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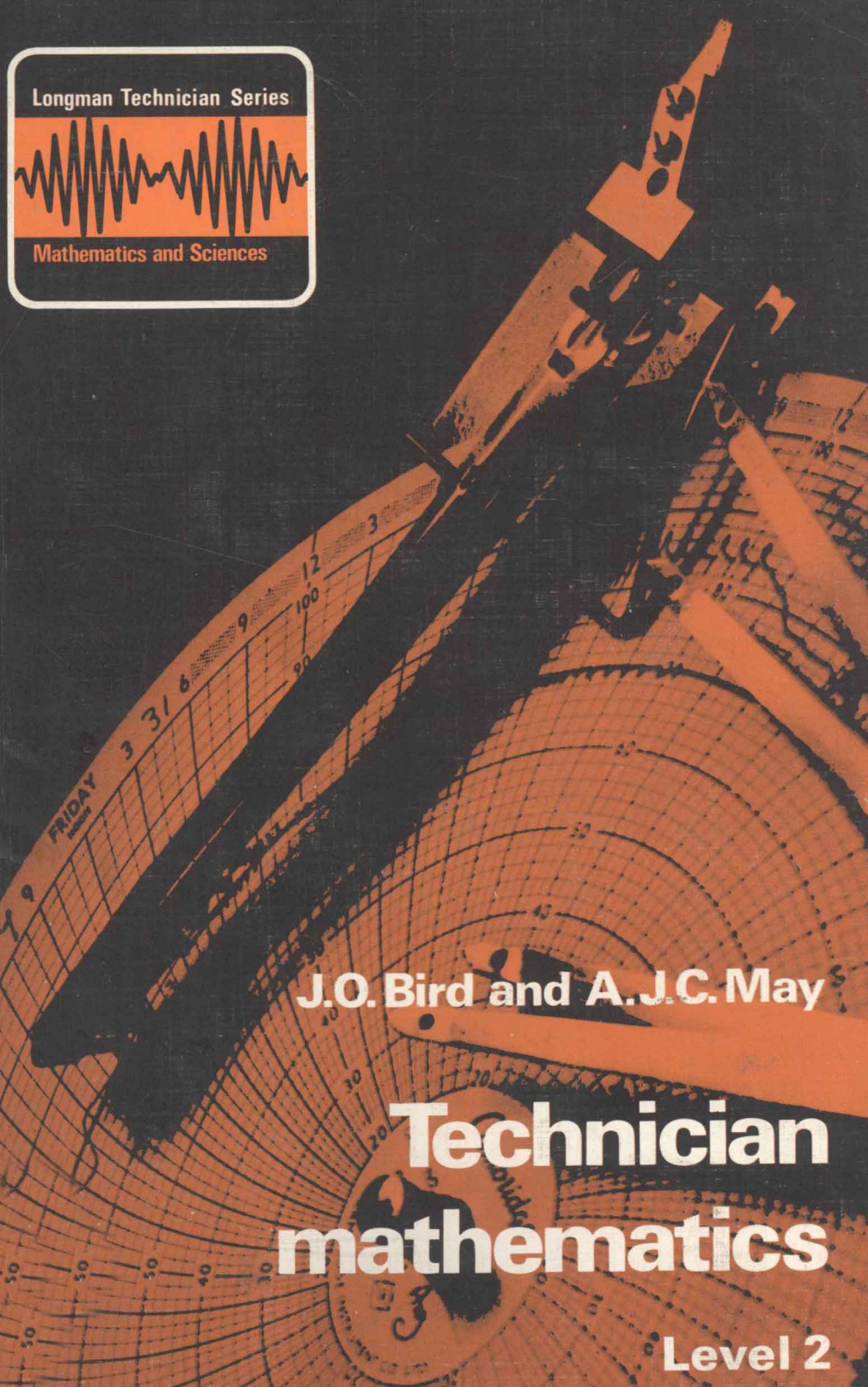

J.O. Bird and A.J.C. May

Technician mathematics

Level 2

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Level 2

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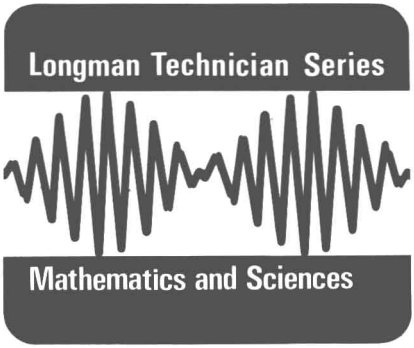

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Preface

This textbook is the second of a series of three which deal simply and carefully with the fundamental Mathematics essential in the development of technicians.

