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THE KINETICS OF SIMPLE MODELS IN THE THEORY OF OSCILLATIONS

Edited by N. G. Basov

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The Kinetics of Simple Models in the Theory of Oscillations

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THE KINETICS OF SIMPLE MODELS IN THE THEORY OF OSCILLATIONS

KINETIKA PROSTYKH MODELEI TEORII KOLEBANII

КИНЕТИКА ПРОСТЫХ МОДЕЛЕЙ ТЕОРИИ КОЛЕБАНИЙ

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PREFACE

This volume deals with models in the theory of oscillations and the prospects for using the associated analytic techniques in a wide variety of scientific problems. It is primarily intended to be pedagogical. Thus, in discussing specific problems which illustrate and explain the general concepts of the theory of oscillations, we shall consider the significance of both the simple models of phenomena introduced at the beginning and the evolution of these models during the course of research. We do not attempt to hide, but, rather, emphasize the incompleteness of the results obtained here and dwell in each case on the difficulties which arise along the way, noting the short-term goals of the work while not overlooking the prospects for the research as a whole.

Both the pedagogical aspects of these problems and the specific results derived here are, in our opinion, of definite value and may hence already be of interest to specialists in the appropriate areas of science. Each chapter has been written in such a way that it may be read separately from the others. Every one of the nine chapters is therefore a small, self-contained review of the work done by its authors over the past few years. We have not tried to link the chapters with subjective and logical "bridges" as we felt such an effort at this stage would be of little use and have thus limited ourselves solely to the "Introduction" written by L. I. Gudzenko. Chapter I was written by L. I. Gudzenko together with V. V. Evstigneev and S. I. Yakovlenko; Chapter II, by L. I. Gudzenko, Yu. I. Syts'ko, and S. I. Yakovlenko; Chapter III, by L. I. Gudzenko with I. S. Lakoba and S. I. Yakovlenko; Chapter IV, by L. I. Gudzenko and V. S. Marchenko; Chapter V, by L. I. Gudzenko and S. I. Yakovlenko; Chapters VI, VIII, and IX, by L. I. Gudzenko and V. E. Chertoprud; and Chapter VII, by L. I. Gudzenko and A. E. Sorkina.

The book consists of two parts. The Introduction discusses the place of the theory of oscillations in modern science and describes the contents of this volume. A principle for isolating dynamic couplings is formulated and then applied in developing a correlation method for analyzing the inverse problems of the theory.

In Part I several related problems in applied physics are used as an example for discussing the simplest variant of the traditional (direct) problems of the theory of oscillations. Then the motion of a dynamic system near a stable stationary point is examined.

The purpose of this part is to seek active media for powerful coherent light sources (plasma lasers at various wavelengths) and to analyze the prospects for building power reactor-lasers. The first chapter discusses the conditions which lead to an inversion in the atomic level populations in a dense supercooled (relative to the free electrons) plasma. In Chapter II a theoretical analysis is made of several methods for producing a supercooled plasma. Chapter III deals with the recombination relaxation of plasmas made up of inert gases and their combinations with other chemical elements. The requirements on the chemical composition and parameters of a plasma which have to be satisfied in order to produce an active medium that will amplify the resonant emission of atoms are discussed in Chapter IV. Finally, in Chapter V the theoretical possibility of obtaining a substantial portion of the energy from power plants (such as nuclear reactors) in the form of laser radiation is discussed.

Part II is more unusual from a methodological standpoint and deals with some rather diverse specific problems. Some chapters are included there which illustrate the usefulness of the correlation technique for solving inverse problems in the theory of oscillations. In this sense, Chapter IX, which does not use this method directly, is an exception. It is essentially a supplement to Chapter VIII (on the cyclic activity of the sun) which arose during the analysis but has an independent significance. Chapter VI deals with a scheme for measuring the probabilities of nonradiative atomic (molecular, ionic) transitions in a low-temperature plasma using the correlation functions of readouts. The elements of a correlation method for medical diagnostics using the pulsations of large blood vessels are discussed in Chapter VII.

