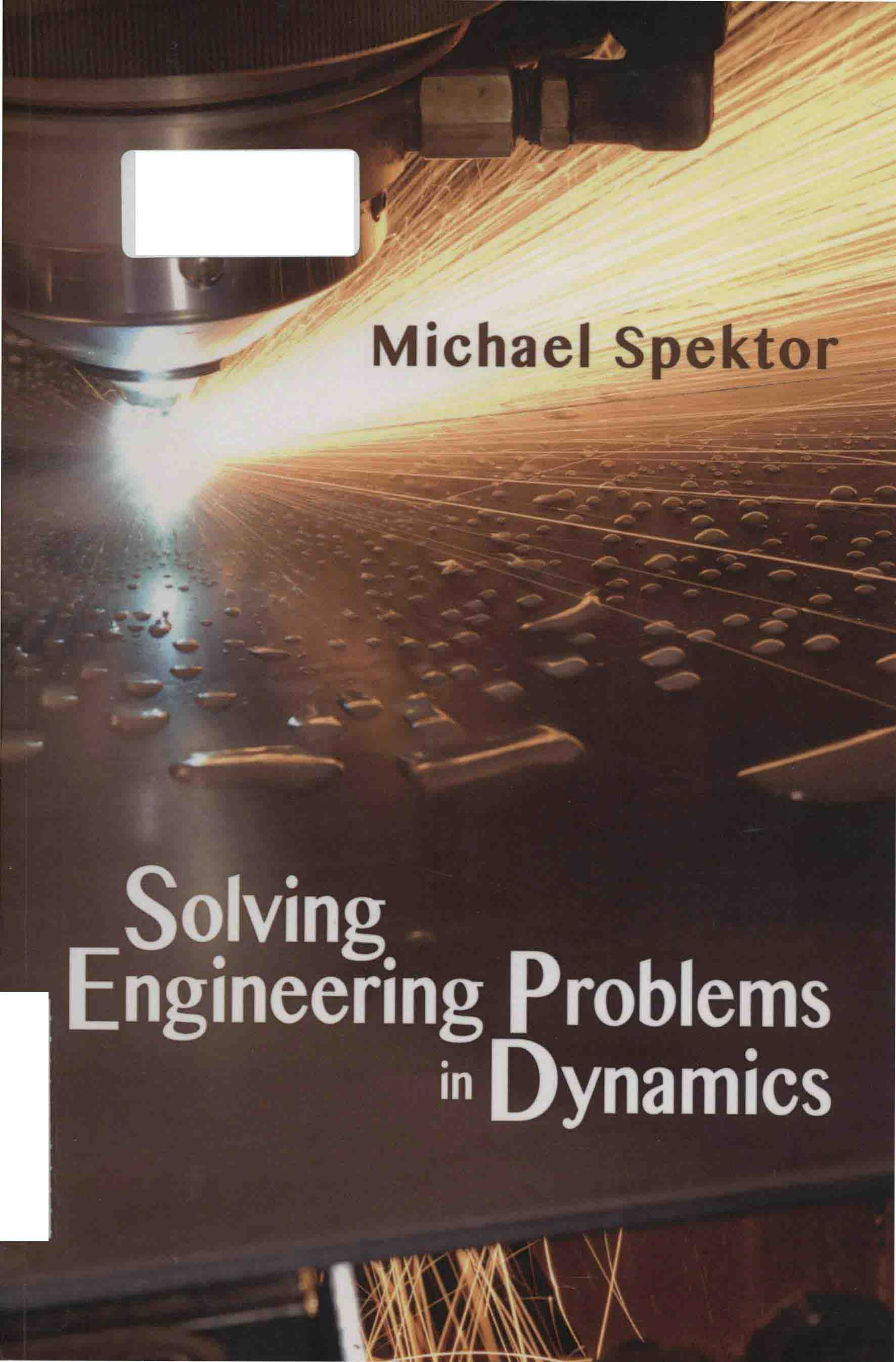

Michael Spektor

Solving Engineering Problems in Dynamics

SOLVING ENGINEERING PROBLEMS IN DYNAMICS

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To My Family

INTRODUCTION

Purposeful control and improvement of how existing mechanical systems perform is an important real-life problem, as is the development of new systems. We can obtain solutions to these problems by investigating the working processes of machines and their units and elements. These investigations should be based on fundamentals of dynamics combined with a variety of related sciences. The working processes that characterize system performance can be described by mathematical expressions that actually represent equations of motion of these systems. Analyzing these equations of motion reveals the relationship between the parameters of the system and their influence on performance and other system characteristics or elements.