Treatment of this Mathematics is developed soundly and logically by authors experienced in various fields of Engineering.

Technician Mathematics Level 2 provides a clear and comprehensive coverage of the new Technician Education Council Level II Mathematics courses (TEC U75/012, 038 and 039).

Each topic considered in the text is presented in a way that assumes in the reader only the knowledge attained at TEC Level I Mathematics. This practical Mathematics book contains over 200 illustrations, nearly 200 detailed worked problems, followed by some 450 further problems with answers.

The authors would like to thank Mr David Browning, Principal Lecturer, Bristol Polytechnic, for his valuable assistance in his capacity as General Editor of the Mathematics and Science Sector of the Longman Technician Series. They would also like to express their appreciation for the friendly cooperation and helpful advice given to them by the publishers.

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Contents

Chapter 1

Trigonometry

1 Trigonometric ratios of acute angles

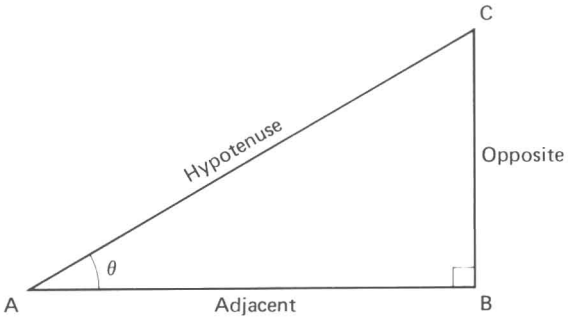

Fig. 1

Trigonometry deals with the measurements of the sides and angles of triangles and their relationships with each other. There are six definitions of trigonometric ratios which apply to any right-angled triangle. Consider a right-angled triangle ABC shown in Fig. 1, and let the angle CAB be denoted by θ. With reference to angle θ, the side BC is called the opposite side, the side AB the adjacent side and the side AC the hypotenuse.

Then by definition, natural sine $\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

natural cosine $\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

natural tangent $\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

natural secant $\theta = \dfrac{\text{hypotenuse}}{\text{adjacent side}}$

natural cosecant $\theta = \dfrac{\text{hypotenuse}}{\text{opposite side}}$

and natural cotangent $\theta = \dfrac{\text{adjacent side}}{\text{opposite side}}$

Using accepted abbreviations and referring to Fig. 1:

$$\sin\theta = \frac{BC}{AC}$$

$$\cos\theta = \frac{AB}{AC}$$

$$\tan\theta = \frac{BC}{AB}$$

$$\sec\theta = \frac{AC}{AB}$$

$$\operatorname{cosec}\theta = \frac{AC}{BC}$$

$$\cot\theta = \frac{AB}{BC}$$

Now $$\frac{\sin\theta}{\cos\theta} = \frac{\frac{BC}{AC}}{\frac{AB}{AC}} = \frac{BC}{AC} \cdot \frac{AC}{AB} = \frac{BC}{AB}$$

But $$\tan\theta = \frac{BC}{AB}$$

Hence $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Also $$\frac{\cos\theta}{\sin\theta} = \frac{\frac{AB}{AC}}{\frac{BC}{AC}} = \frac{AB}{AC} \quad \frac{AC}{BC} = \frac{AB}{BC}$$

But $$\cot\theta = \frac{AB}{BC}$$

Hence $$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

As $\sin\theta = \frac{BC}{AC}$ and $\operatorname{cosec}\theta = \frac{AC}{BC}$

then

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}\left(\text{or } \sin\theta = \frac{1}{\operatorname{cosec}\theta}\right)$$

Similarly, as $\cos\theta = \frac{AB}{AC}$ and $\sec\theta = \frac{AC}{AB}$

then $$\sec\theta = \frac{1}{\cos\theta}\left(\text{or } \cos\theta = \frac{1}{\sec\theta}\right)$$

