Ric Pimentel
Terry Wall

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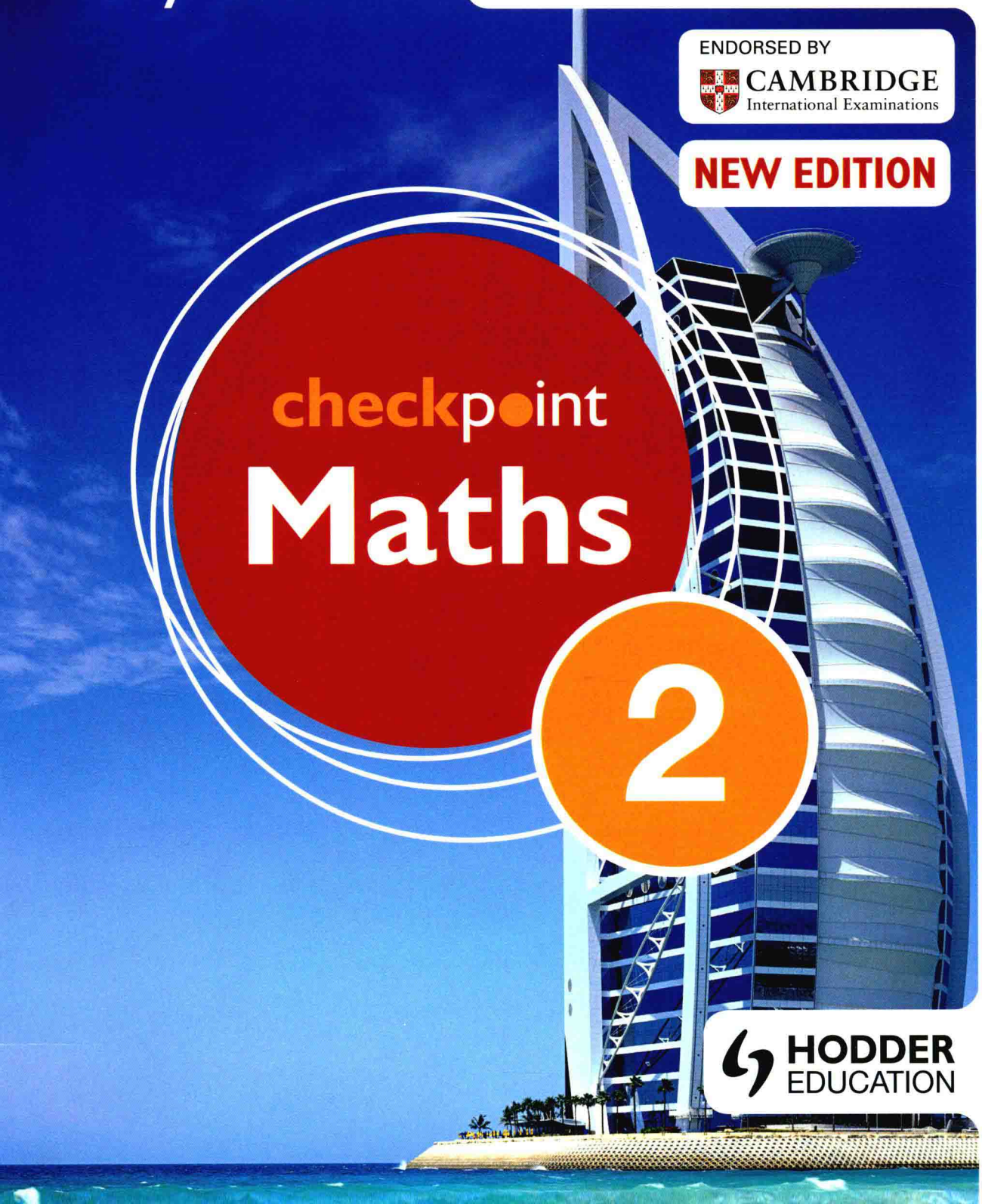

HODDER EDUCATION

Ric Pimentel
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2

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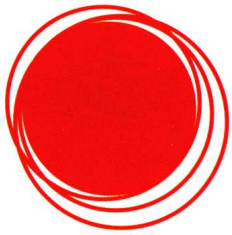

Introduction

This series of books follows the Cambridge Secondary 1 Mathematics Curriculum Framework drawn up by University of Cambridge International Examinations. It has been written by two experienced teachers who have lived or worked in schools in many countries, and worked with teachers from other countries, including England, Spain, Germany, France, Turkey, South Africa, Malaysia and the USA.

