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A CONCISE

INTRODUCTION

TO **LOGIC**

SIXTH EDITION

A CONCISE INTRODUCTION TO LOGIC

Sixth Edition

Patrick J. Hurley
University of San Diego



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PREFACE

The most immediate benefit derived from the study of logic is the skill needed to construct sound arguments of one's own and to evaluate the arguments of others. In accomplishing this goal, logic instills a sensitivity for the formal component in language, a thorough command of which is indispensable to clear, effective, and meaningful communication. On a broader scale, by focusing attention on the requirement for reasons or evidence to support our views, logic provides a fundamental defense against the prejudiced and uncivilized attitudes that threaten the foundations of our democratic society. Finally, through its attention to inconsistency as a fatal flaw in any theory or point of view, logic proves a useful device in disclosing ill-conceived policies in the political sphere and, ultimately, in distinguishing the rational from the irrational, the sane from the insane.

To realize the benefits offered by the study of logic, one must thoroughly understand the central concepts of the subject and be able to apply them in actual situations. To promote the achievement of these goals, this text presents the central concepts of logic clearly and simply. Examples are used extensively, key terms are introduced in boldface type and defined in the glossary/index, and major points are illustrated in graphic boxes. Furthermore, to ensure sufficient practice in applying the basic principles, the book includes over 2,000 exercises selected to illustrate the main points and guard against the most typical mistakes. In most cases, every third exercise is answered in the back of the book.

New to the Sixth Edition

The sixth edition retains the same basic format of its predecessors. One of the principal changes is the introduction of a new Section 4.6, which shows how Venn diagrams apply to the Aristotelian standpoint and provides increased flexibility for instructors. For example, instructors who opt for a simplified treatment of this topic as an introduction to Chapter 5 may assign only the first two subsections and the first set of exercises, while those wanting a fuller treatment may assign the whole section and both sets of exercises. On the other hand, instructors who confine their attention to the Boolean standpoint may skip the entire section without adverse consequences for their students.

In propositional logic, the importance of consistency and inconsistency is given increased emphasis. Inconsistency is added to the list of relations between statements in Section 6.3, and a subsection is added to Section 6.5 that shows how indirect truth tables can be used to test groups of statements for consistency. In regard to natural deduction, a running series of 24 strategies, distributed through the first four sections of Chapter 7, are intended to help students see how to proceed with proofs, and further instruction is given to the technique of working backward from the conclusion.

In Chapter 1, additional instruction is given on distinguishing arguments from explanations, and the context-dependent nature of this distinction is noted. The conditional language (“if the premises are assumed true . . .”) has been removed from the definitions of validity and strength and incorporated into a more fully developed technique for intuitively evaluating deductive and inductive arguments. Also, in Section 1.5 more instruction is given on identifying the form of an argument.

Chapter 3 features a new subsection at the end on avoiding fallacies, a rewritten account of begging the question, improved examples for suppressed evidence, further discussion on identifying causal connections, and a newly drawn distinction between the fallacious appeal to pity and the nonfallacious appeal to compassion. Chapter 5 provides more explicit instruction on applying Venn diagrams from the Aristotelian standpoint, and a newly introduced “superfluous distribution rule” is introduced in Section 5.3 that allows one to identify the critical term in syllogisms that are valid from the Aristotelian standpoint.

Additional improvements include new summaries at the end of each chapter and a new mnemonic device in Section 4.2 for remembering the rule for distribution. To facilitate a transition to this new edition, a complete list of changes by page number is included at the beginning of the Instructor’s Manual.

Robert Burch has updated his *Study Guide*, and I am confident that students will continue to find it useful as a supplementary source of exercises as well as a review for examinations. Also, Nelson Pole has upgraded his popular computer software program, *LogicCoach*, which offers hundreds of

hours of practice on the exercises in the text. A Windows version of this software, as well as a DOS version, is available as of August 1996, and a Macintosh version is planned for December 1996.

