



# Ultrasonic Bioinstrumentation

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**DOUGLAS A. CHRISTENSEN**

# ULTRASONIC BIOINSTRUMENTATION

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**JOHN WILEY & SONS**

*New York • Chichester*

*Brisbane • Toronto • Singapore*

WILEY

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***Library of Congress Cataloging in Publication Data:***

Christensen, Douglas A.

Ultrasonic bioinstrumentation.

Bibliography: p.

Includes index.

1. Diagnosis, Ultrasonic—Instruments. I. Title.

RC78.7.U4C48 1988b 616.07'543 87-34066

ISBN 0-471-60496-8

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

**ULTRASONIC  
BIOINSTRUMENTATION**

To  
Laraine  
and  
Ithaca

# Preface

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The purpose of this text is to present the physical and engineering principles that underlie the use of ultrasound in medical instruments. The text is intended to be helpful to both users of ultrasound in the clinic as well as to designers of ultrasonic instruments. It is felt that only by understanding the mathematical basis for the propagation of waves, including the development and solution of the wave equation and the study of such phenomena as reflection, impedance, and radiation patterns, can the user of ultrasonic bioinstrumentation knowledgeably apply these concepts in clinical practice. Similarly, a bioengineer who is designing new devices needs to obtain a physical understanding of waves and their propagation behavior in order to exploit these ideas in future instruments. The first chapters in this text develop the fundamentals of ultrasound, and later chapters apply these basics to practical instruments.

The level of the text is appropriate to a senior or first-year graduate course. It is assumed that the student has had some training in partial differential equations and has been exposed to the use of vectors. A student who has also had some experience with wave concepts from an undergraduate electromagnetics or optics course will find the analogies between the propagation and reflection of electromagnetic waves and acoustical waves to be reinforcing and reassuring in the study of ultrasound; however, such background is not required for this text.

It should be noted that no attempt has been made to be comprehensive in the wave equation. For example, nonlinear and second-order terms are neglected, as being of small magnitude; that simplification is made in order to concentrate on the primary principles. Also, there has been no attempt to survey completely all current clinical instruments in use today. Such an effort would quickly be outdated by the rapid proliferation of clinical machines seen recently. Instead, typical examples are given of the most general instrument classes, and it is hoped that the basic principles presented can be readily extrapolated to understand instruments now and of the future.

Douglas A. Christensen

# Useful Physical Constants and Conversion Factors

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## USEFUL PHYSICAL CONSTANTS

Planck's constant:  $h = 6.63 \times 10^{-34} \text{ Js} = 4.14 \times 10^{-15} \text{ eVs}$

Speed of light in free space:  $c = 3.0 \times 10^8 \text{ m/s}$

Dielectric constant of free space:  $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

Speed of sound in air (at 1 atm and 20°C):  $c = 344 \text{ m/s}$

Speed of sound in water (at 27°C):  $c = 1501 \text{ m/s}$

Specific heat of water (at 37°C):  $S = 4.18 \text{ J/g}^\circ\text{C} = 0.998 \text{ Cal/g}^\circ\text{C}$

## CONVERSION FACTORS

1 liter (L) =  $0.001000027 \text{ m}^3 \approx 1 \times 10^3 \text{ cm}^3$

1 newton (N) =  $10^5 \text{ dyn}$

= 0.10197 kg weight (at gravitational acceleration  
 $g = 980.665 \text{ cm/s}^2$ )

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## Introduction

The use of ultrasound in the modern-day medical clinic has found a solid niche among the various methods for imaging the body. The reasons for this popularity are many, but they perhaps chiefly derive from the ease and safety associated with its use. Ultrasound is defined as acoustic waves with frequencies above those which can be detected by the ear, from about 20 kHz to several hundred MHz. Medical instrumentation typically uses only the portion of the ultrasound spectrum from 1 MHz to 10 MHz due to the combined needs of good resolution (small wavelength) and good penetrating ability (not too high a frequency). The waves are generated by small acoustic transducers, usually hand-held, that are electrically driven and placed on the surface of the skin. The waves propagate into the tissues of the body where a portion is reflected from the myriad of interfaces between tissue types of different acoustic properties. Some of these interfaces are abrupt, representing major organ boundaries, and some are more gradual.