CONTENTS

INTRODUCTION

Statement of the Problem

The level of development of every natural science is determined by the depth to which the subject has been penetrated by measurement techniques and by the detail with which the relationships among the resulting quantitative expressions have been studied. This is just what is meant when it is said, for example, that genetics, psychology, or linguistics have become competent exact sciences. The center of gravity of these sciences has been moving ever more noticeably from general considerations, allowing a substantial nonuniqueness in the conclusions drawn, toward the discovery of controlling quantitative relationships. Of course, increasing the accuracy of the agreement between theory and experiment requires improvements not only in the measurement techniques used but, especially, in the theory. The aim of a theory is taken to be the analysis of relationships among the observed quantities, starting ultimately with some fundamental irrefutable statements, i.e., laws of nature. In physics (mathematics must be given special consideration from this standpoint), the most developed natural science, it is customary to first examine the simplest models, idealized schemes each of which only involves one or two of these laws. In real experiments an increase in accuracy is always associated with a departure from the initial idealization; however, in physics this has usually led to a gradual improvement in the approximation as the model of the phenomena becomes more complex, rather than to a rejection of the previous description.

The situation changes when from the very beginning one has to take into account many diverse factors which contribute equally to the phenomenon being investigated. The difficulties then arise even at the stage of choosing the simplest model and, in any case, when attempting to solve the resulting equations. This is especially typical of such sciences as astrophysics, biology, or sociology; however, now even in physics and chemistry it more often becomes necessary to deal with processes so far removed from simple systems that a theoretical analysis is possible only as progress is made in high-speed computing techniques and the appropriate computational methods are developed. Digital computers acquire an ever-greater role in the processing of the immense numerical bulk that results. A new research psychology arises which leads, in particular, to computer testing of the possible physical models, that is, to the so-called mathematical experiment. Numerical methods have their disadvantages not only in that a numerical answer is always specific to the parameters of the problem as opposed to analytic treatments (in which a wide class of problems of a single type are examined at once). Even in analyses of the simplest cosmological problems, the now obvious inadequacy of approximations to particular solutions of the equations of celestial mechanics has been noted. Thus, in the three-body problem, numerical integration does not provide an explanation of whether the distance between the bodies remains bounded. Here it is necessary to analyze the motion "as a whole." Similar questions arise in many problems with a comparatively small number of variables and parameters, for example, in studies of the containment of "collisionless" charged particles in accelerator storage units and thermonuclear fusion devices. When there are a large number of variables the results of a computer calculation are practically incomprehensible in that the relationships among the model characteristics are not explicit.

In the first step of a theoretical investigation, the choice of the simplest possible satisfactory model, it is necessary to be able to evaluate the general trends in the behavior of different models under various conditions in order to compare the motion given by the model with the observed changes of state of the object. When the model has been chosen and its equations written down, it is necessary to analyze the solutions in sufficient detail. Often the statement of and methods for treating problems have analogies not only in other branches of physics but in other natural sciences, for example, chemistry, economics, astronomy, etc. Stability analysis of equilibrium states, searching for periodic motions, replacement of a dynamic description of many factors of a single type by stochastic characteristics, evaluation of correlations, interpretation of asymptotic solutions, calculation of the bifurcation parameters at which the entire character of a motion changes, studies of the dependence of the number of significant degrees of freedom on the observation conditions, and many similar problems become impractical if they must be solved anew for every specific scientific problem. These questions lie within the scope of the theory of oscillations.

The description of this science is best begun with a discussion of its approach to the study of phenomena. The theory of oscillations deals chiefly with the qualitative characteristics and laws of motion of an object. For example, in damped oscillations the most important characteristic is not the variable phase and amplitude of the oscillations but the frequency and damping constant characteristic of the motion as a whole (or, at any rate, of the motion over long periods of time) [1]. It may be said that the theory of oscillations analyzes the set of motions of an entire ensemble of identical pendulums under all possible initial conditions rather than the behavior of a given pendulum. It is thus clear how important the methods of the qualitative theory of differential equations were for the theory of oscillations from the very outset and how close the apparatus of the theory of random processes is to it.