Similarly, as $\tan\theta = \frac{BC}{AB}$ and $\cot\theta = \frac{AB}{BC}$

then $$\cot\theta = \frac{1}{\tan\theta}\left(\text{or } \tan\theta = \frac{1}{\cot\theta}\right)$$

Thus cosecant, secant and cotangents are often called the **'reciprocal trigonometric ratios'.**

2 Use of four figure trigonometrical tables for natural secants, cosecants and cotangents

For natural secants, cosecants and cotangents the method of reading values is similar to that for natural sines, cosines and tangents. The angles in degrees are given in the left-hand column, while in the main columns of the page the degrees are subdivided into the intervals of 6 minutes or 0.1 degrees. The mean difference column on the right gives the data which enables angles to be calculated to the nearest one minute.

(a) Natural secant tables

Values of natural secants are shown in Table 1.

It can be seen that the value of a secant increases from 1 at 0° to infinity (i.e. ∞) at 90°. For example, sec 14° is 1.030 6 and sec 76° is 4.133 6. Similarly, sec 23°24′ is 1.089 6 and sec 54°36′ is 1.726 3.

For sec 17°16′:

sec 17°12′	= 1.046 8	
mean difference for 4′	=	4 (adding)
sec 17°16′	= 1.047 2	

After 68° the mean differences cease to be sufficiently accurate. However if a linear relationship is assumed then a small inaccuracy will exist.

For sec 83°38′:

sec 83°36′ = 8.971 1

sec 83°42′ = 9.112 9

Table 1 Natural Secants

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
0	1.0000	0000	0000	0000	0000	0000	0001	0001	0001	0001	0	0	0	0	0
1	1.0002	0002	0002	0003	0003	0003	0004	0004	0005	0006	0	0	0	0	0
2	1.0006	0007	0007	0008	0009	0010	0010	0011	0012	0013	0	0	0	0	0
3	1.0014	0015	0016	0017	0018	0019	0020	0021	0022	0023	0	0	1	1	1
4	1.0024	0026	0027	0028	0030	0031	0032	0034	0035	0037	0	0	1	1	1
5	1.0038	0040	0041	0043	0045	0046	0048	0050	0051	0053	0	1	1	1	1
6	1.0055	0057	0059	0061	0063	0065	0067	0069	0071	0073	0	1	1	1	2
7	1.0075	0077	0079	0082	0084	0086	0089	0091	0093	0096	0	1	1	2	2
8	1.0098	0101	0103	0106	0108	0111	0114	0116	0119	0122	0	1	1	2	2
9	1.0125	0127	0130	0133	0136	0139	0142	0145	0148	0151	0	1	1	2	2
10	1.0154	0157	0161	0164	0167	0170	0174	0177	0180	0184	1	1	2	2	3
11	1.0187	0191	0194	0198	0201	0205	0209	0212	0216	0220	1	1	2	2	3
