of the cylinder strength of the concrete may be assumed. The bending moments and forces that act on statically indeterminate structures are calculated assuming linear-elastic behavior.

Ultimate Strength Design

The sections of the members of the structures are designed taking inelastic strains into account to reach ultimate (maximum) strength (i.e., the concrete at maximum strength and usually the steel yielding) when an ultimate load, equal to the sum of each service load multiplied by its respective load factor, is applied to the structure. Typical load factors used in practice are 1.4 for dead load and 1.7 for live load. The bending moments and forces that act on statically indeterminate structures at the ultimate load are calculated assuming linear-elastic behavior of the structure up to the ultimate load. Alternatively, the bending moments and forces are calculated taking some account of the redistribution of actions that may occur because of the non-linear relationships that exist between the actions and deformations in the members at high loads.

Some of the reasons for the trend towards ultimate strength design are as follows:

1. Reinforced concrete sections behave inelastically at high loads; hence elastic theory cannot give a reliable prediction of the ultimate strength of the members because inelastic strains are not taken into account. For structures designed by the working stress method, therefore, the exact load factor (ultimate load/service load) is unknown and varies from structure to structure.

2. Ultimate strength design allows a more rational selection of the load factors. For example, a low load factor may be used for loading known more exactly, such as dead load, and a higher load factor for less certain loads, such as live load.

3. The stress-strain curve for concrete is nonlinear and is time dependent. For example, the creep strains for concrete under constant sustained stress may be several times the initial elastic strain. Therefore, the value of the modular ratio (ratio of the elastic modulus of steel to that of concrete) used in working stress design is a crude approximation. Creep strains can cause a substantial redistribution of stress in reinforced concrete sections, and this means that the stresses that actually exist at the service loads often bear little relation to the design stresses. For example, the compression steel in columns may reach the yield strength during the sustained application of service