Students and teachers in these countries come from a variety of cultures and speak many different languages as well as English. Sometimes cultural and language differences make understanding difficult. However, mathematics is largely free from these problems. Even a maths book written in Japanese will include algebra equations with x and y.

We should also all be very aware that much of the mathematics you will learn in these books was first discovered, and then built upon, by mathematicians from all over the world, including China, India, Arabia, Greece and Western countries.

Most early mathematics was simply game play and problem solving. Later this maths was applied to building, engineering and sciences of all kinds. Mathematicians study maths because they enjoy it.

We hope that you will enjoy the work you do, and the maths you learn in this series of books. Sometimes the ideas will not be easy to understand at first. That should be part of the fun. Ask for help if you need it, but try hard first. Write down what you are thinking so that others can understand what you have done and help to correct your mistakes. Most students think that maths is about answers, and so it is, but it is also a way to exercise our brains, whether we find the solution or not. Some questions throughout this book are starred (✪). This means that these questions go slightly beyond the content of the curriculum at this level and will be an enjoyable challenge for those of you who try them.

Ric Pimentel and Terry Wall

Contents

The chapters in this book have been arranged to match the Cambridge Secondary 1 Mathematics Curriculum Framework for Stage 8 as follows:

- Number
- Algebra
- Geometry
- Measure
- Handling data
- Calculation and mental strategies
- Problem solving

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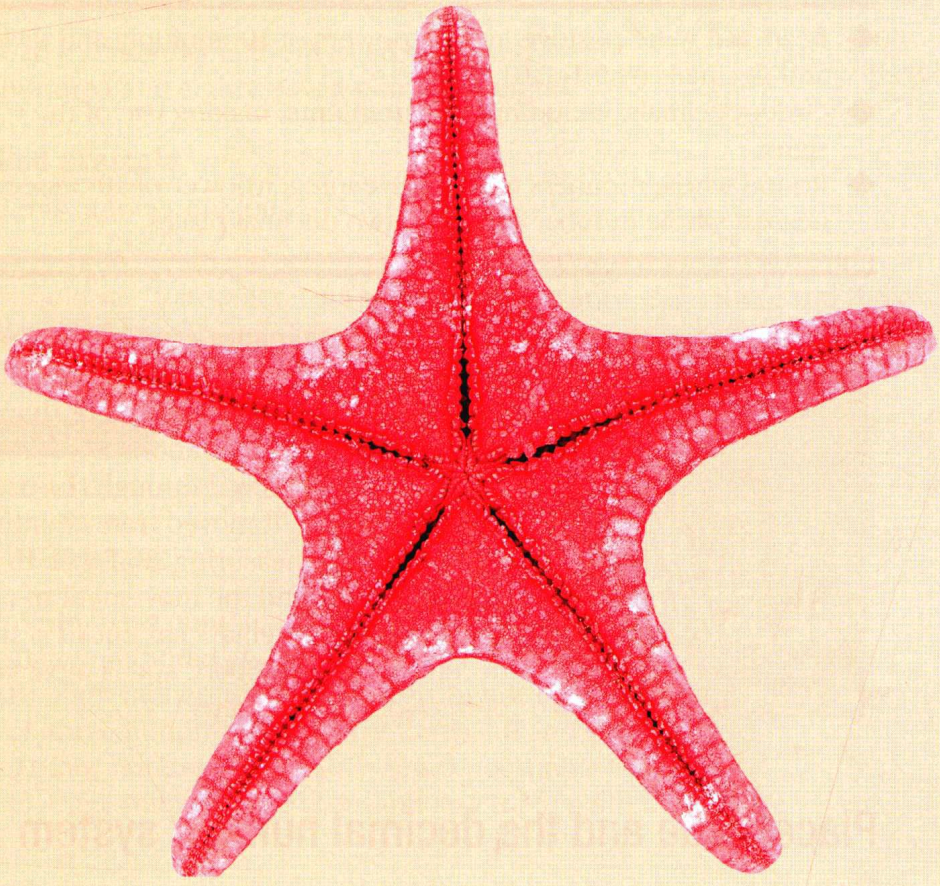
SECTION 1