12	1.0223	0227	0231	0235	0239	0243	0247	0251	0255	0259	1	1	2	3	3
13	1.0263	0267	0271	0276	0280	0284	0288	0293	0297	0302	1	1	2	3	4
14	1.0306	0311	0315	0320	0324	0329	0334	0338	0343	0348	1	2	2	3	4
15	1.0353	0358	0363	0367	0372	0377	0382	0388	0393	0398	1	2	3	3	4
16	1.0403	0408	0413	0419	0424	0429	0435	0440	0446	0451	1	2	3	3	4
17	1.0457	0463	0468	0474	0480	0485	0491	0497	0503	0509	1	2	3	4	5
18	1.0515	0521	0527	0533	0539	0545	0551	0557	0564	0570	1	2	3	4	5
19	1.0576	0583	0589	0595	0602	0608	0615	0622	0628	0635	1	2	3	4	5
20	1.0642	0649	0655	0662	0669	0676	0683	0690	0697	0704	1	2	3	5	6
21	1.0711	0719	0726	0733	0740	0748	0755	0763	0770	0778	1	2	4	5	6
22	1.0785	0793	0801	0808	0816	0824	0832	0840	0848	0856	1	3	4	5	6
23	1.0864	0872	0880	0888	0896	0904	0913	0921	0929	0938	1	3	4	6	7
24	1.0946	0955	0963	0972	0981	0989	0998	1007	1016	1025	1	3	4	6	7
25	1.1034	1043	1052	1061	1070	1079	1089	1098	1107	1117	2	3	5	6	8
26	1.1126	1136	1145	1155	1164	1174	1184	1194	1203	1213	2	3	5	6	8
27	1.1223	1233	1243	1253	1264	1274	1284	1294	1305	1315	2	3	5	7	9
28	1.1326	1336	1347	1357	1368	1379	1390	1401	1412	1423	2	4	5	7	9
29	1.1434	1445	1456	1467	1478	1490	1501	1512	1524	1535	2	4	6	8	9
30	1.1547	1559	1570	1582	1594	1606	1618	1630	1642	1654	2	4	6	8	10
31	1.1666	1679	1691	1703	1716	1728	1741	1753	1766	1779	2	4	6	8	10
32	1.1792	1805	1818	1831	1844	1857	1870	1883	1897	1910	2	4	7	9	11
33	1.1924	1937	1951	1964	1978	1992	2006	2020	2034	2048	2	5	7	9	12
34	1.2062	2076	2091	2105	2120	2134	2149	2163	2178	2193	2	5	7	10	12
35	1.2208	2223	2238	2253	2268	2283	2299	2314	2329	2345	3	5	8	10	13
36	1.2361	2376	2392	2408	2424	2440	2456	2472	2489	2505	3	5	8	11	13
37	1.2521	2538	2554	2571	2588	2605	2622	2639	2656	2673	3	6	8	11	14
38	1.2690	2708	2725	2742	2760	2778	2796	2813	2831	2849	3	6	9	12	15
39	1.2868	2886	2904	2923	2941	2960	2978	2997	3016	3035	3	6	9	12	16
40	1.3054	3073	3093	3112	3131	3151	3171	3190	3210	3230	3	7	10	13	10
41	1.3250	3270	3291	3311	3331	3352	3373	3393	3414	3435	3	7	10	14	17
42	1.3456	3478	3499	3520	3542	3563	3585	3607	3629	3651	4	7	11	14	18
43	1.3673	3696	3718	3741	3763	3786	3809	3832	3855	3878	4	8	11	15	19
44	1.3902	3925	3949	3972	3996	4020	4044	4069	4093	4118	4	8	12	16	20