In one common instrument configuration, the B-mode, the transducer is pulsed so that the reflected waves come as a series of various amplitude echoes whose arrival times after the transmitted pulse represent the depth of the reflecting boundary. In another configuration, known as Doppler velocimetry, the reflecting surfaces are moving (such as red blood cells in a pulsating blood vessel) and the reflected sound waves are shifted in frequency proportionally to the velocity of these scatterers. In the following chapters the principles of these instruments and more will be discussed in detail.

Since the sound waves are generated external to the body and no foreign substances need to be introduced into the body to interact with the waves, ultrasound is considered to be a *noninvasive* technique. This is opposed to other techniques, such as indwelling sensors placed in position surgically, which require piercing the body's outer surface. That there is no need for surgical intervention represents an important advantage of ultrasound. When coupled with the relatively straightforward electronic apparatus required for transmission and display, ultrasound is noted for its ease of use.

Ultrasonic imaging is often compared to radiography (imaging with X-rays) as to the effectiveness and accuracy of the images produced. Two major differences are evident. The first relates to the safety of the procedure. X-Radiation has a well-documented hazard associated with its use, depending upon the dosage required, which means that a judgment must be made as to whether the benefit obtained outweighs the risk. This is especially true for obstetrical and gynecological cases, as X-radiation is most harmful to cells in the dividing phase. Ultrasound, on the other hand, is generally considered safe, even for pregnancy scans, at the low power levels used in routine imaging. (At much higher levels, some cell damage occurs with ultrasound, and more study needs to be done on its long-term hazard at high powers.)

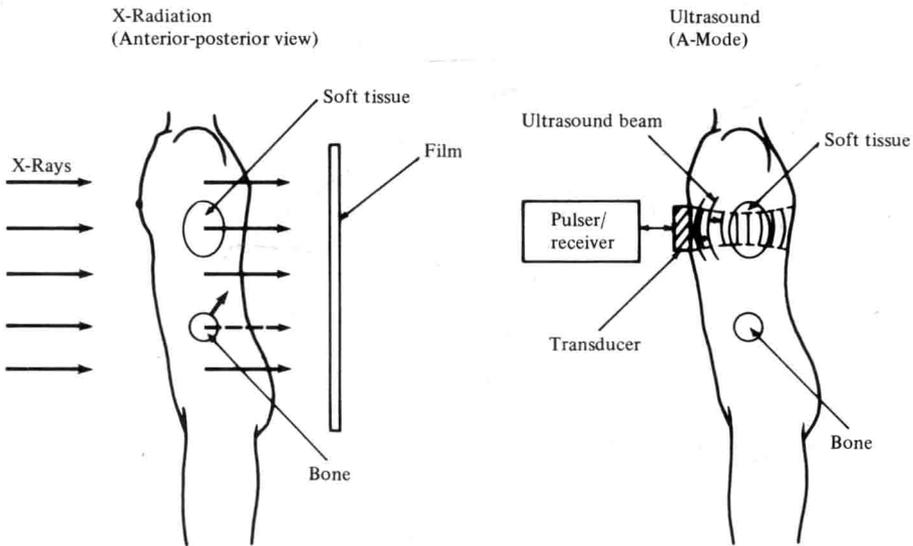
The other major difference between the two techniques relates to the characteristics of the tissue that is actually imaged by each. X-Radiation is attenuated (absorbed and scattered away from a straight-line path) in a power law relationship according to the density of electrons in the tissues encountered along the X-ray's path between the generator and the film, so a traditional X-ray view shows differences in tissue density as differences in exposure levels on the film. Because of this, dense tissue such as bone is delineated nicely from other less dense tissue such as muscle. But unfortunately, there is only a slight difference in X-ray attenuation between most types of soft tissue and therefore a radiographic view will not differentiate very well between various soft tissue types. For example, a soft tumor may be hard to see when it is located internal to a soft tissue organ, unless a radio-opaque contrast dye is invasively injected into the region.

Ultrasonic waves, on the other hand, are easy to launch through soft tissue and liquid, and, as mentioned earlier, they are partially reflected at interfaces between different soft tissue types. Thus, an ultrasound scan may be more sensitive to variations in soft tissue type than a radiograph, which is another reason why it is often used in obstetrical scans. It must be noted, however, that the advantage that ultrasound possesses in soft tissue scans is counterbalanced by the fact that ultrasound will not penetrate bony areas or air spaces readily, which makes it impractical for scanning the lungs (at least at the higher frequencies) and for imaging regions blocked by bone, such as the brain behind skull. These points are discussed in a later chapter.

