Springer International
Student Edition

Rudolf Zurmühl

# Numerical Analysis for Engineers and Physicists

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# Translators' Preface

We are happy to have been asked to translate into English this well-known book by Prof. Dr. Zurmühl. The popularity and usefulness which this book has enjoyed amongst the German-knowing public have provided enough confidence that it is bound to be of great value to the English-knowing students of Mathematics.

Every translator is faced with a dilemma, namely, whether he should be literally faithful to the original or aim at the correct idiom of the language into which the translation is made at the risk of deviating, though slightly, from the original. We have all along tried to strike a balance between these conflicting objectives and accepted a 'least square fit' as the best solution.

This book should find a place on the desks of all those who are interested in applying numerical methods to practical problems. An overview of the salient features of the book is contained in the author's prefaces. We cannot possibly do better than referring the reader to them.

In a book of this size the number of printing errors located (and of course corrected!) by us is very small. Suggestions for improvement of the translation in all its aspects are welcome.

We thank the authorities of the Indian Institute of Technology, Madras for having permitted us to undertake this project. A few of our colleagues have read through portions of the translation and offered their suggestions, for which we are thankful to them. Mr. Venkateswaran did an excellent job in typing the entire manuscript. It is a pleasure to express our appreciation to M/s. Springer-Verlag and in particular to Mr. N. K. Mehra and Allied Publishers for their co-operation.

R. SUBRAMANIAN
P. ACHUTHAN
K. VENKATESAN

Madras 600036
January 1977

# Preface to the Fifth Edition

In spite of the short period of time, since the appearance of the last edition, it appeared to me that a revision of many sections of the book was desirable and this was essentially due to the increasing influence of computers in all the areas of numerial mathematics. As before, the book will be an introduction to the fundamentals of numerical methods; it is not a textbook on computer science and programming. Still I felt it useful to include a brief introduction to the programming language ALGOL, which nowadays is an indispensable tool for an engineer to precisely formulate and solve his numerical problems and which gives him an easy access to the computers.

Besides many small improvements, the following parts of the book have been revised for this edition; the major deciding factor being easy adaptation to computer calculations. As a counterpart to Newton's method for solution of equations, the *Regula falsi* is introduced in the form of an algorithm with an almost-quadratic convergence; this approach has the essential advantage that it does not necessitate any differentiation. For iteration with linear convergence—this idea will be clarified later—Aitken's process for accelerating the convergence will be adopted, as it has proved to be an important general principle. The section on the Routh criterion is new and a method of Collatz (based on it) for solution of polynomial equations has been extended. In numerical integration, the method of Gauss has been presented at length on account of its fundamental importance.

The fifth chapter has been rewritten under the new title "Approximation". From the approximation in the least-squares sense, introduced at the beginning, the trigonometric approximation is derived. There is a new section on uniform approximation, an important problem today, for which is needed the powerful approximation method based on the trigonometric interpolation.

For the differential equations the automatic (computer) step-control of the Runge-Kutta method is specially noteworthy. For eigenvalue problems the multipoint ("Mehrstellen") difference method is based on the new approach of Falk. Finally, a new section brings the Ritz method to a schematic form—which can be used for general variational problems of ordinary differential equations and through which a computer solution of even complicated eigenvalue problems is made possible. By judicious omissions, I have attempted to maintain the original size of the book.

I have to thank my Chief Engineer, Mr. D. Stephan, for his valuable help in writing the introduction to ALGOL. I also have to thank Messrs

H. J. Amtsberg and H. Weirich for carrying out numerical calculations and for help in proof-reading. I thank Mrs. H. Heydebreck for typing the manuscript. Finally, my thanks are due to Springer-Verlag for the fine get-up of the book—for which they are well known—and for their ready compliance with all my wishes.

Berlin 33, Summer, 1965
Trabener Str. 42

RUDOLF ZURMÜHL

## From the Preface to the Second Edition

This book is conceived of as a supplement and continuation of the basic course on Mathematics in technical schools. It is intended to stimulate the young students of Engineering, to pursue those branches of mathematics that are fundamental to the numerical treatment of engineering problems of all kinds, i.e. to numerical methods in practical mathematics. It introduces the theory and practical aspects of these methods, equal emphasis being placed on lucid development of the basic ideas and on the details of numerical calculation. The book can also be used by the practising engineer, when, in solving his problems, he has to apply numerical methods.

A book meant for engineers and physicists must be different in many respects from the one meant specifically for the mathematicians. But it should certainly be equally reliable and accurate. While the mathematicians should be dedicated to developing new methods, physicists and engineers learn the practical aspects of the methods in the first place, in order to employ them as tools in their specific professional work. For the application of these methods in a meaningful and correct fashion, one should certainly understand their mathematical basis. A mere collection of recipes will not be useful. Only a person, who has understood well the principle of a method, will be in a position to estimate its limitations and range of applicability and to choose the best from among the many methods. It is, therefore, also one of the principal objectives of this book to clearly work out the method underlying the mathematical formulation of the problems and to show, in individual cases, the way that leads from the problem to the solution. Otherwise, in consonance with the purpose of the book, such discussions as will serve the practical solution of the problems will be spotlighted, while problems which are of interest more to the mathematician will be relegated to the background.

The book, therefore, does not completely exhaust all the methods developed for a given problem. Rather, it is considered to be one of its main tasks to make a careful selection of the methods to be treated—a selection, which is naturally determined by personal tastes.

Finally, a mathematical text-book, meant for the engineer, must also give a comparatively well-informed preparatory training in the calculation, —which has been often neglected in the past. As a prerequisite, the reader is assumed to have a knowledge of the basic course in mathematics and for the first part of the book only the material covered in the first two semesters. Additional material will be given in the later parts. The presentation is deliberately wide and detailed, so that, I hope, even the reader who is not well versed in mathematical literature can follow it with some co-operation and patience on his part; he has only to have enough inclination for, and interest in, the subject.

Darmstadt
March, 1957

RUDOLF ZURMÜHL

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# Contents

## Introduction. Preliminaries



Chapter I

## Equations

Chapter III

**Interpolation and Integration**

## Chapter IV

**Statistics and the theory of estimation**

Chapter V

**Approximation**



Chapter VI

**Differential equations: Initial value problems**

## Chapter VII

### Differential equations—Boundary and eigenvalue problems