Table 1 Natural Secants (*cont.*)

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
45	1.4142	4167	4192	4217	4242	4267	4293	4318	4344	4370	4	8	13	17	21
46	1.4396	4422	4448	4474	4501	4527	4554	4581	4608	4635	4	9	13	18	22
47	1.4663	4690	4718	4746	4774	4802	4830	4859	4887	4916	5	9	14	19	23
48	1.4945	4974	5003	5032	5062	5092	5121	5151	5182	5212	5	10	15	20	25
49	1.5243	5273	5304	5335	5366	5398	5429	5461	5493	5525	5	10	16	21	26
50	1.5557	5390	5622	5055	5088	5721	5755	5788	5822	5856	6	11	17	22	28
51	1.5890	5925	5959	5994	6029	6064	6099	6135	6171	6207	6	12	18	24	29
52	1.6243	6279	6316	6353	6390	6427	6464	6502	6540	6578	6	12	19	25	31
53	1.6616	6655	6694	6733	6772	6812	6852	6892	6932	6972	7	13	20	26	33
54	1.7013	7054	7095	7137	7179	7221	7263	7305	7348	7391	7	14	21	28	35
55	1.7434	7478	7522	7566	7610	7655	7700	7745	7791	7837	7	15	22	30	37
56	1.7883	7929	7976	8023	8070	8118	8166	8214	8263	8312	8	16	24	32	40
57	1.8361	8410	8460	8510	8561	8612	8663	8714	8766	8818	9	17	26	34	43
58	1.8871	8924	8977	9031	9084	9139	9194	9249	9304	9360	9	18	27	36	45
59	1.9416	9473	9530	9587	9645	9703	9702	9821	9880	9940	10	19	29	39	49
60	2.0000	0061	0122	0183	0245	0308	0371	0434	0498	0562	10	21	31	42	52
61	2.0627	0692	0757	0824	0890	0957	1025	1093	1162	1231	11	22	34	45	56
62	2.1301	1371	1441	1513	1584	1657	1730	1803	1877	1952	12	24	36	48	61
63	2.2027	2103	2179	2256	2333	2412	2490	2570	2650	2730	13	26	39	52	66
64	2.2812	2894	2976	3060	3144	3228	3314	3400	3486	3574	14	28	43	57	71
65	2.3662	3751	3841	3931	4022	4114	4207	4300	4395	4490	15	31	46	62	77
66	2.4586	4683	4780	4879	4978	5078	5180	5282	5384	5488	17	34	50	67	84
67	2.5593	5699	5805	5913	6022	6131	6242	6354	6466	6580	18	37	55	73	92
68	2.6695	6811	6927	7046	7165	7285	7407	7529	7653	7778	20	40	60	81	101
69	2.7904	8032	8161	8291	8422	8555	8688	8824	8960	9099					
70	2.9238	9379	9521	9665	9811	9957	3.0106	3.0256	3.0407	3.0561					
71	3.0716	0872	1030	1190	1352	1515	1681	1848	2017	2188	Mean differences				
72	3.2361	2535	2712	2891	3072	3255	3440	3628	3817	4009	cease to be sufficiently				
73	3.4203	4399	4598	4799	5003	5209	5418	5629	5843	6060	accurate.				
74	3.6280	6502	6727	6955	7186	7420	7657	7897	8140	8387					
75	3.8637	8890	9147	9408	9672	9939	4.0211	4.0486	4.0765	4.1048					
76	4.1336	1627	1923	2223	2527	2837	3150	3469	3792	4121					
77	4.4454	4793	5137	5486	5841	6202	6569	6942	7321	7706					
78	4.8097	8496	8901	9313	9732	5.0159	5.0593	5.1034	5.1484	5.1942					
79	5.2408	2883	3367	3860	4362	4874	5396	5928	6470	7023					
80	5.7588	8164	8751	9351	9963	6.0589	6.1227	6.1880	6.2546	6.3228					
81	6.3925	4637	5366	6111	6874	7655	8454	9273	7.0112	7.0972					
82	7.1853	2757	3684	4635	5611	6613	7642	8700	9787	8.0905					
83	8.2055	3238	4457	5711	7004	8337	9711	9.1129	9.2593	9.4105					
84	9.5668	9.728	9.895	10.07	10.25	10.43	10.63	10.83	11.03	11.25					
85	11.47	11.71	11.95	12.20	12.47	12.75	13.03	13.34	13.65	13.99					
86	14.34	14.70	15.09	15.50	15.93	16.38	16.86	17.37	17.91	18.49					
87	19.11	19.77	20.47	21.23	22.04	22.93	23.88	24.92	26.05	27.29					
88	28.65	30.16	31.84	33.71	35.81	38.20	40.93	44.08	47.75	52.09					
89	57.30	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					
90	∞														

6 The difference between 8.971 1 and 9.112 9 is 0.141 8. If a linear relationship is assumed then 0.141 8 represents 6′ in equal increments of $\frac{0.141\ 8}{6}$ or 0.023 6.

Hence 2′ represents 2 (0.023 6) or 0.047 2

sec 83°36′	= 8.971 1
mean difference for 2′	≏ 0.047 2 (adding)
sec 83°38′	≏ 9.018 3

The following values of secants may be checked:

sec 5°46′ = 1.005 1	sec 62°09′ = 2.140 7
sec 32°11′ = 1.181 6	sec 78°19′ ≏ 4.938 3
sec 48°57′ = 1.522 7	sec 87°23′ ≏ 21.91

Reverse use of natural secant tables

To find the angle whose secant is 1.277 2:

The nearest number **less** than 1.277 2 in the main columns is 1.276 0, which corresponds to 38°24′. The number 276 0 is 12 less than 277 2. 12 in the mean difference column corresponds to 4′. Therefore the angle whose secant is 1.277 2 is 38°24′ + 4′, that is, 38°28′.