This book contains comprehensive methods for analyzing the motion of engineering systems and their components. The analysis covers three basic phases: 1) composing the differential equation of motion, 2) solving the differential equation of motion, and 3) analyzing the solution. Engineering education provides the fundamental skills for completing these three phases. However, many engineers would benefit from additional training in using these fundamentals to solve real-life engineering problems. This book provides this training by describing in a step-by-step order the methods related to each of these three phases.

When assembling a differential equation of motion, it is essential to completely understand the components of this equation as well as the system's working process. This book describes all possible components of the differential equation of motion and all possible factors of the working process. In mechanical engineering, all these components and factors represent forces and moments. The characteristics of all these loading factors and their application to particular differential equations of motion are presented in this book.

This book also introduces a straightforward universal methodology for solving differential equations of motion by using the Laplace Transform. This approach replaces calculus with conventional algebraic procedures that do not represent any difficulties for engineers. Using the Laplace methodology to solve differential equations of motion does not require memorizing the fundamentals of the Laplace Transform. Instead, this book presents an appropriate table of Laplace Transform pairs. It then explains how to use the pairs to convert differential equations into algebraic equations and then how to invert the solutions of these algebraic equations into conventional equations representing the functions of displacement of time.

Analyzing the solutions of differential equations of motion reveals the role of the system's parameters, the influence of these parameters on each other, and how to control the performance of the system.

The motion of a mechanical system is characterized by its displacement, velocity, and acceleration. These three characteristics are the three basic parameters of the system's motion. All other characteristics of the working processes can be determined by analyzing these three parameters. The equation of motion represents the displacement of the system as a function of time. The other two parameters — velocity and acceleration — are respectively the first and second derivatives from the displacement. Thus, the equation of motion is the basis for solving the mechanical engineering problem.

The equations of motion represent the solutions of differential equations of motion that reflect the real working processes of the systems. When we assemble these differential equations of motion, we use methodologies that are built on a close interaction between theoretical and applied sciences. Rapidly advancing technology stimulates intensive searches for more sophisticated engineering solutions. Therefore, we must be familiar with the methodologies for solving actual mechanical engineering problems.

This text can help you achieve the level of competence you need to successfully analyze real mechanical systems. An engineering educational background is sufficient to comprehend the contents of this text. We develop a comprehensive, step-by-step guide to solving mechanical engineering problems. Numerous examples demonstrate the methodologies that enable us to control the parameters of real systems. A wide range of readers can benefit from this book. Accounting for the different levels of their backgrounds, the step-by-step approach begins with the simplest examples and then gradually increases the complexity of the problems.

The text consists of six chapters. Let us consider briefly the contents of each.

1. Differential Equations of Motion

Our analysis of problems associated with dynamics is based on the laws of motion. These laws (or equations) of motion are the subject of Chapter 1. They represent displacement (the dependent variable, the function) as a function of running time (the independent variable, the argument). In general, motion has three phases: acceleration, uniform motion, and deceleration.

Displacement in uniform motion is a product of multiplying a constant velocity by the running time. This formula is known from basic physics; it is applicable to any uniformly moving object. Analysis of this formula, however, adds very limited help in understanding the working process and performance of an actual mechanical system.

Solutions that lead to performance control can be obtained from the expressions that describe acceleration and deceleration — equations representing the displacement, velocity, and acceleration as functions of time. For the plurality of real problems, there are no readily available formulas for these three parameters. Instead, mathematical expressions of these three parameters can often be obtained from solutions of corresponding differential equations of motion.

For each case, we should assemble an appropriate differential equation of motion that reflects the physical nature of the problem. As the book will show, composing differential equations of motion is not a trivial procedure.

A differential equation of motion is a second order differential equation made up of the second and first derivatives, the function, the argument, and, the constant terms. The structure of a second order differential equation is based on principles of mathematics without any dependence on laws of motion. The same approach is applicable to all mathematical rules used for practical calculations in different fields. The characteristics of motion include the second derivative (acceleration), the first derivative (velocity), the function (displacement), the argument (running time), and the constant terms. A natural linkage exists between the second order differential equation and the parameters of motion — the process of motion is described by a second order differential equation. (The second order differential equation is also applicable to electrical circuits and other physical phenomena; this text can be used for electrical engineering as well.)