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1 Place value, ordering and rounding

- Read and write positive integer powers of 10; multiply and divide integers and decimals by 0.1, 0.01.
- Order decimals, including measurements, making use of the =, ≠, < and > signs.
- Round whole numbers to a positive integer power of 10 or decimals to the nearest whole number or one or two decimal places.

The word 'mathematics' comes from the Greek 'mathematika'. It is the study of patterns, including number patterns, and of shape and its structure, measurement and transformations.

Mathematics evolved through the use of logical reasoning. It moved from counting to calculating and measuring, and then to the study of shapes and the movement of objects.

The Greek mathematician Euclid is shown on the left. He lived in 300BC and is most famous for his work on geometry.

Place value and the decimal number system

Our number system is said to be 'base 10'. The position of each digit in a number determines its value. For example, 6287.48 can be placed in a table like this:

Thousands	Hundreds	Tens	Units	Tenths	Hundredths
6	2	8	7	4	8

the decimal point

Looking at the table, we can see that:

the 6 is worth	6000
the 2 is worth	200
the 8 is worth	80
the 7 is worth	7
the 4 is worth	0.4 or $\frac{4}{10}$
the 8 is worth	0.08 or $\frac{8}{100}$

We read the number 6287.48 as 'six thousand two hundred and eighty-seven point four eight'.

We do NOT say '... point forty-eight'.

Worked example

Read this number out loud:

4 628 370.52

You should say 'four million six hundred and twenty-eight thousand three hundred and seventy point five two'.

EXERCISE 1.1

1 Read these numbers out loud to a partner.

a) 684 000 **b)** 3 420 000 **c)** 726 845.66

d) 63 972.83 **e)** 543 210.98

2 Write these numbers in words.

a) two hundred and fifty thousand

b) eight million ten thousand six hundred

c) three hundred and twelve thousand and six point two

d) four hundred and eighty-seven point nine two six

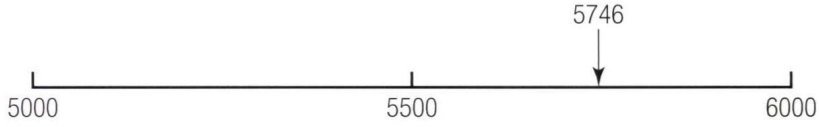
e) three million three thousand and three point three three

Rounding

Rounding to powers of 10

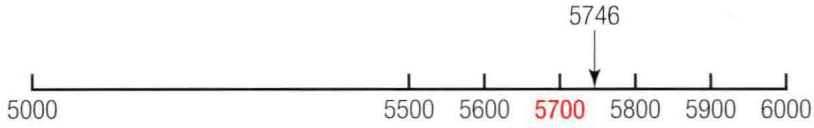
For integer values (whole numbers), it is often not necessary to give the exact number.

For example, if a college has 5746 students, this figure can be rounded (approximated) in several ways by showing its position on a number line.

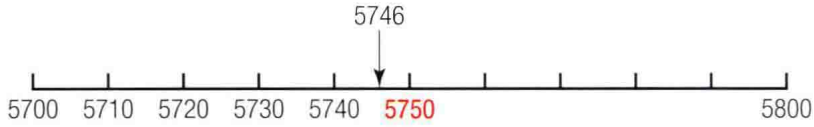


If 5746 is rounded to the nearest **thousand**, then it is written as 6000, because 5746 is closer to 6000 than it is to 5000.

If 5746 is rounded to the nearest **hundred**, then it is written as 5700, because 5746 is closer to 5700 than it is to 5800.



If 5746 is rounded to the nearest **ten**, then it is written as 5750, because 5746 is closer to 5750 than it is to 5740.



If a number is half way, then it is rounded up. For example, 5745 rounded to the nearest ten would be 5750 even though it is half way between 5740 and 5750.