To find the angle whose secant is 2.229 7:

Given value of secant	= 2.229 7
sec 63°18′	= 2.225 6 (subtracting)
	4 1

From the mean difference column 41 corresponds closest to an increase of 3′. Hence the angle is 63°21′.

Note that 'arcsec θ' is a short way of writing 'the angle whose secant is equal to θ'.

Using the above method it may be shown that:

arcsec 1.073 1 = 21°16′
arcsec 1.395 7 = 44°14′
arcsec 2.617 1 = 67°32′

(b) **Natural cosecant tables**

Values of natural cosecants are shown in Table 2. It can be seen that the value of a cosecant decreases from infinity at 0° to 1 at 90°. That is, as the angle increases the cosecant decreases (which is directly opposite to that of a secant). For example, cosec 22° is 2.669 5 and cosec 59° is 1.166 6.

Similarly, cosec 6°42′ is 8.571 1 and cosec 78°54′ is 1.019 1.

For cosec 27°34′:

cosec 27°30′	= 2.165 7
mean difference for 4′	= 4 8 (subtracting)
cosec 27°34′	= 2.160 9

With cosecant tables the mean difference is subtracted since the cosecants are decreasing in value as the angles are increasing. For values of cosecants less than 18° the mean differences cease to be sufficiently accurate but approximate values may be obtained within this range assumimg a linear relationship.

The following values of cosecants may be checked:

cosec 4°44′ ≏ 12.12
cosec 21°58′ = 2.673 0
cosec 39°02′ = 1.587 9
cosec 51°11′ = 1.283 4
cosec 68°14′ = 1.076 8
cosec 85°22′ = 1.003 3

Reverse use of natural cosecant tables

To find the angle whose cosecant is 1.583 2:

The nearest number **greater** than 1.583 2 in the main columns is 1.585 6 which corresponds to 39°6′. The number 585 6 is 24 greater than 583 2. 24 in the mean difference column corresponds closest to 4′. Therefore the angle whose cosecant is 1.583 2 is 39°6′ + 4′, that is, **39°10′**.

To find the angle whose cosecant is 1.0687:

Given value of cosecant = 1.068 7
cosec 69°18′ = 1.069 0 (subtracting)
−3

3 in the mean difference column corresponds to 3′, the minus sign indicating an increase in the angle. Hence the angle is **69° 21′.**

Note that 'arccosec θ' is a short way of writing 'the angle whose cosecant is equal to θ'.

Using the above method it may be shown that:

arccosec 2.204 8 = 26°58′
arccosec 1.444 5 = 43°49′
arccosec 1.125 7 = 62°40′

(c) **Natural cotangent tables**

Values of natural cotangents are shown in Table 3.

It can be seen that the value of a cotangent decreases from infinity at 0° to 1 at 45° and then to 0 at 90°.

For example, cot 12° is 4.704 6 and cot 74° is 0.286 7.
Similarly, cot 15°48′ is 3.533 9 and cot 54°30′ is 0.713 3.
Fot cot 31°15′:

cot 31°12′ = 1.651 2
mean difference for 3′ = 3 2 (subtracting)
cot 31°15′ = 1.648 0

Table 2 Natural Cosecants

(Numbers in difference columns to be subtracted, not added.)