Motions which repeat themselves to some degree or other, in particular, periodic ones, are taken to be oscillatory. This property is no longer the basic defining feature of the theory of oscillations. From this standpoint the name "theory of oscillations" does not even reflect the significance of this science although in its time the fruitfulness of a unified approach to the study of oscillatory processes of different kinds was the reason for its isolation as an independent branch of physics. From its very foundation it was clear that the generality of the terminology and methods of the theory of oscillations makes it possible to transfer results from mechanics to electrodynamics and acoustics or conclusions from electronics to mechanics and optics. Over the past half century the range of topics in the theory of oscillations has increased greatly. Its principal aim has become the isolation and study of macroscopic models of the same type belonging to widely different areas of knowledge.

From the above it is already clear that the theory of oscillations must not be regarded as a mathematical discipline although it is often treated that way in textbooks and monographs. The characteristic problems of the theory of oscillations are not analyzed using one logic alone and proceeding from a chosen system of postulates. Experiment plays an important, if not defining, role† in these problems, at least at the model building stage. Besides selecting known and developing new mathematical methods the theory of oscillations is characterized by a distinct realization that the actual analysis is always associated with an idealization, a radical simplification of the phenomena. An analysis of the relationship between models and phenomena and a discussion of the dependence of the number of significant degrees of freedom of an object on the experimental conditions characterize the theory of oscillations to no less a degree than do the mathematical methods used in it [2].

† This is an indication of the arbitrariness in the first part of the term "theory of oscillations."

It is difficult at present to evaluate fully the role of the ideas and concrete results of the theory of oscillations in modern science, a role which appears literally in all areas. It is sufficient to simply list some fields which have very recently sprung up from the theory of oscillations, such as radioastronomy and radiospectroscopy, quantum electronics, and nonlinear optics, to make the importance of its role obvious. There is no doubt about the progress over the last few decades in the individual fields and in the theory of oscillations itself, for example, in the theory of dynamic Hamiltonian systems and in certain aspects of the statistical theory of oscillations. However, if we judge from the scientific literature, interest in the general problems of the theory of oscillations has fallen noticeably over this period. This is unjustified since their significance to science is far from exhausted but continues to grow.

In traditional scientific publications some initial discussion of the timeliness of the topic and the shortcomings of previous work is followed by the principal content which deals with a measurement (or computational) technique and the results obtained from it. It is nevertheless important from time to time to discuss in sufficient detail the research strategy, including, in particular, the conditions which stimulated new directions in the research and the reasons for choosing one or another model of the processes being analyzed. There are, it is true, very few such books and they are written by mathematicians or about mathematics [3-5]. The role of this type of writing is no less important in other widely different fields of science. The need for an analysis of only recently accumulated experience with a reasonable choice of models is perhaps indisputable in the case of "young" exact sciences which have only recently entered (or are still entering) the path of "extensive mathematization," such as physiology psychology, or economics. But even in long-established exact sciences such as physics, chemistry, or astrophysics, progress in important areas is frequently slowed down precisely due to the absence of appropriate models at a given time. Equally objectionable is the irrational expenditure of time and energy in an attempt to solve problems which have not yet made corresponding progress in neighboring fields. It is clear that the choice of a model is determined not only by the intended degree of detail in the investigation but also by the purpose and level of knowledge.