When one considers the different physical tissue properties that are imaged by ultrasound as opposed to X-radiation, it can be seen that ultrasound and radiography should not be viewed as competitors. Rather, they are complementary procedures, since both may image regions of the body from different viewpoints, being sensitive to different tissue properties. What one technique misses, the other might expose. Bone fractures are best viewed with X-rays; pregnancy scans are best done with ultrasound; suspected abdominal tumors or heart valve abnormalities may call for both techniques. Figure 1.1 summarizes the unique characteristics of each modality.

### Summary of Chapters

The first portion of this text develops the mathematics of ultrasonic wave propagation and discusses some of the physical principles involved. In Chapter 2 the lossless wave equation is derived describing the propagation of the important compressional waves through a supporting medium. So-



#### ADVANTAGES

1. Excellent resolution
2. Distinguishes bone boundaries well

#### DISADVANTAGES

1. Hazard (esp. to dividing cells)
2. Won't differentiate soft tissues well

#### ADVANTAGES

1. Noninvasive, safe at low powers
2. Differentiates soft tissues

#### DISADVANTAGES

1. Resolution not as good as X-rays
2. Won't penetrate air or bone areas

**Figure 1.1** A comparison of imaging by X-radiation to ultrasonic imaging.

lutions to this differential equation are presented and are examined for wave behavior, including a discussion of the interaction of phase velocity, wavelength, and frequency. Chapter 3 examines the critical topic of reflection of acoustic waves at boundaries, and equations are derived to predict the amount of reflected and transmitted power at any given tissue interface.

Chapter 4 begins with a brief survey of tissue types found in the body. A quantitative description of the acoustic properties of these various tissue types is presented, allowing the numerical calculation of returned powers from a realistic model of an ultrasonic imager. At this point an improvement in the wave equation is made, adding terms which represent the loss mechanism in biological tissue and examination is then made of the effects of this ultrasonic attenuation in tissue.

Chapter 5 deals with the acoustic transducers that generate sound waves from an electrical signal, and vice versa. Particular attention is paid to their frequency dependence and to the resulting beam patterns. The important concept of spatial resolution (both axial and lateral) is derived here.

The next two chapters deal with the design of practical ultrasonic bioinstruments. Chapter 6 discusses the classification of imaging instruments into general categories depending upon display mode and transducer placement. Examples are given of each of the categories. Nonimaging uses of ultrasound in medicine are also introduced.

Chapter 7 describes the use of ultrasound to measure flowrates in the body. The chapter starts with a derivation of the important Doppler principle, then applies this to practical Doppler blood flowmeter design. Other ultrasonic flowmeters, such as the transit time flowmeter, are also analyzed.

The last chapter covers what is known today regarding the safety of ultrasound, and describes various techniques for measuring its exposure power levels.

The reader should pay particular attention throughout this study to a characteristic of ultrasonic instrument design that is found again and again throughout the entire realm of engineering design, namely, the trade-off that usually needs to be negotiated between conflicting instrument requirements. A good example of this is the tradeoff between the need for an adequate depth of penetration of the ultrasonic wave into the body and the requirement for good resolution in the resulting image. It will be seen that a compromise in the ultrasound frequency is needed here. Other examples of design optimization will be pointed out as the principles evolve.

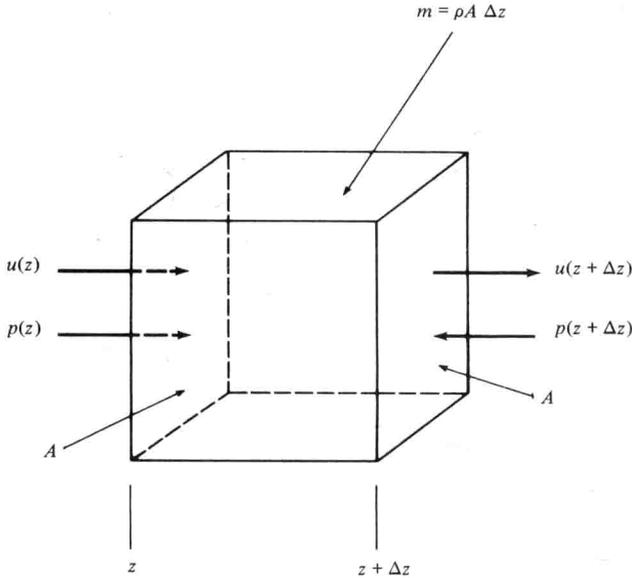
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# The Wave Equation and Its Solutions