Finally, new test materials have been placed in the *Instructor's Manual*, and they are available in computerized format from Wadsworth Publishing Co. Also, the two supplements, "Critical Thinking and Writing" and "Truth Trees," have been updated, and they will be supplied free when bundled with new copies of the textbook or for a nominal charge independently of the textbook.

Alternative Course Approaches to the Textbook

Depending on the instructor's preferences, this textbook can be approached in several ways. The following chart presents possible approaches for three different kinds of course.

In general, the material in each chapter is arranged so that certain later sections can be skipped without affecting subsequent chapters. For example, those wishing a brief treatment of natural deduction in both propositional and predicate logic may want to skip the last three sections of Chapter 7 and the last four (or even five) sections of Chapter 8. Chapter 2 can be skipped altogether, although some may want to cover the first section of that chapter as an introduction to Chapter 3. Finally, the five sections of Chapter 9 depend only slightly on earlier chapters, so these sections can be treated in any order one chooses.

TYPE OF COURSE

	Traditional logic course	Informal logic course, critical reasoning course	Course emphasizing modern formal logic
Recommended material	Chapter 1 Chapter 3 Chapter 4 Chapter 5 Chapter 6 Sections 7.1–7.4	Chapter 1 Chapter 2 Chapter 3 Chapter 4 Sections 5.1–5.3 Sections 5.5–5.6 Sections 6.1–6.4 Section 6.6 Section 9.1 Sections 9.4–9.5 Writing Supplement	Chapter 1 Sections 4.1–4.3 Section 4.7 Sections 6.1–6.5 Chapter 7 Chapter 8 Truth Tree Supplement
Optional material	Chapter 2 Sections 7.5–7.7 Chapter 9	Section 5.4 Section 5.7 Section 6.5 Sections 9.2–9.3	Chapter 3 Sections 4.4–4.6 Sections 5.1–5.2 Section 5.7 Section 6.6

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1

BASIC CONCEPTS

1.1 ARGUMENTS, PREMISES, AND CONCLUSIONS

Logic may be defined as the science that evaluates arguments. All of us encounter arguments in our day-to-day experience. We read them in books and newspapers, hear them on television, and formulate them when communicating with friends and associates. The aim of logic is to develop a system of methods and principles that we may use as criteria for evaluating the arguments of others and as guides in constructing arguments of our own. Among the benefits to be expected from the study of logic is an increase in confidence that we are making sense when we criticize the arguments of others and when we advance arguments of our own.

An **argument**, as it occurs in logic, is a group of statements, one or more of which (the premises) are claimed to provide support for, or reasons to believe, one of the others (the conclusion). All arguments may be placed in one of two basic groups: those in which the premises really do support the conclusion and those in which they do not, even though they are claimed to. The former are said to be good arguments (at least to that extent), the latter bad arguments. The purpose of logic, as the science that evaluates arguments, is thus to develop methods and techniques that allow us to distinguish good arguments from bad.

As is apparent from the above definition, the term “argument” has a very specific meaning in logic. It does not mean, for example, a mere verbal fight,

as one might have with one's parent, spouse, or friend. Let us examine the features of this definition in greater detail. First of all, an argument is a group of statements. A **statement** is a sentence that is either true or false—in other words, typically a declarative sentence or a sentence component that could stand as a declarative sentence. The following sentences are statements:

Aluminum is attacked by hydrochloric acid.
Broccoli is a good source of vitamin A.
Argentina is located in North America.
Napoleon prevailed at Waterloo.
Rembrandt was a painter and Shelley was a poet.

The first two statements are true, the second two false. The last one expresses two statements, both of which are true. Truth and falsity are called the two possible **truth values** of a statement. Thus, the truth value of the first two statements is true, the truth value of the second two is false, and the truth value of the last statement, as well as that of its components, is true.

