

LINEAR CONTROL SYSTEMS

**MODELING,
ANALYSIS,
AND DESIGN**

JAMES R. ROWLAND

LINEAR

CONTROL SYSTEMS: MODELING, ANALYSIS, AND DESIGN

JAMES R. ROWLAND

Professor of Electrical and Computer Engineering
University of Kansas

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PREFACE

Dynamic in coverage and emphasis, the field of control engineering has progressed through several distinct stages in recent decades. At one time dealing primarily with the analysis and design of servomechanisms, feedback control applications have expanded to such areas as economics, sociology, transportation, energy, and ecology. Concurrently, a shift in emphasis has taken place from classical transfer function procedures to state variable techniques. Renewed interests in the transfer function approach and its relationship to state variable design have been exhibited recently. This book presents an integrated treatment of linear control system modeling, analysis, and design based on relating and interpreting concepts from both transfer function and state variable viewpoints.

This book is intended for a one-semester or one-quarter introductory feedback course at the undergraduate level for seniors (or possibly advanced juniors) in electrical engineering and related engineering fields. It should also be useful for self-study, or as a reference, for graduate engineers on the job or in graduate school. Although both transform and state variable techniques are presented and integrated throughout the book, a conscious effort has been made to develop the transform procedures somewhat more completely. This choice is based on the realization that subsequent graduate courses usually emphasize the state variable approach and that introductory feedback control courses traditionally emphasize classical transform procedures. Relating and interpreting concepts from both viewpoints with a firm development of the classical approach appears to be a suitable blend for a modern textbook at the senior undergraduate level.

A prerequisite knowledge of the analysis of linear circuits is assumed. A background including the use of Laplace transforms to solve linear ordinary differential equations and some familiarity with frequency response methods and matrix fundamentals are also recommended, though these concepts are covered as needed within the book.

This book is organized into parts on modeling (Part I: Chapters 1, 2, and 3), analysis (Part II: Chapters 4, 5, and 6), and design (Part III: Chapters 7 and 8). Points of integration between classical transform and state variable viewpoints appear in Chapters 2 to 8 as alternative approaches within the same section (e.g., Sections 2.5, 3.3, or 3.4), in neighboring sections (e.g., Sections 2.4 and 2.6), and in related sections in different chapters (e.g., Sections 7.3 and 8.3). Computational methods within the book include the use of iterative formulas for determining the roots of a polynomial (e.g., Sections 4.2 and 6.6) and step-by-step procedures for designing series compensation networks (Chapter 7). These computational methods emphasize the numerical algorithms being used and not their specialization to a particular calculator or digital computer.

I wish to express my appreciation to the reviewers for their detailed evaluations in helping to shape the scope, content, and tone of this book. Initial reviewers

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were Dean K. Frederick, Rensselaer Polytechnic Institute; J. B. Cruz, Jr., University of Illinois; Robert N. Clark, University of Washington; and Richard Roberts, University of Colorado. Final reviewers were Arild Thowsen, Iowa State University; Alexander Nauda, Bucknell University; and Paul E. Russell, Arizona State University. Special thanks go to Charles M. Bacon, Oklahoma State University, for his personal and administrative encouragement; to Daniel D. Lingelbach, Gary E. Young, James H. Taylor, and Lynn R. Ebbesen for helpful suggestions while teaching from manuscripts for this book at Oklahoma State University; and to Robert J. Mulholland, J. Mark Richardson, and John M. Acken for reviewing selected chapters.

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James R. Rowland

ABOUT THE AUTHOR

James R. Rowland, Ph.D., P.E., received his doctorate in electrical engineering from Purdue University. He is currently Professor and Chairman of the Department of Electrical and Computer Engineering at the University of Kansas. He has previous faculty experience at Oklahoma State University and Georgia Institute of Technology and consulting and industrial experience at Lockheed-Georgia, the U.S. Army Missile Command, and Sandia National Laboratories. Teaching and research interests have included control systems theory, stochastic modeling, digital signal processing, and estimation theory. He is a Senior Member of the Institute of Electrical and Electronics Engineers and was the recipient of an IEEE Centennial Medal. He has served the IEEE as Education Society President and as Awards and Recognition Chairman of the Educational Activities Board. A member of the American Society for Engineering Education and the National Society of Professional Engineers, he has served as a program accreditation visitor for the Accreditation Board for Engineering and Technology.

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PART I

INTRODUCTION AND CONTROL SYSTEM MODELING

1

Introduction to Automatic Control

Ingenuity in devising new and better controllers for physical dynamic systems has helped to stimulate rapid technological progress in this century. These improvements have been enhanced by the continual development of new devices for use as system components and by the expanding capabilities of computers for use in modeling, analysis, and design tasks. The marketplace of the world has come to depend on the products of the control engineer. Familiar control applications range from autopilots to air conditioners, elevators to ecosystems, spacecraft to solar trackers, and from nuclear submarines to sensor-equipped industrial robots. Some of these systems that employ feedback are shown in Figure 1.1.

A control engineer's first step is the formation of a suitable model of the dynamic system or process to be controlled. This model may be validated by analyzing its performance for realistic input conditions and then by comparing with field test data taken from the dynamic system in its operating environment. Model validation is often achieved with the aid of computer simulations in which the effects of varying parameters can be determined more easily. Further analysis of the simulated model is usually necessary to obtain the model response for different feedback configurations and parameter settings. Once an acceptable controller has been designed and tested on the model, the feedback control strategy is then applied to the actual system to be controlled. In this chapter we present introductory concepts as a motivation for a further appreciation and understanding of these modeling, analysis, and design steps.