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
0	Inf.	573.0	286.5	191.0	143.2	114.6	95.49	81.85	71.62	63.66					
1	57.30	52.09	47.75	44.08	40.93	38.20	35.81	33.71	31.84	30.16					
2	28.65	27.29	26.05	24.92	23.88	22.93	22.04	21.23	20.47	19.77					
3	19.11	18.49	17.91	17.37	16.86	16.38	15.93	15.50	15.09	14.70					
4	14.34	13.99	13.65	13.34	13.03	12.75	12.47	12.20	11.95	11.71					
5	11.47	11.25	11.03	10.83	10.63	10.43	10.25	10.07	9.895	9.728					
6	9.5668	9.4105	9.2593	9.1129	8.9711	8.8337	8.7004	8.5711	8.4457	8.3238					
7	8.2055	8.0905	7.9787	7.8700	7.7642	7.6613	7.5011	7.4635	7.3684	7.2757	Mean differences				
8	7.1853	7.0972	7.0112	6.9273	6.8454	6.7655	6.6874	6.6111	6.5366	6.4637	are not sufficiently				
9	6.3925	6.3228	6.2546	6.1880	6.1227	6.0589	5.9963	5.9351	5.8751	5.8164	accurate.				
10	5.7588	5.7023	5.6470	5.5928	5.5396	5.4874	5.4362	5.3866	5.3367	5.2883					
11	5.2408	5.1942	5.1484	5.1034	5.0593	5.0159	4.9732	4.9313	4.8901	4.8496					
12	4.8097	4.7706	4.7321	4.6942	4.6569	4.6202	4.5841	4.5486	4.5137	4.4793					
13	4.4454	4.4121	4.3792	4.3469	4.3150	4.2837	4.2527	4.2223	4.1923	4.1627					
14	4.1336	4.1048	4.0765	4.0486	4.0211	3.9930	3.9672	3.9408	3.9147	3.8890					
15	3.8637	8387	8140	7897	7657	7420	7186	6955	6727	6502					
16	3.6280	6060	5843	5629	5418	5209	5003	4799	4598	4399					
17	3.4203	4009	3817	3628	3440	3255	3072	2891	2712	2535					
18	3.2361	2188	2017	1848	1681	1515	1352	1190	1030	0872	27	55	82	105	137
19	3.0716	0561	0407	0256	0106	2.9957	2.9811	2.9665	2.9521	2.9379	25	49	74	99	123
20	2.9238	9099	8960	8824	8688	8555	8422	8291	8161	8032	22	44	67	89	111
21	2.7904	7778	7653	7529	7407	7285	7165	7046	6927	6811	20	40	60	81	101
22	2.6695	6580	6466	6354	6242	6131	6022	5913	5805	5699	18	37	55	73	92
23	2.5593	5488	5384	5282	5180	5078	4978	4879	4780	4683	17	34	50	67	84
24	2.4586	4490	4395	4300	4207	4114	4022	3931	3841	3751	15	31	46	62	77
25	2.3662	3574	3486	3400	3314	3228	3144	3060	2976	2894	14	28	43	57	71
26	2.2812	2730	2650	2570	2490	2412	2333	2256	2179	2103	13	26	39	52	65
27	2.2027	1952	1877	1803	1730	1657	1584	1513	1441	1371	12	24	36	48	60
28	2.1301	1231	1162	1093	1025	0957	0890	0824	0757	0692	11	22	34	45	56
29	2.0627	0562	0498	0434	0371	0308	0245	0183	0122	0061	10	21	31	42	52
30	2.0000	1.9940	1.9880	1.9821	1.9762	1.9703	1.9645	1.9587	1.9530	1.9473	10	19	29	39	49
31	1.9416	9360	9304	9249	9194	9139	9084	9031	8977	8924	9	18	27	36	45
32	1.8871	8818	8766	8714	8663	8612	8561	8510	8460	8410	8	17	25	34	42
33	1.8361	8312	8263	8214	8166	8118	8070	8023	7976	7929	8	16	24	32	40
34	1.7883	7837	7791	7745	7700	7655	7610	7566	7522	7478	7	15	22	30	37
35	1.7434	7391	7348	7305	7263	7221	7179	7137	7095	7054	7	14	21	28	35
36	1.7013	6972	6932	6892	6852	6812	6772	6733	6694	6655	7	13	20	26	33
37	1.6616	6578	6540	6502	6464	6427	6390	6353	6316	6279	6	12	19	25	31
38	1.6243	6207	6171	6135	6099	6064	6029	5994	5959	5925	6	12	18	23	29
39	1.5890	5856	5822	5788	5755	5721	5688	5655	5622	5590	6	11	17	22	28
40	1.5557	5525	5493	5461	5429	5398	5366	5335	5304	5273	5	10	16	21	26
41	1.5243	5212	5182	5151	5121	5092	5062	5032	5003	4974	5	10	15	20	25
42	1.4945	4916	4887	4859	4830	4802	4774	4746	4718	4690	5	9	14	19	23
43	1.4663	4635	4608	4581	4554	4527	4501	4474	4448	4422	4	9	13	18	22
44	1.4396	4370	4344	4318	4293	4267	4242	4217	4192	4167	4	8	13	17	21