In mathematics, the components of differential equations are dimensionless. In dynamics, each component of a differential equation should have the same physical units. Differential equations of motion are made up of loading factors that represent forces or moments whereas differential equations of electrical circuits include components that represent voltage.

The three basic parameters of motion are not loading factors — they have different units. These parameters cannot be directly included in a differential equation of motion. Each parameter should be multiplied by appropriate coefficients in such a way that the products have the units of loading factors, which cause the motion of objects.

Both the structure and the solution of the differential equations of motion are absolutely identical for rectilinear and rotational motions; their parameters are completely similar. Thus, the examples are presented just for rectilinear motion. Keep in mind that, if necessary, forces should be replaced by moments while the masses should be replaced by moments of inertia; the rectilinear parameters of motion should be replaced by the corresponding angular parameters. All this will not change the structure of the differential equation of motion and its solution. All considerations regarding forces are completely applicable to moments.

Particular attention is paid in Chapter 1 to explaining the structure of differential equations of motion and assembling them.

2. Analysis of Forces

The structure of the differential equation of motion is absolutely similar for rectilinear and rotational motion. So too is the process of composing the equation. To avoid redundant explanations, our analysis of loading factors focuses just on forces. However, the same characteristics and considerations are completely applicable to moments.

The left side of the differential equation of motion consists of resisting forces, whereas the right side consists of active forces. The resisting forces are variables (inertia, damping, and stiffness) and constants (e.g., dry friction, gravity, and plastic deformation). The force of inertia is present in all differential equations of motion. The resisting forces should be identified depending on the functionality and on the structure of the mechanical system as well as on the nature of the environment in which the motion occurs.

As variables, the force of inertia depends on acceleration, the damping force on velocity, and the stiffness force on displacement. These resisting forces can be linear or non-linear and their characteristics are determined by their coefficients. The coefficient of the inertia force is the mass, which is usually a constant value; consequently, the inertia force is linear. Non-linear inertia forces are not considered in this book. The damping and stiffness coefficients can also be constant or variable. If constant, the differential equation of motion is linear. If even one of these coefficients is a variable, the differential equation of motion is non-linear.

In certain mechanical systems, resisting forces could appear that represent some functions of time. However the majority of conventional mechanical systems do not have any obvious factors pointing to the existence of time-depending resisting forces which, therefore, are not discussed in this book.

In the majority of cases, the characteristics of active forces are predetermined. For conventional mechanical systems, these active forces include: constant forces, sinusoidal forces exerted by vibrators, and forces depending on time, velocity, or displacement. These last three can be linear or non-linear.

Chapter 2 looks closely at the characteristics and peculiarities of the resisting and active forces.

3. Solving Differential Equations of Motion Using Laplace Transforms

In solving the differential equations of motion, our goal is to obtain an expression for displacement as a function of time. This expression is also called the law of motion. Finding the best method for solving various linear differential equations can be challenging. However, the Laplace Transform represents a straightforward universal method for solving all linear differential equations.

The Laplace Transform lets us convert differential equations into algebraic equations whose solutions can be achieved by conventional algebraic procedures. We can apply the Laplace Transform without addressing the mathematical principles on which it is built. It provides a straightforward methodology regardless of the characteristics of the equation's components or its initial conditions.

Chapter 3 reviews the steps of this methodology; they are identical for each differential equation. First, we convert the differential equation of motion from the time domain form into the Laplace domain form, working with a table of Laplace Transform conversion pairs compiled for this text. The second step of the methodology deals with the Laplace domain solution of the differential equation of motion. This step, based on ordinary algebraic procedures, results in an algebraic equation that represents the dependant variable (e.g., displacement) as a function of the independent variable (e.g., running time). Both variables are in the same Laplace domain. The Laplace Transform eliminates the need of calculus to solve the differential equation of motion. Therefore, we obtain an algebraic equation with the dependent variable in the left side of the equation, and a sum of algebraic expressions (proper fractions) on the right.

In the last step, we invert all the terms of the solution from the Laplace domain into the time domain form. This inversion represents the solution of the differential equation of motion. All three steps of this methodology are demonstrated in the text by solving numerous examples.

In some cases, there will be terms in the right side of the Laplace domain solution that do not have representations in this text's table, or even in other, more comprehensive tables. For these cases, Chapter 3 discusses a method of decomposition used to resolving these expressions.