### 2.1 DERIVATION OF THE WAVE EQUATION

Acoustic (sound) waves are merely the organized vibrations of the molecules or atoms of a medium that is able to support the propagation of these waves. When the frequency of the vibration is above the audible hearing range, the waves are known as ultrasonic radiation. As their frequency is increased, the wavelength of these waves gets progressively smaller, and this small size accounts for some of the unique resolution capabilities of ultrasound when compared to ordinary sound waves.

To develop the acoustic wave equation, consider an incremental volume element of the supporting material as shown in Figure 2.1. Assume that this volume size is small compared to the wavelength of the waves so variations of quantities throughout the volume are slight, but that is large enough to contain many molecules or atoms so the material can be considered to be made up of continuous “particles.” The length of the tiny volume along the longitudinal axis is  $\Delta z$ , and the area of the faces perpendicular to the longitudinal axis is  $A$ . Since the density of the material is given by  $\rho$ , the mass of this volume is given by  $m = \rho A \Delta z$ . Since the volume size is small compared to a wavelength,  $\rho$  undergoes only a small change throughout the volume and thus may be approximated here by a single quantity. There is a variation in the pressure in the longitudinal direction in the material, so that the excess pressure above static pressure at any  $z$  position is given by  $p(z)$ . In conjunction with the pressure field and coupled to it, there is a longitudinal velocity field  $u(z)$  representing the local motion of the particles of the medium.



**Figure 2.1** Pressure and velocities in an incremental volume of material.

It is important to note that by restricting both the particle velocity and the variation of this velocity to be solely in the longitudinal direction, we are considering here only so-called longitudinal waves, also known as compressional waves, and are ignoring initially any transverse motion which describes transverse, or shear, waves. We do this because compressional waves are the most important kind of ultrasound waves for biological purposes (shear waves damp out quickly in all tissue except bone), and an understanding of most of the wave principles can be obtained by analyzing the simpler compressional waves. Figure 2.2 shows that for compressional waves, the motion of the particles is in the same direction as the propagation of the wave, leading to adjacent areas of compression and rarefaction in the material (hence the name compressional waves), while for shear waves the particle motion is perpendicular to the wave propagation direction. Also, since the only variation considered for our compressional waves is in the  $z$  direction, the wave equation that results is known as a *one-dimensional* wave equation.

Newton's force equation can be applied to the volume element. In partial differential form using Eulerian coordinates,\*

\* Eulerian coordinates are assumed fixed in space such that the material particles flow past the coordinate positions, as opposed to Lagrangian coordinates, which follow the displacement of a specific particle.

$$F = m \frac{du}{dt} = m \left( \frac{\partial u}{\partial t} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial t} \right) \quad (2.1)$$

Since  $\partial z / \partial t = u$ , Equation (2.1) becomes

$$F = m \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right) \quad (2.2)$$

Since pressure is force per unit area, the net force on the volume is

$$F = [p(z) - p(z + \Delta z)]A \quad (2.3)$$

Substituting Equation (2.3) and the expression for mass into Equation (2.2) gives

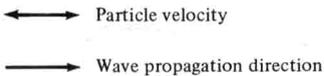
$$\frac{p(z) - p(z + \Delta z)}{\Delta z} = \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right) \quad (2.4)$$

Taking the limit as  $\Delta z \rightarrow 0$  gives the differential equation

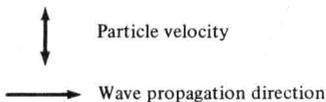
$$-\frac{\partial p}{\partial z} = \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right) \quad (2.5)$$

Now it is assumed (with good justification for the power levels normally encountered in imaging instruments; see Problem 3.5) that the *variation* in density  $\rho$  of the material due to the action of the waves is a very small

Compressional or longitudinal waves



Shear or transverse waves



**Figure 2.2** Directions of particle velocity and wave propagation for two types of ultrasonic waves, compressional and shear. Compressional waves have much less loss than shear waves in soft tissue and are the only type analyzed in the text.