Unlike statements, many sentences cannot be said to be either true or false. Questions, proposals, suggestions, commands, and exclamations usually cannot, and so are not usually classified as statements. The following sentences are not statements:

What is the atomic weight of carbon?	(question)
Let's go to the park today.	(proposal)
We suggest that you travel by bus.	(suggestion)
Turn to the left at the next corner.	(command)
All right!	(exclamation)

The statements that make up an argument are divided into one or more premises and one and only one conclusion. The **premises** are the statements that set forth the reasons or evidence, and the **conclusion** is the statement that the evidence is claimed to support or imply. In other words, the conclusion is the statement that is claimed to follow from the premises. Here is an example of an argument:

All crimes are violations of the law.
Theft is a crime.
Therefore, theft is a violation of the law.

The first two statements are the premises; the third is the conclusion. (The claim that the premises support or imply the conclusion is indicated by the word "therefore.") In this argument the premises really do support the conclusion, and so the argument is a good one. But consider this argument:

Some crimes are misdemeanors.
Murder is a crime.
Therefore, murder is a misdemeanor.

In this argument the premises do not support the conclusion, even though they are claimed to, and so the argument is not a good one.

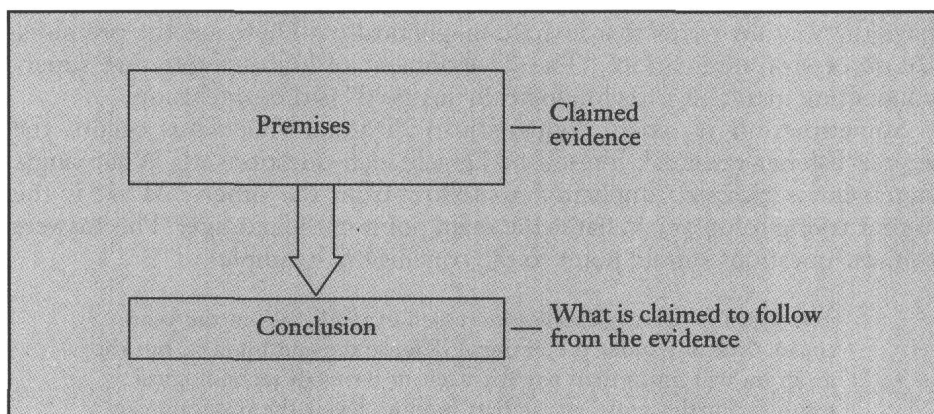
One of the most important tasks in the analysis of arguments is being able to distinguish premises from conclusion. If what is thought to be a conclusion is really a premise, and vice versa, the subsequent analysis cannot possibly be correct. Frequently, arguments contain certain indicator words that provide clues in identifying premises and conclusion. Some typical **conclusion indicators** are

therefore	hence	whence
wherefore	thus	so
accordingly	consequently	it follows that
we may conclude	we may infer	implies that
entails that	it must be that	as a result

Whenever a statement follows one of these indicators, it can usually be identified as the conclusion. By process of elimination the other statements in the argument are the premises. Example:

Corporate raiders leave their target corporation with a heavy debt burden and no increase in productive capacity. Consequently, corporate raiders are bad for the business community.

The conclusion of this argument is “Corporate raiders are bad for the business community,” and the premise is “Corporate raiders leave their target corporation with a heavy debt burden and no increase in productive capacity.”



If an argument does not contain a conclusion indicator, it may contain a premise indicator. Some typical **premise indicators** are

since	in that	seeing that
as indicated by	may be inferred from	for the reason that
because	as	inasmuch as
for	given that	owing to

Any statement following one of these indicators can usually be identified as a premise. Example:

Expectant mothers should never use recreational drugs, since the use of these drugs can jeopardize the development of the fetus.

The premise of this argument is “The use of these drugs can jeopardize the development of the fetus,” and the conclusion is “Expectant mothers should never use recreational drugs.”