Worked examples

77 648 people attended a recent football match in Milan.

a) This was reported in the programme as 80 000 attendance.
b) The club estimated that there were 78 000 people in the crowd.
c) A closer estimate in a newspaper was 77 650.

All of these estimates are acceptable. However, they are to different degrees of accuracy. Write down the degree of accuracy for each one.

a) nearest ten thousand
b) nearest thousand
c) nearest ten

EXERCISE 1.2A

1 Round each of these numbers to the degree of accuracy shown in brackets.

a) 4865 (nearest ten)
b) 7843 (nearest hundred)
c) 18 695 (nearest thousand)
d) 14 295 (nearest hundred)
e) 684 (nearest ten)

f) 7346 (nearest hundred)
g) 899200 (nearest hundred thousand)
h) 487653 (nearest hundred thousand)
i) 3478000 (nearest million)
j) 4562700 (nearest million)

2 A survey showed that 14627 cars went through a village on one Saturday.
a) Round this number **(i)** to the nearest ten, **(ii)** to the nearest hundred and **(iii)** to the nearest thousand.
b) Which number might be used by a local newspaper campaigning for a safe crossing to be built in the village?

3 A village hall holds 567 people.
a) Round this number to the nearest hundred.
b) What would you say is its **safe capacity** to the nearest hundred?

4 The rock band Muse played in front of 106348 people in China.
Round this number to the nearest hundred thousand.

5 From London to Boston is 4827km.
Round this distance to the nearest 100km.

6 A company made a profit of $687250.
Round this amount to the nearest $10000.

7 The table shows the capacities of some of the largest sports stadiums in the world. With a partner, round the number for the capacity of each stadium to the nearest thousand.

Stadium	Sport	Capacity
Rungrado May Day Stadium – Pyongyang, North Korea	Association football	150000
Salt Lake Stadium – Kolkata, India	Association football	119750
Michigan Stadium – Ann Arbor, USA	American football	109901
Estadio Azteca – Mexico City, Mexico	Association football	100607
Melbourne Cricket Ground – Melbourne, Australia	Cricket, Association football	100018
Azadi Stadium – Tehran, Iran	Association football	100126
Camp Nou – Barcelona, Spain	Association football	98787
FNB Stadium – Johannesburg, South Africa	Association football	94700
Gelora Bung Karno Stadium – Jakarta, Indonesia	Association football	88306
Bukit Jalil National Stadium – Kuala Lumpur, Malaysia	Association football	87411
Estádio do Maracanã – Rio de Janeiro, Brazil	Association football	82238

Decimal places

If 6287.48 is rounded to the nearest whole number, then it is written as 6287, and to one decimal place it rounds up to 6287.5.

This method can be extended in order to round a number to two decimal places.

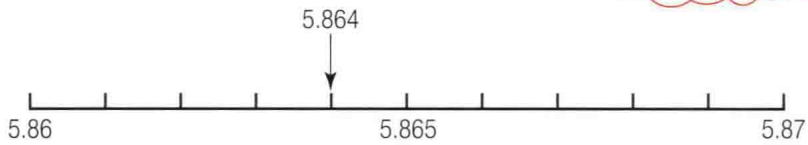
For example, if the number 6287.483 is rounded to two decimal places, the third digit after the decimal point is 3 so it rounds down to 6287.48. But in the number 6287.487, the third digit after the decimal point is 7 so it rounds up to 6287.49.

Numbers can be rounded to any number of decimal places. This can be shown clearly by using a number line.

Worked example

The length of a model car is 5.864 cm.
Write 5.864 to two decimal places.
Draw a number line to help you.

A number written to two decimal places has two digits after the decimal point.



5.864 is closer to 5.86 than it is to 5.87, so 5.864 written to two decimal places is 5.86.

To round to the nearest whole number (integer) or to a certain number of decimal places, look at the next digit after the one in question. If that digit is 5 or more, round up. If it is 4 or less, round down.