A rather detailed discussion of the reasons for choosing a given model, and perhaps even a valid discussion of why it was necessary to avoid choosing other descriptive schemes (which at first seemed natural), cannot, of course, serve as a recipe for the investigation of new problems, and soon there must be some assistance from that secret feature of the researcher himself which we may refer to as "model intuition" (or simply intuition?). Such books should, it seems, occasionally be written by the greatest scientists in, shall we say, the form of "scientific-methodological memoirs." But this does not happen. Thus, we have tried here to describe our limited experience and the ways which gradually led us to a choice of model which "worked" in several fields. This applies to the traditional direct problems. Primarily we are speaking here of relaxation of extremely nonequilibrium (supercooled) dense plasmas as possible media for efficient amplification of light (including x rays). This also refers to the question now plaguing makind of the rational use of available energy sources. In fact, even of itself the behavior of a dense decaying plasma is complex in the simplest imaginable conditions. It is determined by a large number of interrelated parameters. Solving the corresponding equations directly makes it possible to analyze the simplest situations only with the aid of a high-speed computer. If we try to study the prospects for effective conversion (in nuclear reactor-lasers) of the energy from fissions of nuclei into directed light extracted from the apparatus, the difficulties become practically insuperable, at least in the near future. Besides nuclear processes, we have here to include recombination relaxation of the plasma formed by the fission fragments, the rapid chemical reactions taking place in such a medium, heat transfer to the walls of a vessel filled with gaseous fissionable material, etc.

One direct problem discussed here very briefly is an exception. In the final chapter we discuss the dynamic character of feedback in the phase of oscillations from the well-known relaxation oscillator with a gaseous discharge lamp. The statistical limitation in the growth of the phase-shift dispersion in this device was first observed in analyses of the cyclic activity of the sun. This effect served for a time as the basis of an erroneous concept of the mechanism for this activity and even later, after the discovery of the meaning of this effect, gave substantial support to the development of models for the processes which control cyclic activity. In this book the effect is discussed in the framework of a simple experiment which could be done in the laboratory.

The ideas and methods of the theory of oscillations have clearly come from the study of processes which are nearly periodic. Between "oscillatory" and "nonoscillatory" motions there is no significant fundamental boundary starting at which we would have to place only motions with components which regularly change sign in the domain of the theory of oscillations. It is unreasonable to separate oscillation and limiting relaxation problems, setting, for example, a critical value of damping as the boundary for the theory of oscillations. Thus, the oscillatory problems would include only dynamic systems lying in the regions of their phase spaces adjacent to limiting cycles or to focus or center stationary points. It would be no less artificial to require the existence of oscillatory motions among the trajectories since that would exclude consideration of some parameter intervals of the system. But even the simplest variants of nonoscillatory motions are often associated with striking effects which can be analyzed naturally by the methods of the theory of oscillations. Of the wide variety of such problems, in the first section we have chosen to study the intense recombination of a supercooled dense plasma in order to construct lasers with high energy densities. A shortage of reliable data on the probabilities of a number of elementary events controlling the population kinetics has limited further progress in this important field. In one chapter of the second part we discuss an inverse problem for the theory of oscillations which arises from this situation – an experimental technique for determining transition probabilities.

A discussion of inverse problems in the theory of oscillations is the main subject of two other chapters in this part of the book. In one chapter a general scheme for these inverse problems is applied to the mechanism for the cyclic activity of the sun, and in the other chapter the possibility of medical diagnosis by means of statistical analysis of the external pulsations of large blood vessels is discussed.

Direct and Inverse Problems

The approaches to research in the natural sciences are grouped under direct or indirect schemes. Both in mathematics and in other exact sciences the discussion of the direct problems was begun much earlier; hence, they are more familiar than the inverse problems. Briefly speaking, the direct problems of mathematics consist of seeking the solutions of a given equation, while trying to find the equation itself from known solutions is the object of the inverse problems. An analogous division into direct and indirect schemes holds for other branches of science. Essentially all schemes that have been worked out in detail belong to the traditional (direct) problems. The examination of the inverse schemes has begun very recently, the terminology has not yet been settled, and the statement of the main problems is not always clear. This by no means indicates that the inverse problems are of secondary significance. Precisely because of the existence of direct methods for analyzing them new problems in science are often studied using an excessive number of models. It is clear that this method is not always effective. A gradual realization of this has led in the last ten or fifteen years to the appearance of a number of publications (including review articles and monographs) on the statement and analysis of inverse problems in mathematical physics, cybernetics, and the theory of oscillations.

We must note that the concept of inverse problems often used now in applied mathematics does not completely coincide with that noted in the previous paragraph and used in this book.