The examples in this chapter begin with a solution of a very simple differential equation. The complexity of the solutions gradually increases; ultimately, the examples include a range of diversified differential equations of motion of actual mechanical systems.

4. Analysis of Typical Mechanical Engineering Systems

Assembling the different equation of motion is a very important step when investigating the dynamics of a mechanical system. The differential equation should reflect the peculiarities of the real working process. This chapter discusses the considerations that are relevant to the process of assembling differential equations of motion. These considerations are associated with real-life problems of typical mechanical systems. We start with composing the appropriate differential equation of motion. The following step focuses on this equation's solution. In the last step, our analysis of the solution reveals the system's performance characteristics: energy consumption, required power, acting forces, and others. The complexity of the examples increases from example to example, and can be very helpful in solving actual problems.

5. Piece-Wise Linear Approximation

Chapters 3 and 4 are devoted to solving linear differential equations of motion. In reality, many loading factors that are included in these equations are actually non-linear. However, the non-linearity of these factors is often not essential; it is then justifiable to consider them as being linear. There are no currently established methodologies for solving non-linear differential equations in general terms.

Many specific non-linear differential equations can be solved using particular mathematical investigations, and there are catalogs where these solutions can be found. However, these solutions have a very limited applicability to non-linear differential equations of motion.

In a significant number of real life problems, the non-linearity of the loading factors cannot be ignored. Neglecting the strong non-linearity of these factors results in essential quantitative errors; yet some important qualitative characteristics of the process could be misunderstood or not revealed at all.

The method of piece-wise linear approximation allows us — with an appropriate accuracy — to investigate problems that include non-linear loading factors. The characteristics of these factors can be represented by corresponding graphs whose curvatures reflect the extent of the factors' non-linearity.

Piece-wise linear approximation consists of replacing the curve by a broken line. For instance, if the curve is replaced by a broken line including three straight segments, the process of motion can be divided into three intervals. For each interval, a linear differential equation will be composed with the initial conditions of motion equal to the conditions of motion at the end of the previous interval. The shorter the length of the segments, the more accurate the results of the solution will be. A reasonable compromise will decide the number and values of the replacement increments that would satisfy the goal of the investigation. The application of the piece-wise linear approximation to the solutions of real-life problems comprising non-linear loading factors is presented in a detailed way in this chapter.

6. Dynamics of Two-Degree-of-Freedom Systems

Numerous mechanical engineering systems are made up by several separate masses connected among themselves by specific links. These links allow for motion of these masses relative to each other. Each motion is described by its mass's differential equation. The amount of these masses defines the number of degrees-of-freedom of the system. Of the actual multiple-degree-of-freedom mechanical systems, the majority have just two masses — therefore, this text is limited to considering two-degree-of-freedom structures. Two types of links allow relative motion of the connected masses: the elastic link (spring) and the hydraulic link (dashpot). The masses could be connected by a hydraulic or elastic link, or by both links acting in parallel. A simultaneous system of two differential equations of motion should be assembled in order to describe the motion of the two masses.

Chapter 6 contains a detailed discussion of the structures of the differential equations of motion and also of the considerations for composing these equations. It also includes typical examples that demonstrate the methods for investigating two-degree-of-freedom systems.

A General Note

These chapter descriptions indicate that the analysis of an actual mechanical system is a complex process engaging an interaction among several sciences.

During the first steps of the analysis, we should pay particular attention to the characteristics of the damping and stiffness resisting forces. In the majority of practical cases, these forces could be linear or non-linear whereas the rest of the forces are usually linear. Information regarding the characteristics of the actual damping and stiffness forces for a specific case should be based on the results of the investigations; these results are usually presented in graphs or can be found in corresponding sources.

Normally, our analysis of the solutions of the differential equation of motion provides the information needed to make appropriate engineering decisions. This text includes all the steps necessary for a complete analysis of actual problems in mechanical engineering dynamics.

Numerous software programs are available for computing the parameters of motion of mechanical engineering systems. These programs can be used when the differential equations of motion are already available. When investigating real life problems, the first steps are associated with composing the differential equations of motion. This text is intended to help you assemble these equations. In many practical situations, you may need to analyze the working process of a mechanical engineering system in order to estimate the influence of the parameters on each other and to reveal their specific roles. For these cases, we present the analysis in general terms without any use of related numerical data. This book will also be useful for performing this kind of analyses.

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