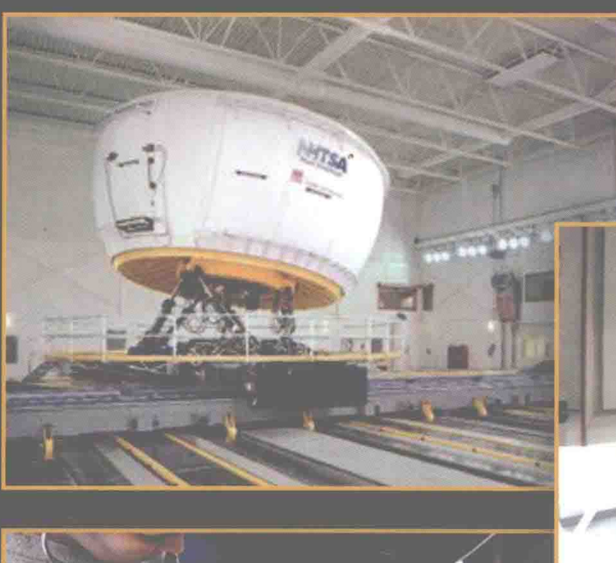
Simulation of Dynamic Systems

with MATLAB® and Simulink®



SECOND EDITION

Harold Klee
Randal Allen

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To Andrew, Cassie and in loving memory
of their mother and devoted wife, Laura.

Harold Klee

To Dave Lundquist and Steve Roemerman who believed in me.

Randal Allen

Foreword

As the authors point out in the preface, there is not yet extant a universally accepted definition of the term *simulation*. Another approach to defining the field would be "the art of reproducing the behavior of a system for analysis without actually operating that system." The authors have written a seminal text covering the simulation design and analysis of a broad variety of systems using two of the most modern software packages available today. The material is presented in a particularly adept fashion enabling students new to the field to gain a thorough understanding of the basics of continuous simulation in a single semester and providing, at the same time, a more advanced treatment of the subject for researchers and simulation professionals. The authors' extensive treatment of continuous and discrete linear system fundamentals opens the door to simulation for individuals without formal education in a traditional engineering curriculum.

However defined, simulation is becoming an increasingly important component of curricula in engineering, business administration, the sciences, applied mathematics, and the like. This text will be a valuable resource for study in courses using simulation as a tool for understanding processes that are not amenable to study in other ways.

Chris Bauer, PhD, PE, CMSP
Orlando, Florida

Simulation has come a long way since the days analog computers filled entire rooms. Yet, it is more important than ever that simulations be constructed with care, knowledge, and a little wisdom, lest the results be gibberish or, worse, reasonable but misleading. Used properly, simulations can give us extraordinary insights into the processes and states of a physical system. Constructed with care, simulations can save time and money in today's competitive marketplace.

One major application of simulation is the simulator, which provides interaction between a model and a person through some interface. The earliest simulator, Ed Link's Pilot Maker aircraft trainer, did not use any of the simulation techniques described in this book. Modern simulators, however, such as the National Advanced Driving Simulator (NADS), cannot be fully understood without them.

The mission of the NADS is a lofty one: to save lives on U.S. highways through safety research using realistic human-in-the-loop simulation. This is an example of the importance simulation has attained in our generation. The pervasiveness of simulation tools in our society will only increase over time; it will be more important than ever that future scientists and engineers be familiar with their theory and application.

The content for *Simulation of Dynamic Systems with MATLAB® and Simulink®* is arranged to give the student a gradual and natural progression through the important topics in simulation. Advanced concepts are added only after complete examples have been constructed using fundamental methods. The use of MATLAB and Simulink provides experience with tools that are widely adopted in industry and allow easy construction of simulation models.

May your experience with simulation be enjoyable and fruitful and extend throughout your careers.

Chris Schwarz, PhD
Iowa City, Iowa

Preface

In the first article of *SIMULATION* magazine in the Fall of 1963, the editor John McLeod proclaimed simulation to mean "the act of representing some aspects of the real world by numbers or symbols which may be easily manipulated to facilitate their study." Two years later, it was modified to "the development and use of models for the study of the dynamics of existing or hypothesized systems." More than 40 years later, the simulation community has yet to converge upon a universally accepted definition. Either of the two cited definitions or others that followed convey a basic notion, namely, that simulation is intended to reinforce or supplement one's understanding of a system. The definitions vary in their description of tools and methods to accomplish this.