EXERCISE 1.2B

Round each of the following numbers **(i)** to the nearest whole number, **(ii)** to one decimal place and **(iii)** to two decimal places.

a) 8.757	**b)** 7.492	**c)** 18.685	**d)** 0.1238	**e)** 4.856
f) 9.470	**g)** 12.3852	**h)** 9.2563	**i)** 14.7777	**j)** 0.6666

Rounding up or rounding down

The usual rule for rounding is that if the digit after the one in question is 5 or more, you round up. If it is 4 or less, you round down. For example,

1.7 to the nearest whole number is 2.
5.72 to one decimal place is 5.7.
5.75 to one decimal place is 5.8.
0.07 to one decimal place is 0.1.

But in Student's Book 1 you saw that you need to use common sense too.

Worked example

A bridge is 4.7 m high. A sign needs to be put on the bridge to warn drivers.

Would you say that 'Maximum height 5 m' or 'Maximum height 4 m' is the best sign?

Rounded to the nearest whole number, the maximum height would be given as 5 m.

But somebody driving a bus 4.8 m tall would not be helped by rounding up. The bus would crash.

So 'Maximum height 4 m' is a more useful sign.

EXERCISE 1.2C

1 A bridge can safely carry a weight of 11.75 tonnes.
Round this weight sensibly to a whole number of tonnes.
Write a warning sign.

2 A pilot needs to know the height of a mountain the aircraft is to fly over.
The mountain is 6.35 km high. Round this height appropriately:
a) to a whole number of kilometres
b) to one decimal place.

3 An aqualung will work safely to a depth of 67.3 metres.
What rounded figure would you use as the safe working depth?

4 A stadium has 76 890 seats.
What would you say was its maximum capacity:
a) to a whole thousand **b)** to a whole hundred?

5 A bridge is to be built over a river 2315 m wide.
Write down a sensible estimate of its length, to a whole 100 m, for working out how much steel will be needed.

When dividing, you may need to decide whether it is more sensible to round up or to round down.

Worked example

A car transporter can carry eight cars.
How many transporters are needed for 60 cars?

$60 \div 8 = 7.5$

Therefore 8 transporters will be needed.

In fact, 8 transporters will be needed for any number of cars from 57 up to and including 64.

EXERCISE 1.2D

1 A plane can carry 8.5 tonnes of cargo. Each container weighs 1.75 tonnes.
How many containers can the plane carry safely?

2 A baker makes loaves weighing 600 g.
How many loaves can she make from 12.9 kg of dough?

3 Wooden houses are being built on a concrete base.
Each base needs 400 kg of concrete.
A cement truck carries 9 tonnes of concrete.
How many bases can be laid from one truck-load of concrete?

4 Houses are built with wooden panels. Each house needs 15 panels.
How many houses can be built from 1000 panels?

5 A ship can carry 12 000 tonnes of cargo. A container weighs 14 tonnes.
How many containers can the ship carry safely?

Multiplying and dividing integers by 0.1 and 0.01

You know that $0.1 = \frac{1}{10}$.

So multiplying by 0.1 is the same as multiplying by $\frac{1}{10}$. For example,

$$740 \times 0.1 = 740 \times \frac{1}{10} = 74$$

But $740 \div 10 = 74$.

So multiplying by 0.1 is the same as multiplying by $\frac{1}{10}$ and as dividing by 10.

Similarly, $0.01 = \frac{1}{100}$.

So multiplying by 0.01 is the same as multiplying by $\frac{1}{100}$ and as dividing by 100.

Worked examples

a) Work out

450×0.1

$450 \times 0.1 = 450 \div 10 = 45$

b) Work out

352×0.1

$352 \times 0.1 = 352 \div 10 = 35.2$

c) Work out

8370×0.01

$8370 \times 0.01 = 8370 \div 100 = 83.7$

d) Work out

524×0.01

$524 \times 0.01 = 524 \div 100 = 5.24$

EXERCISE 1.3A

Work out the following multiplications without using a calculator.

1	360×0.1	**2**	850×0.1	**3**	1460×0.1
4	5600×0.1	**5**	8760×0.1	**6**	2360×0.01
7	1850×0.01	**8**	5600×0.01	**9**	1460×0.01
10	875×0.01	**11**	62×0.01	**12**	153×0.01
13	7244×0.01	**14**	27×0.1	**15**	38×0.01

Since $0.1 = \frac{1}{10}$, dividing by 0.1 is the same as dividing by $\frac{1}{10}$.