Thus, we shall discuss some definitions, starting with some examples from "pure" mathematics in which we limit ourselves solely to some illustrative terminology.

The solution $\mathbf{x} \equiv (x_1, x_2, \ldots, x_p)$ of the equation $\mathbf{f_A}(\mathbf{x}) = \mathbf{y}$, where $\mathbf{A} \equiv (A_1, A_2, \ldots, A_q)$ is a parameter and $\mathbf{y} \equiv (y_1, y_2, \ldots, y_r)$ is the right-hand side, will be written in the form $\mathbf{x} = \mathbf{f}_{\mathbf{A}}^{-1}(\mathbf{y})$. For various values of $\mathbf{a}$ the operation $\mathbf{f_a}(\mathbf{x})$ forms a family, $\{\mathbf{f_a}(\mathbf{x})\}$, of transformations of the solution $\mathbf{x}$ into the right side $\mathbf{y}$. We shall write this in the form

$$x, A \xrightarrow{\{f_a\}} y. \tag{1}$$

Finding the behavior of the inverse transformation

$$y, A \xrightarrow{\{f_a^{-1}\}} x \tag{2}$$

is equivalent to finding the solution of the equation. The inverse problem is now another transformation of Eq. (1) which involves finding the parameter $\mathbf{A}$ from known values of the image $\mathbf{y}$ and object $\mathbf{x}$:

$$x, y \xrightarrow{\{f_a\}} A; \qquad y, x \xrightarrow{\{f_a^{-1}\}} A. \tag{3}$$

If for given values of $\mathbf{x}$ and $\mathbf{y}$ it is not possible to find the parameter (due to nonuniqueness or inconsistency of the corresponding equations) then the problem (3) is said to be incorrectly stated.

The meaning of $\mathbf{A}$ and the right-hand side, $\mathbf{y}$, in scientific applications of this scheme varies. Staying within a mathematical framework we now illustrate this with a more specific example. In the theory of ordinary differential equations the direct problem is the Cauchy problem. The equation

$$\frac{d^q x}{dt^q}(t) + D_A\left[x(t), \frac{dx}{dt}(t), \ldots, \frac{d^{q-1}}{dt^{q-1}}x(t)\right] = y(t)$$

with the initial condition

$$\frac{d^k x}{dt^k}(t_0) = x_0^{(k)}, \qquad k = 0, 1, \ldots, q-1 \quad \text{or} \quad x(t_0) = x_0$$

maps $x(t)$ into the function $y(t)$:

$$x(t), A, (x_0) \xrightarrow{\{D_a\}} y(t). \tag{4}$$

The reverse of this transformation,

$$y(t), A \xrightarrow{\{D_a^{-1}\}} x(t), \tag{5}$$

is a solution of the Cauchy problem. The inverse problem is to find the parameter $\mathbf{a} \pm \mathbf{A}$ which distinguishes that differential operator within a given family, $\{D_a\}$, of operators which transforms $x(t)$ into another, also known, function $y(t)$. The two variants of the inverse problem are written symbolically in the form

$$\begin{aligned} x(t), y(t) &\xrightarrow{\{D_a\}} A, \\ y(t), x(t) &\xrightarrow{\{D_a^{-1}\}} A. \end{aligned} \tag{3'}$$

Problem (3') may also not have solutions if the chosen family of operators $\{D_a\}$ does not work for the given functions $x(t)$ and $y(t)$ or, we say, is too narrow. If the family $\{D_a\}$ is too wide, it impedes the search for the desired value of the parameter and may make the solution to (3') nonunique.

The cybernetic "black box" analysis technique is akin to the two preceding examples. For a set of known effect y_n fed into the box and the reactions x_n received at the output of the box it is necessary to find an operator D_A from among a class of operators $\{D_a\}$ which transforms the responses x_n of the object into values y' which are sufficiently close to the signals, y_n, at the input to the box. The operator D_A, which transforms x into y', serves in effect as a description of the object being studied, which remains enclosed in the black box, showing itself to the researcher in no other way than through its responses x. For a reasonable choice of classes $\{D_a\}$ which sort the functions y_n it is possible in principle by constantly improving the accuracy of the operators $\{D_A\}$ relative to the responses x_n to construct a detailed mathematical model of the object which will permit prediction of its behavior under new conditions with sufficient accuracy. The ultimate goal of cybernetic techniques is just this kind of description of an object, independently of the specific (physical, chemical, biological) processes actually taking place within it.