The field of simulation is experiencing explosive growth in importance because of its ability to improve the way systems and people perform, in a safe and controllable environment, at a reduced cost. Understanding the behavior of complex systems with the latest technological innovations in fields such as transportation, communication, medicine, aerospace, meteorology, etc., is a daunting task. It requires an assimilation of the underlying natural laws and scientific principles that govern the individual subsystems and components. A multifaceted approach is required, one in which simulation can play a prominent role, both in validation of a system's design and in training of personnel to become proficient in its operation.

Simulation is a subject that cuts across traditional academic disciplines. Airplane crews spend hours flying simulated missions in aircraft simulators to become proficient in the use of onboard subsystems during normal flight and possible emergency conditions. Astronauts spend years training in shuttle and orbiter simulators to prepare for future missions in space. Power plant and petrochemical process operators are exposed to simulation to obtain peak system performance. Economists resort to simulation models to predict economic conditions of municipalities and countries for policymakers. Simulations of natural disasters aid in preparation and planning to mitigate the possibility of catastrophic events.

While the mathematical models created by aircraft designers, nuclear engineers, and economists are application specific, many of the equations are analogous in form despite the markedly different phenomena described by each model. Simulation offers practitioners from each of these fields the tools to explore solutions of the models as an alternative to experimenting with the real system.

This book is meant to serve as an introduction to the fundamental concepts of continuous system simulation, a branch of simulation applied to dynamic systems whose signals change over a continuum of points in time or space. Our concern is with mathematical models of continuous-time systems (electric circuits, thermal processes, population dynamics, vehicle suspension, human physiology, etc.) and the discrete-time system models created to simulate them. The continuous system mathematical models consist of a combination of algebraic and ordinary differential equations. The discrete-time system models are a mix of algebraic and difference equations.

Systems that transition between states at randomly occurring times are called discrete-event systems. Discrete-event simulation is a complementary branch of simulation, separate from continuous system simulation, with a mathematical foundation rooted in probability theory. Examples of discrete-event systems are facilities such as a bank, a tollbooth, a supermarket, or a hospital emergency room, where customers arrive and are then serviced in some way. A manufacturing plant involving multiple production stages of uncertain duration to generate a finished product is another candidate for discrete-event simulation.

Discrete-event simulation is an important tool for optimizing the performance of systems that change internally at unpredictable times due to the influence of random events. Industrial engineering programs typically include a basic course at the undergraduate level in discrete-event simulation.

Not surprisingly, a number of excellent textbooks in the area have emerged for use by the academic community and professionals.

In academia, continuous simulation has evolved differently than discrete-event simulation. Topics in continuous simulation such as dynamic system response, mathematical modeling, differential equations, difference equations, and numerical integration are dispersed over several courses from engineering, mathematics, and the natural sciences. In the past, the majority of courses in modeling and simulation of continuous systems were restricted to a specific field like mechanical, electrical, and chemical engineering or scientific areas like biology, ecology, and physics.

A transformation in simulation education is underway. More universities are beginning to offer undergraduate and beginning graduate courses in the area of continuous system simulation designed for an interdisciplinary audience. Several institutions now offer master's and PhD programs in simulation that include a number of courses in both continuous and discrete-event simulation. A critical mass of students are now enrolled in continuous simulation–related courses and there is a need for an introductory unifying text.

The essential ingredient needed to make simulation both interesting and challenging is the inclusion of real-world examples. Without models of real-world systems, a first class in simulation is little more than a sterile exposition of numerical integration applied to differential equations.

Modeling and simulation are inextricably related. While the thrust of this text is continuous simulation, mathematical models are the starting point in the evolution of simulation models. Analytical solutions of differential equation models are presented, when appropriate, as an alternative to simulation and a simple way of demonstrating the accuracy of a simulated solution. For the most part, derivations of the mathematical models are omitted and references to appropriate texts are included for those interested in learning more about the origin of the model's equations.