One premise indicator not included in the above list is “for this reason.” This indicator is special in that it comes immediately *after* the premise that it indicates. “For this reason” (except when followed by a colon) means for the reason (premise) that was just given. In other words, the premise is the statement that occurs immediately *before* “for this reason.” One should be careful not to confuse “for this reason” with “for the reason that.”

Sometimes a single indicator can be used to identify more than one premise. Consider the following argument:

The development of high-temperature superconducting materials is technologically justified, for such materials will allow electricity to be transmitted without loss over great distances, and they will pave the way for trains that levitate magnetically.

The premise indicator “for” goes with both “Such materials will allow electricity to be transmitted without loss over great distances” and “They will pave the way for trains that levitate magnetically.” These are the premises. By process of elimination, “The development of high-temperature superconducting materials is technologically justified” is the conclusion.

Sometimes an argument contains no indicators. When this occurs, the reader/listener must ask himself or herself such questions as: What single statement is claimed (implicitly) to follow from the others? What is the arguer trying to prove? What is the main point in the passage? The answers to these questions should point to the conclusion. Example:

The space program deserves increased expenditures in the years ahead. Not only does the national defense depend upon it, but the program will more than pay for itself in terms of technological spinoffs. Furthermore, at current funding levels the program cannot fulfill its anticipated potential.

The main point of this argument is that the space program deserves increased expenditures in the years ahead. All the other statements provide support for this statement. This example reflects the pattern of most (but not all) clear-cut arguments that lack indicator words: The conclusion is the first statement. When the argument is restructured according to logical principles, however, the conclusion is always listed *after* the premises:

- P₁: The national defense is dependent upon the space program.
- P₂: The space program will more than pay for itself in terms of technological spinoffs.
- P₃: At current funding levels the space program cannot fulfill its anticipated potential.
- C: The space program deserves increased expenditures in the years ahead.

When restructuring arguments such as this, one should remain as close as possible to the original version, while at the same time attending to the requirement that premises and conclusion be complete sentences that are meaningful in the order in which they are listed.

Note that the first two premises are included within the scope of a single sentence in the original argument. For the purposes of this chapter, compound arrangements of statements in which the various components are all claimed to be true will be considered as separate statements.

Passages that contain arguments sometimes contain statements that are neither premises nor conclusion. Only statements that are actually intended to support the conclusion should be included in the list of premises. If a statement has nothing to do with the conclusion or, for example, simply makes a passing comment, it should not be included within the context of the argument. Example:

Socialized medicine is not recommended because it would result in a reduction in the overall quality of medical care available to the average citizen. In addition, it might very well bankrupt the federal treasury. This is the whole case against socialized medicine in a nutshell.

The conclusion of this argument is "Socialized medicine is not recommended," and the two statements following the word "because" are the premises. The last statement makes only a passing comment about the argument itself and is therefore neither a premise nor a conclusion.

Closely related to the concepts of argument and statement are those of inference and proposition. An **inference**, in the technical sense of the term, is the reasoning process expressed by an argument. As we will see in the next section, inferences may be expressed not only through arguments but through conditional statements as well. In the loose sense of the term, "inference" is used interchangeably with "argument."

Analogously, a **proposition**, in the technical sense, is the meaning or information content of a statement. For the purposes of this book, however, “proposition” and “statement” are used interchangeably.

Note on the History of Logic

The person who is generally credited as being the father of logic is the ancient Greek philosopher Aristotle (384–322 B.C.). Aristotle’s predecessors had been interested in the art of constructing persuasive arguments and in techniques for refuting the arguments of others, but it was Aristotle who first devised systematic criteria for analyzing and evaluating arguments. Aristotle’s logic is called **sylogistic logic** and includes much of what is treated in Chapters 4 and 5 of this text. The fundamental elements in this logic are *terms*, and arguments are evaluated as good or bad depending on how the terms are arranged in the argument. In addition to his development of syllogistic logic, Aristotle cataloged a number of informal fallacies, a topic treated in Chapter 3 of this text.