Completing our illustration of the difference between the inverse problem and the inverse transformation, we conclude that where the goal of the inverse problem is the elucidation of the characteristics of the object itself, the significance of the inverse transformation is to find an external effect on the object. Inverse transformations correspond to the problem of reducing observational data when it is necessary to reproduce the actual signal entering the receiver and the distorting of the receiving and recording apparatus are known. This problem is usually complicated by noise, inaccurate knowledge of the characteristics of the apparatus, and so on. What we have said here is probably sufficient to clarify the difference in the concepts of direct and inverse problems as used in the theory of oscillations and in mathematical physics. In the latter, the direct problem refers to solving a given equation, or more generally, to finding the consequences of a known cause. In mathematical physics, the inverse problem means not only looking for the operator (which describes the object) in the left-hand side of the equation, but also determining (for a known form of operator) the right-hand side which describes an external effect. Both these problems have in common a general principle, finding the cause of a known consequence, as well as a number of similar difficulties in their solution. This, of course, is sufficient for the two problems to be considered jointly. However, the difference between them is so great that they merit separate names. Inverse problems are more characteristic of research problems while inverse transformations are more characteristic of direct processing of measurement results. In this book the term "inverse problem" refers everywhere to a search for a description of a phenomenon or (speaking up to now only of the mathematical side) to an analysis of the structure of an object in terms of its signal. In cybernetics such problems are referred to as "object identification problems." We must now discuss the difference in the approach to inverse problems in the theory of oscillations and cybernetics.

The traditional method of studying new phenomena may be divided into several stages:

1. Isolation of the most important processes under conditions similar to those in the object, taking the interactions of these processes into account, and deriving equations which describe the chosen arrangement of phenomena.
2. Analyzing the solution of the equations for various supplementary conditions (boundary, initial, etc., conditions) leading to uniqueness in the mathematical problem. If the model has been chosen correctly these solutions will yield an acceptable picture of the phenomenon. However, the appropriateness of the idealization chosen is not known in advance. Hence the following stage is necessary.
3. Comparing the resulting solutions with the observed behavior of the object.

If there is qualitative disagreement between the theoretical and observed motions the model is scrapped. Then it is generally necessary to begin everything from the first stage. This research approach in fact involves an excess of different descriptions. On the other hand, experience with successful models of similar phenomena usually allows us to reduce the number of variants attempted.

This approach, which is typical for direct problems in the theory of oscillations, is inappropriate when too little is known about the processes producing the observed properties of the object. Then it is natural to change the order of doing things. Without specifying the model too much at first it is possible to try to find the equations directly from the signal by analyzing the signal numerically in order to then interpret these equations in terms of specific processes. We note the stages of research in the inverse problems of the theory of oscillations in greater detail as follows:

1. Mobilization of a priori information about the object in order to narrow as much as possible the class of operators (equations) $\{D_a\}$ used to describe it.
2. Analysis of the signal from the object in order to isolate from the class $\{D_a\}$ the operator D_A which agrees best with the observations, i.e., construction of a mathematical model of the phenomenon.
3. Construction of a concrete model by interpreting the mathematical description in terms of the specific processes which produce this phenomenon.

The last stage is evidently the least amenable to any kind of regular recipe and thereby the most difficult, but in the theory of oscillations it is just this stage which is the actual goal. And, perhaps, only this stage is capable of evoking the enthusiasm necessary for laborious problem (meaning, in general, nonapplied problems, that is, those which promise no early practical yield) solving in order to surmount the first two less creative stages. This third stage is not included in the cybernetic "black box" scheme.