Simulation is best learned by doing. Accordingly, the material is presented in a way that permits the reader to begin exploring simulation, starting with a mathematical model in Chapter 1. A detailed derivation of the mathematical model of a tank with liquid flowing in and out leads to a simulation model in the form of a simple difference equation. The simulation model serves as the vehicle for predicting the tank's response to various inputs and initial conditions. Additionally, the derivation illustrates the process of obtaining a mathematical model based on the natural laws of science.

Chapters 2 and 4 present a condensed treatment of linear, continuous-time, and discrete-time dynamic systems, normally covered in an introductory linear systems course. Coverage is limited to basic topics that should be familiar to a simulation practitioner. Section 2.7 is extended to include a discussion of additional common nonlinear elements, namely, dead zone, quantization, relay, and saturation. The instructor can skip some or all of the material in these chapters if the students' background includes a course in signals and systems or linear control theory.

Numerical integration is at the very core of continuous system simulation. Instead of treating the subject in one exhaustive chapter, coverage is distributed over three chapters. Elementary numerical integration in Chapter 3 is an informal introduction to the subject, which includes discussion of several elementary methods for approximating the solutions of first-order differential equations. The material in Chapters 2 through 4 is a prerequisite for understanding general purpose, continuous simulation programs that are popular in the engineering and scientific community.

Simulink®, from The MathWorks, is the featured simulation program because of its tight integration with MATLAB®, the de facto standard for scientific and engineering analysis, and data visualization software. Chapter 5 takes the reader through the basic steps of creating and running Simulink models. Section 5.5 includes new material related to simulation implementation of nonlinear systems using specific blocks from the Simulink library. Due to the popularity of the Kalman filter, a case study has been added in Section 5.12 on this topic. The continuous-time Kalman filter equations are developed and modeled in Simulink, including simulated output. Subsequently, the steady-state continuous-time Kalman filter equations are developed and modeled in Simulink. The steady-state results are compared with the continuous-time results. Finally, the discrete-time Kalman filter equations are developed and modeled in Simulink. The discrete-time results are compared with the continuous-time results.

Chapter 6 delves into intermediate-level topics of numerical integration, including a formal presentation of One-Step (Runge–Kutta) and multistep methods, adaptive techniques, truncation errors, and a brief mention of stability.

Chapter 7 highlights some advanced features of Simulink useful in more in-depth simulation studies. A new section (Section 7.5) on S-blocks is introduced and an example is presented showing how to make the discrete-time Kalman filter available for drag-and-drop from the Simulink library. Other simulation programs offer similar features and the transition from Simulink to other simulation software is straightforward.

Chapter 8 is for those interested in more advanced topics on continuous simulation. Coverage includes a discussion of dynamic errors, stability, real-time compatible numerical integration, and multi-rate integration algorithms for simulation of systems with fast and slow components. Due to the popularity of Lego's Mindstorms™ NXT, a case study has been added in Section 8.7 on this topic.

All but two chapters conclude with a case study illustrating one or more of the topics discussed in that chapter. The featured text examples and case studies are analyzed using MATLAB script files and Simulink model files, all of which are available from CRC Press.

The text has been field-tested in the classroom for several years in a two-semester sequence of continuous simulation courses. Despite numerous revisions based on the scrutiny and suggestions of students and colleagues, it is naïve to think the final product is free of errors. Further suggestions for improvement and revelations of inaccuracies can be brought to the attention of the authors at rallen397@cfl.rr.com and klee@mail.ucf.edu.

Numerous individuals deserve our thanks and appreciation for helping to make this book possible. In particular, a sincere "thank you" to Nora Konopka at Taylor & Francis/CRC Press for committing to the second edition and seeing it through to fruition.

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Dr. Harold Klee received his PhD in systems science from Polytechnic Institute of Brooklyn in 1972, his MS in systems engineering from Case Institute of Technology in 1968, and his BSME from The Cooper Union in 1965.

Dr. Klee has been a faculty member in the College of Engineering at the University of Central Florida (UCF) since 1972. During his tenure at UCF, he has been a five-time recipient of the college's Outstanding Teacher Award. He has been instrumental in the development of simulation courses in both the undergraduate and graduate curricula. He is a charter member of the Core Faculty, which is responsible for developing the interdisciplinary MS and PhD programs in simulation at UCF. Dr. Klee served as graduate coordinator in the Department of Computer Engineering from 2003 to 2006. Two of his PhD students received the prestigious Link Foundation Fellowship in Advanced Simulation and Training. Both are currently enjoying successful careers in academia.