After Aristotle’s death, another Greek philosopher, Chrysippus (279–206 B.C.), one of the founders of the Stoic school, developed a logic in which the fundamental elements were *whole propositions*. Chrysippus treated every proposition as either true or false and developed rules for determining the truth or falsity of compound propositions from the truth or falsity of their components. In the course of doing so, he laid the foundation for the truth functional interpretation of the logical connectives presented in Chapter 6 of this text and introduced the notion of natural deduction, treated in Chapter 7.

For thirteen hundred years after the death of Chrysippus, relatively little creative work was done in logic. The physician Galen (A.D. 129–C. 199) developed the theory of the compound categorical syllogism, but for the most part philosophers confined themselves to writing commentaries on the works of Aristotle and Chrysippus. Boethius (c. 480–524) is a noteworthy example.

The first major logician of the Middle Ages was Peter Abelard (1079–1142). Abelard reconstructed and refined the logic of Aristotle and Chrysippus as communicated by Boethius, and he originated a theory of universals that traced the universal character of general terms to concepts in the mind rather than to “natures” existing outside the mind, as Aristotle had held. In addition, Abelard distinguished arguments that are valid because of their form from those that are valid because of their content, but he held that only formal validity is the “perfect” or conclusive variety. The present text follows Abelard on this point.

After Abelard, the study of logic during the Middle Ages blossomed and flourished through the work of numerous philosophers. It attained its final

expression in the writings of the Oxford philosopher William of Occam (c. 1285–1349). Occam devoted much of his attention to **modal logic**, a kind of logic that involves such notions as possibility, necessity, belief, and doubt. He also conducted an exhaustive study of forms of valid and invalid syllogisms and contributed to the development of the concept of a metalanguage—that is, a higher-level language used to discuss linguistic entities such as words, terms, propositions, and so on.

Toward the middle of the fifteenth century, a reaction set in against the logic of the Middle Ages. Rhetoric largely displaced logic as the primary focus of attention; the logic of Chrysippus, which had already begun to lose its unique identity in the Middle Ages, was ignored altogether, and the logic of Aristotle was studied only in highly simplistic presentations. A reawakening did not occur until two hundred years later through the work of Gottfried Wilhelm Leibniz (1646–1716).

Leibniz, a genius in numerous fields, attempted to develop a symbolic language or “calculus” that could be used to settle all forms of disputes, whether in theology, philosophy, or international relations. As a result of this work, Leibniz is sometimes credited with being the father of symbolic logic. Leibniz’s efforts to symbolize logic were carried into the nineteenth century by Bernard Bolzano (1781–1848).

With the arrival of the middle of the nineteenth century, logic commenced an extremely rapid period of development that has continued to this day. Work in symbolic logic was done by a number of philosophers and mathematicians, including Augustus DeMorgan (1806–1871), George Boole (1815–1864), William Stanley Jevons (1835–1882), and John Venn (1834–1923), some of whom are popularly known today by the logical theorems and techniques that bear their names. At the same time, a revival in inductive logic was initiated by the British philosopher John Stuart Mill (1806–1873), whose methods of induction are presented in Chapter 9 of this text.

Toward the end of the nineteenth century, the foundations of modern mathematical logic were laid by Gottlob Frege (1848–1925). His *Begriffsschrift* sets forth the theory of quantification presented in Chapter 8 of this text. Frege’s work was continued into the twentieth century by Alfred North Whitehead (1861–1947) and Bertrand Russell (1872–1970), whose monumental *Principia Mathematica* attempted to reduce the whole of pure mathematics to logic. The *Principia* is the source of much of the symbolism that appears in Chapters 6, 7, and 8 of this text.

During the twentieth century, much of the work in logic has focused on the formalization of logical systems and on questions dealing with the completeness and consistency of such systems. A now-famous theorem proved by Kurt Gödel (1906–1978) states that in any formal system adequate for number theory there exists an undecidable formula—that is, a formula such that neither it nor its negation is derivable from the axioms of the system. Other developments include multivalued logics and the formalization of