It would be incorrect to think that only the presence of the third stage differentiates the inverse-problem method for the theory of oscillations from the cybernetic-object-identification problem. Often the first and, almost in every case, the second stages in the theory of oscillations are substantially different from the cybernetic treatment, since isolation of the operator D_A is determined not only by the signal from the object but also by the class of operators and, most importantly for the question at hand, by the criterion for optimizing the operator. The choice of criterion is dictated by the purpose of the discussion and these purposes, finding the most exact mathematical description of the signal (in cybernetics) and constructing a concrete process model of the phenomenon (in the theory of oscillations), do not immediately coincide.

Isn't something incompatible here? Isn't the purpose of any theory to find the most exact description of the observed motion and then to predict the behavior of the object under new conditions? Isn't the approach of cybernetics then the most reasonable? This point of view ("the direct way is always the shortest") simplifies the actual situation. It has been repeatedly noted that the generality of the methods and the "universality of the language" of the theory of oscillations permits the use of analogies. It is appropriate to begin a brief listing of the degrees of analogy from our standpoint by recalling the analogy between a phenomenon and the models which approximate it. A generalization of the principle widely used in the theory of similarity allows us to transfer the results of a specific study not only to phenomena at different scales but also to different scientific fields. The agreement among the dimensionless equations corresponds both to the mathematical isomorphism and to the similarity of the concrete models of various phenomena. Similar dimensionless complexes of parameters and variables are thereby defined. Yet richer is the following degree of analogy in which qualitatively similar models are discussed. This refers to dimensionless equations which are not identical but have "phase portraits" of the same type.

From this listing there comes still one more very common degree of analogy. It follows from the similarity of component processes of a single type which make up fairly diverse phenomena. This isolation of similar processes makes it possible to classify them in terms of physical, chemical, biological, social, and other processes. Within physics, for example, this classification distinguishes electromagnetic, nuclear, gravitational and other processes. In more detail, electromagnetic processes may be divided, for example, into those which take place in plasmas, gases, liquids, and solids. A still more detailed classification distinguishes relaxing free plasmas, electric discharges, shock waves in plasmas, etc. The greatest detail makes it possible to do a unique theoretical analysis. Thus, if the parameters of a free plasma are known at $t = t_0$, it is possible to predict their variation for $t > t_0$. The level of detail in classifying processes which makes it possible to analyze their later variation corresponds to the term "specific (or concrete) processes" used in this book.

Of course, a general statement that it is necessary to explain the specific processes in every case would be wrong. Thus, for example, if in biology basic interest is usually in interpreting specific mechanisms, then in medicine it would be an inadmissible luxury to make a complete analysis of the biological processes which are taking place every time, while avoiding use of a prescription which would reliably lead to recovery from a dangerous disease until this analysis was made.

But it is no less obvious that knowledge of the controlling processes in the end always aids in obtaining an exact quantitative description of a phenomenon. At the same time, until identification and analysis of the most important processes, excessive detail and accuracy in the analysis are most often an obstacle to discovering the essence of the observed phenomenon and constructing an appropriate (at first crude) model. Clearly, even the first physical laws (for example, the laws of dynamics) could not be discovered without that understanding of the unity of the specific processes which made it possible for the first time to avoid using both a series of distinctly different factors and the less important factors. It is just this isolation of the specific component processes of phenomena which has led to the present highly accurate agreement between experiment and theory in physics. And where has there been a science able to analyze every phenomenon anew with a mathematically optimal description, that is, by closely following the cybernetic scheme described above? Thus, in medicine, having not yet found other ways to objectively diagnose heart ailments in detail, we most often turn to purely "mechanical" methods in interpreting cardiograms and electrocardiograms. The results would undoubtedly be improved if there were reasonable preliminary preparation of the material to be analyzed. For example, knowing that the heart plays the basic role of a pump in the body to drive blood through two blood circulation loops, it is natural to first use a method for obtaining (for example, from the electrocardiogram traces of the major blood vessels) the individual equations of each person's "own pump." It is certain that with these equations it is easier to set up a computer diagnosis (although with the same means of identifying the features in charts) than to do this directly from unprepared electrokymograph traces, i.e., almost blindly. The principle that a "smart machine should understand everything by itself" is as yet premature.