But dividing by $\frac{1}{10}$ is the same as multiplying by 10.

So dividing by 0.1 is the same as dividing by $\frac{1}{10}$ and as multiplying by 10.

Similarly, $0.01 = \frac{1}{100}$, so dividing by 0.01 is the same as dividing by $\frac{1}{100}$.

But dividing by $\frac{1}{100}$ is the same as multiplying by 100.

So dividing by 0.01 is the same dividing by $\frac{1}{100}$ and as multiplying by 100.

Worked examples

a) Work out

$7.3 \div 0.1$

$7.3 \div 0.1 = 7.3 \times 10 = 73$

b) Work out

$84.62 \div 0.01$

$84.62 \div 0.01 = 84.62 \times 100 = 8462$

c) Work out

$5.02 \div 0.1$

$5.02 \div 0.1 = 5.02 \times 10 = 50.2$

d) Work out

$0.39 \div 0.01$

$0.39 \div 0.01 = 0.\ 39 \times 100 = 39$

EXERCISE 1.3B

Work out the following divisions without using a calculator.

1	$5.9 \div 0.1$	**2**	$17.2 \div 0.1$	**3**	$0.6 \div 0.1$
4	$0.87 \div 0.1$	**5**	$14.03 \div 0.1$	**6**	$18.82 \div 0.01$
7	$5.43 \div 0.01$	**8**	$0.792 \div 0.01$	**9**	$5.3 \div 0.1$
10	$0.3 \div 0.1$	**11**	$0.03 \div 0.1$	**12**	$0.07 \div 0.01$
13	$6 \div 0.1$	**14**	$20 \div 0.01$	**15**	$0.6666 \div 0.01$

16 A 10 cent coin is worth \$0.10.
How many 10 cent coins are there in \$8?

17 A piece of wood 2 m 30 cm long is marked off into 0.1 m lengths.
How many 0.1 m lengths are there?

18 A car travels 0.5 km on 0.01 litre of fuel.
How many kilometres will it travel on 40 litres of fuel?

Inequalities

You are familiar with the sign $=$, which means 'is equal to'. Another sign is $\neq$, which means 'is not equal to'. Two other useful signs are $>$ meaning 'is greater than' and $<$ meaning 'is less than'.

These signs can be used to describe the relationship between numbers and quantities.

For example,

$4 \times 5 < 3 \times 8$ means 4×5 is less than 3×8. *Since $20 < 24$*

EXERCISE 1.4A

Copy and complete the following statements, writing one of the signs $=$, $>$ or $<$ to make each statement true.

1	5×3 _____ 7×2	**2**	$6 + 5$ _____ 4×3
3	$8 + 8$ _____ 4×4	**4**	70 cm _____ 1 m
5	70 cm _____ 0.7 m	**6**	10 cm _____ 1000 mm
7	10 m _____ 100 cm	**8**	4 tonnes _____ 3500 kg
9	$5 \times 5 \times 5$ _____ 20×5	**10**	8×0.1 _____ $8 \div 10$

Ordering integers and decimals

Worked example

Write the following numbers in order of size, smallest first.
Use the symbol $<$ (less than).

0.4, 0.04, 4, 3.8, 0.38, 0.06, 0.038

The order can be found by writing the numbers in a place value table.

Tens	Units	Tenths	Hundredths	Thousandths
	0	4		
	0	0	4	
	4			
	3	8		
	0	3	8	
	0	0	6	
	0	0	3	8

Working from left to right in the table, the correct order is:

$0.038 < 0.04 < 0.06 < 0.38 < 0.4 < 3.8 < 4$

EXERCISE 1.4B

Write each of the following sets of numbers in order of size, smallest first.
Use the symbol for 'less than' in your answers.

1 0.8, 0.6, 0.84, 0.09, 1, 0.49

2 0.36, 3.6, 3, 6.3, 0.6, 0.09

3 4, 4.4, 0.44, 0.4, 44, 0.04

4 6, 0.6, 6.6, 0.66, 0.06, 0.066

5 3.3, 0.3, 0.33, 3.33, 3.03, 0.333

Make a place value table if necessary.