Dr. Klee has served as the director of the UCF Driving Simulation Lab for more than 15 years. Under the auspices of the UCF Center for Advanced Transportation Systems Simulation, the lab operates a high-fidelity motion-based driving simulator for conducting traffic engineering–related research. He also served as editor-in-chief for the *Modeling and Simulation* magazine for three years, a publication for members of the Society for Modeling and Simulation International.

Dr. Randal Allen is an aerospace and defense consultant working under contract to provide 6DOF aerodynamic simulation modeling, analysis, and design of navigation, guidance, and control systems. His previous experience includes launch systems integration and flight operations for West Coast Titan-IV missions, propulsion modeling for the Iridium satellite constellation, and field applications engineering for MATRIXx. He also chairs the Central Florida Section of the American Institute of Aeronautics and Astronautics (AIAA).

Dr. Allen is certified as a modeling and simulation professional (CMSP) by the Modeling and Simulation Professional Certification Commission (M&SPCC) under the auspices of the National Training and Simulation Association (NTSA). He is also certified to deliver FranklinCovey's Focus and Execution track, which provides training on achieving your highest priorities.

Dr. Allen's academic background includes a PhD in mechanical engineering from the University of Central Florida, an engineer's degree in aeronautical and astronautical engineering from Stanford University, an MS in applied mathematics, and a BS in engineering physics from the University of Illinois (Urbana-Champaign). He also serves as an adjunct professor at the University of Central Florida in Orlando, Florida.

Contents

1 Mathematical Modeling

1.1 INTRODUCTION

1.1.1 Importance of Models

Models are an essential component of simulation. Before a new prototype design for an automobile braking system or a multimillion dollar aircraft is tested in the field, it is commonplace to "test drive" the separate components and the overall system in a simulated environment based on some form of model. A meteorologist predicts the expected path of a tropical storm using weather models that incorporate the relevant climatic variables and their effect on the storm's trajectory. An economist issues a quantitative forecast of the U.S. economy predicated based on key economic variables and their interrelationships with the help of computer models. Before a nuclear power plant operator is "turned loose" at the controls, extensive training is conducted in a model-based simulator where the individual becomes familiar with the plant's dynamics under routine and emergency conditions. Health care professionals have access to a human patient simulator to receive training in the recognition and diagnosis of disease. Public safety organizations can plan for emergency evacuations of civilians from low-lying areas using traffic models to simulate vehicle movements along major access roads.

The word "model" is a generic term referring to a conceptual or physical entity that resembles, mimics, describes, predicts, or conveys information about the behavior of some process or system. The benefit of having a model is to be able to explore the intrinsic behavior of a system in an economical and safe manner. The physical system being modeled may be inaccessible or even nonexistent as in the case of a new design for an aircraft or automotive component.

Physical models are often scaled-down versions of a larger system of interconnected components as in the case of a model airplane. Aerodynamic properties of airframe and car body designs for high-performance airplanes and automobiles are evaluated using physical models in wind tunnels. In the past, model boards with roads, terrain, miniaturized models of buildings, and landscape, along with tiny cameras secured to the frame of ground vehicles or aircraft, were prevalent for simulator visualization. Current technology relies almost exclusively on computer-generated imagery.

In principle, the behavior of dynamic systems can be explained by mathematical equations and formulae, which embody either scientific principles or empirical observations, or both, related to the system. When the system parameters and variables change continuously over time or space, the models consist of coupled algebraic and differential equations. In some cases, lookup tables containing empirical data are employed to compute the parameters. Equations may be supplemented by mathematical inequalities, which constrain the variation of one or more dependent variables. The aggregation of equations and numerical data employed to describe the dynamic behavior of a system in quantitative terms is collectively referred to as a mathematical model of the system.

Partial differential equation models appear when a dependent variable is a function of two or more independent variables. For example, electrical parameters such as resistance and capacitance are distributed along the length of conductors carrying electrical signals (currents and voltages). These signals are attenuated over long distances of cabling. The voltage at some location x measured from an arbitrary reference is written $v(x, t)$ instead of simply $v(t)$, and the circuit is modeled accordingly.