The poverty of the pragmatic, transient cybernetic approach is especially obvious in those areas of science without practical significance such as astrophysics or cosmology in which the meaning of the mathematical description usually appears in the confirmation or refutation of specific process models of phenomena. These branches of science, which have grown rapidly in the last few decades, are also noteworthy in that the objects of the analysis are not subject to intervention by the scientist. To use the terminology of cybernetics, we may say the black box has no input in this case. In such fields it is appropriate to use the word "observation" in place of "experiment." About ten years ago a statistical method in the framework of an inverse problem in the theory of oscillations was proposed for the study of uncontrollable objects [6]. Its later development and application to several astrophysical objects led to a number of instructive, and at times unexpected, results.

Complex logical structures which begin from precise definitions and postulates are typical of modern theories. Usually the corresponding apparatus comes from mathematics in more or less complete form. In other cases the apparatus initially developed for analyzing a specific problem is gradually abstracted from its scientific specifics and becomes a new mathematical discipline of itself. This entwinement with mathematics is one of the reasons for the traditional deductive exposition of theories in monographs and textbooks. It produces a distorted concept of the significance of various methods and results in science. The impression is created that adequate statements of scientific problems together with rigorous schemes for quantitatively analyzing them always lead to the analysis of some equations, most often to one of the modifications of the Cauchy problem. Meanwhile, the fundamental achievements of natural science are usually associated with inductive rather than deductive approaches. Nature most often presents the researcher with an inverse problem rather than a direct one. In fact, only in the case of the motion of a known mechanism under given external circumstances is the solution of a direct problem applicable. In investigating new phenomena one always deals with a signal (in a rather general sense) about the equations of which very little is known in advance. Thus, rapidly developing areas of the exact sciences which are solving fundamental problems must correspond more fully to the logic of inverse problems, methods of problem solving which, if they were developed as much as the traditional methods, would be of great benefit. It is to be hoped that the growing understanding of the significance of inverse problems will lead to intensive discussion of the questions which arise in connection with them. The notion of inverse problems is gaining a deserved place in the most varied sciences, including the theory of oscillations, which deals more and more with general methods for understanding and analyzing dynamic laws.

A Principle for Isolating Dynamic Couplings

Separating the characteristics of an object into stochastic and dynamic characteristics is often fairly arbitrary at the start.† It would seem possible to reason as follows: Let x(t) be the signal from our object. If its behavior is sufficiently accurately described by the equation $D[x] = F(t)$, where D is a dynamic operator and F(t) is a fluctuation, then D plays the role of a dynamic description of the phenomenon and F(t) is a stochastic perturbation. But without special refinements such a description is nonunique even in the simplest situations. Let the dynamic operator D belong to a class L_q of linear homogeneous differential operators of the q-th order with constant parameters. Then, besides the equation $D[x] = F$, we can always write $D_1[x] = F_1$, where D_1 is any other operator from the same class L_q and $F_1 \equiv F(t) + D_1[x(t)] - D[x(t)]$ is again a fluctuating process. What kind of limitations need to be placed on D and F to ensure uniqueness of these concepts?

The problem of uniquely isolating the stochastic component loses its sharpness when the fluctuations in the signal are sufficiently small and can be neglected when analyzing the phenomenon. If the class of equations for the dynamic model has been determined, then it is possible in principle to find an effective description for any sufficiently complete set of external perturbations of the object in whose responses it is possible to recognize all the degrees of freedom of the observed phenomenon. We now illustrate this with an elementary example. Let the signal x(t) from an autonomous object be described by an ordinary linear differential equation

$$\frac{d^q x}{dt^q}(t) + \sum_{m=0}^{q-1} A_m \frac{d^m x}{dt^m}(t) = 0 \qquad (6)$$

† Here and in the following it is understood that the phenomena under consideration are macroscopic.