1.1 FEEDBACK AND ITS SIGNIFICANCE

Regulatory processes in nature can be regarded as natural feedback that provides an automatic control feature for the natural system. Often assumed to be a relatively simple structure, the underlying regulatory process and its relationship to the total natural system might be quite complex. Familiar examples are skin temperature control through perspiration cooling (homeostasis) and the ecological

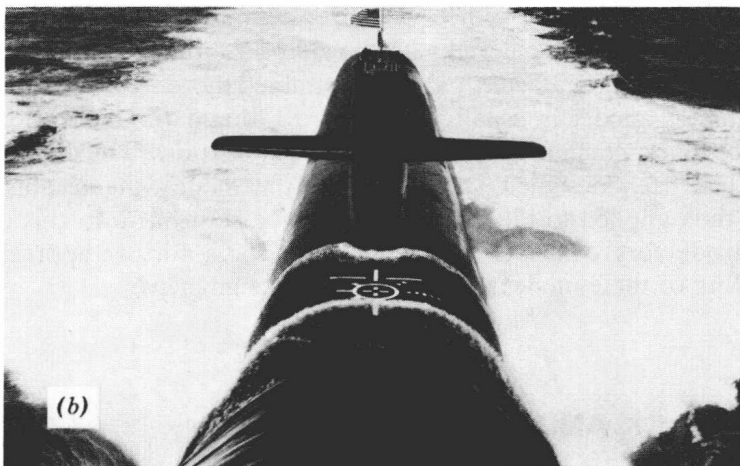


Figure 1.1 Common control system applications include (a) spacecraft control, (b) the Trident submarine USS Alabama.

Source: Courtesy of (a) Hughes Aircraft Company, (b) McDonnell Douglas.

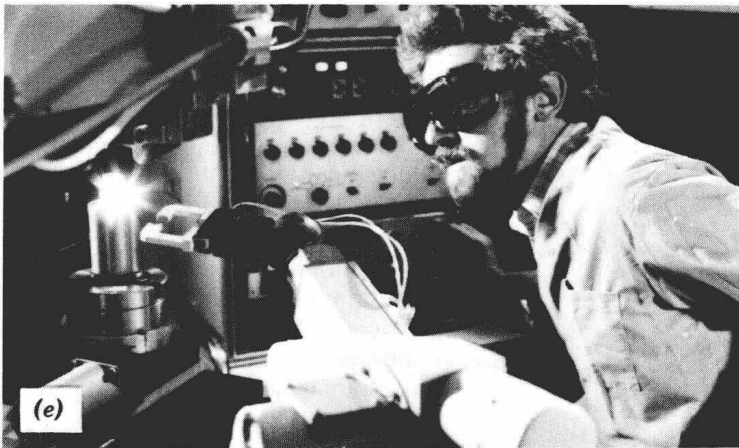
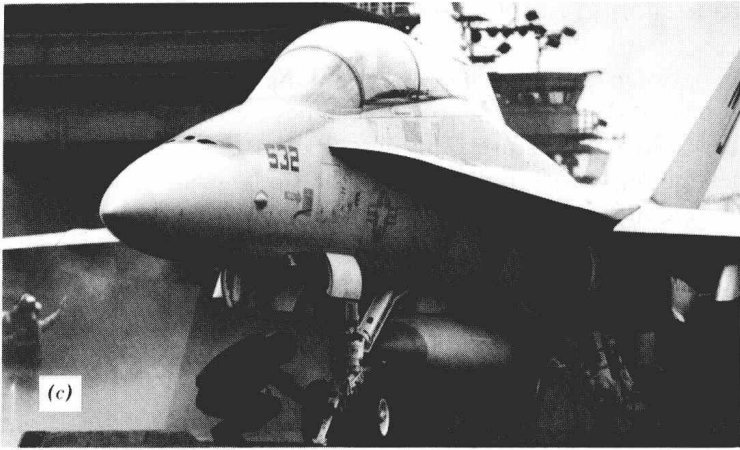


Figure 1.1 (Continued) (c) the F/A-18 Hornet fighter aircraft, (d) PRÖVOX[®] boiler control instrumentation, and (e) a prototype robot arm used in conjunction with laser welding.

(c) General Dynamics, (d) Fisher Controls, and (e) Northrop Corporation.

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balance between predator and prey. In the first case, the perspiration, which appears on the skin when the body becomes overheated, begins to evaporate and lower the skin temperature. The perspiration that forms and then cools makes possible the regulatory process, and the body itself is the natural system that needs temperature control. In the second case, if a large number of prey (e.g., rabbits) is available in a wildlife system, the predators (e.g., wolves) increase in number and consequently consume more of the prey. Since fewer of the prey are then available, the number of predators automatically decreases, causing a larger

TABLE 1.1 Typical Feedback Applications

Categories	Specific Applications
Ecological	Wildlife management and control; control of plant chemical wastes via monitoring lakes and rivers; air pollution abatement; water control and distribution; flood control via dams and reservoirs; forest growth management
Medical	Medical instrumentation for monitoring and control; artificial limbs (prosthesis)
Home appliances	Home heating, refrigeration, and airconditioning via thermostatic control; electronic sensing and control in clothes dryers; humidity controllers; temperature control of ovens
Power/energy	Power system control and planning; feedback instrumentation in oil recovery; optimal control of windmill blade and solar panel surfaces; optimal power distribution via power factor control
Transportation	Control of roadway vehicle traffic flows using sensors; automatic speed control devices on automobiles; propulsion control in rail transit systems; building elevators and escalators
Manufacturing	Sensor-equipped robots for cutting, drilling, die casting, forging, welding, packaging, and assembling; chemical process control; tension control windup processes in textile mills; conveyor speed control with optical pyrometer sensing in hot steel rolling mills
Aerospace and military	Missile guidance and control; automatic piloting; spacecraft control; tracking systems; nuclear submarine navigation and control; fire-control systems (artillery)
