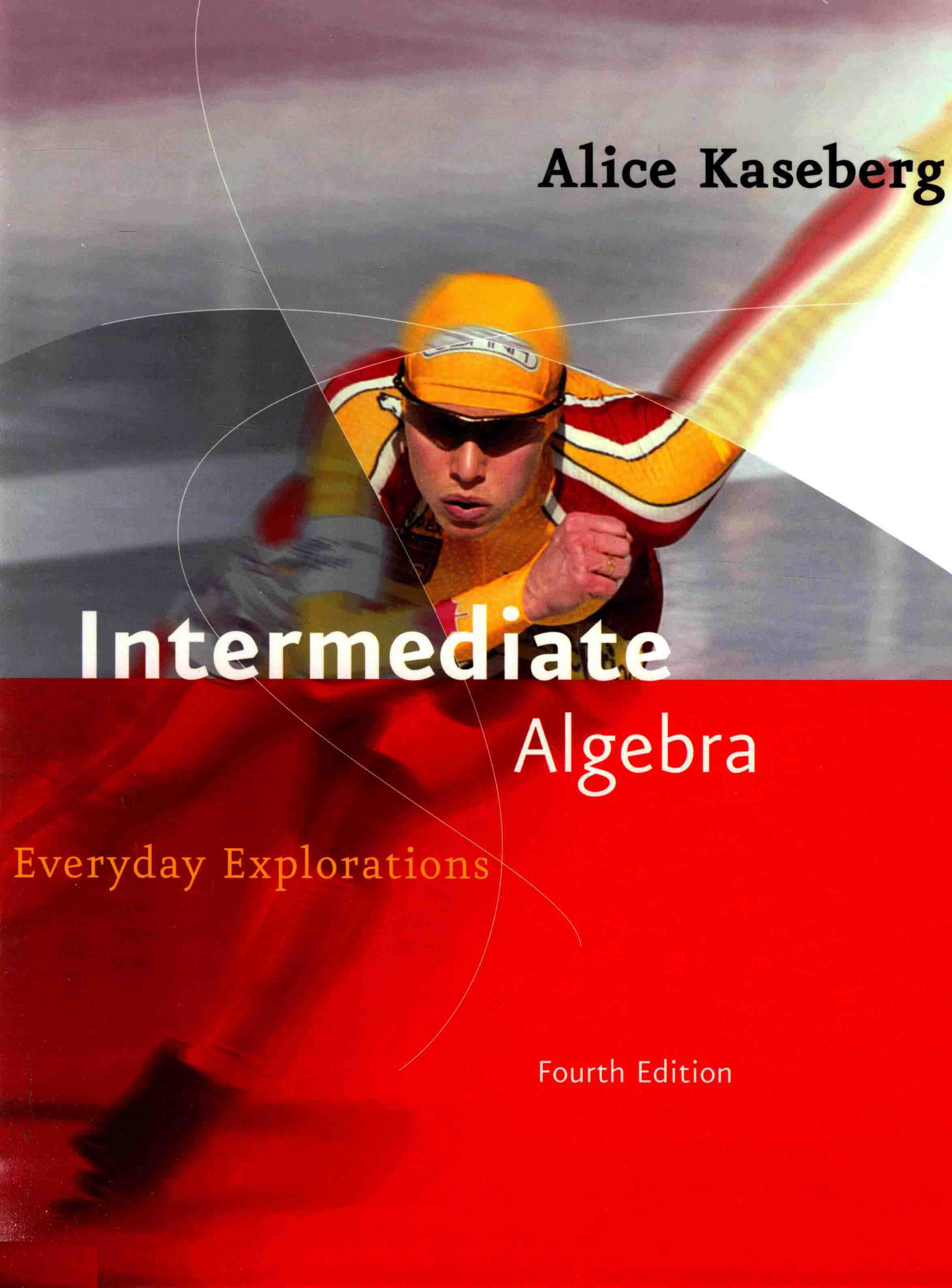

Alice Kaseberg

# Intermediate Algebra

## Everyday Explorations

Fourth Edition

# INTERMEDIATE ALGEBRA
## Everyday Explorations
### Fourth Edition

**ALICE KASEBERG**
formerly of Lane Community College

HOUGHTON MIFFLIN COMPANY    BOSTON    NEW YORK

**About the Cover**

When you hear familiar music, see a color you like, or recognize the name of a person, you focus on more of the accompanying information. My attention was caught when an Olympic Games announcer mentioned that a speed skater tilts 45 to 50 degrees off vertical when cornering. It is amazing that the tilt is more than halfway to horizontal. (Think about how many degrees are in a right angle.) Sometime, when you are standing on a soft surface, see how far from vertical you can lean before falling over. It might be safer to hang on to a rope and tilt 45 to 50 degrees.

Listening to details of sports events is an easy way to see and hear mathematics in the everyday world. If we start looking for mathematics relationships and patterns, we can see them everywhere.

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## How to Evaluate, Simplify, and Solve

The most common directions in algebra are *evaluate*, *simplify*, and *solve*.

**EVALUATE**

To find the value (usually numerical) of an expression.
To replace variables and constants with numbers in a formula or expression.
To replace the function variable with numbers or another expression.

**SIMPLIFY**

To change an expression, making it less complicated.

The meaning of *simplify* expands as you progress through the book.

CHAPTER 1

Do the operations shown, such as $+$, $-$, $\times$, and $\div$.
Do the operations shown, taking into account the order of operations.
Use the distributive property to eliminate parentheses from an expression.
Do the operations shown using number properties, such as the associative properties of $+$ and $\times$ or the commutative properties of $+$ and $\times$.
Eliminate common factors in a fraction, leaving the fraction in lowest terms.
Add (or subtract) like terms.
Carry out any of the above steps on the expressions on either side of an equation before solving the equation.

CHAPTER 2

Do the operations needed, after substituting numbers into an expression or function.

CHAPTER 4

Add (or subtract) like terms in a polynomial.
Arrange terms in descending order of exponents on whichever variable is first alphabetically.
Do the operations shown with power expressions, leaving no zero or negative exponents.
Do the operations shown with power expressions, using the properties of exponents.

CHAPTER 5

Find the square root of square factors under a radical sign.
Apply the product and quotient properties of square roots.
Change even square roots of negative numbers to the product of a real number and *i*.
Do the operations shown with complex numbers.

CHAPTER 6

Eliminate common units (feet, inches, etc.) from the numerator and denominator of a fraction.
Eliminate common factors in a ratio, leaving the ratio in lowest terms.
Eliminate common units from a unit analysis and do the operations.
Factor rational expressions and eliminate common factors.
Apply the properties of fractions to adding, subtracting, multiplying, and dividing rational expressions.

CHAPTER 7

Do the operations shown with power expressions, using the properties of exponents.
Do the operations shown in and with square root expressions.
Apply properties of square roots to variable expressions.
Rationalize the denominator of an expression.

CHAPTER 8

Use the properties of exponents on the definition of a logarithm.

**SOLVE**

To isolate the specified variable on just one side of an equation or inequality.
To find the common ordered pair in tables for the left and right sides of an equation.
To find the intersection of the graphs of the left and right sides of an equation.
To draw on a number line the solution set to a linear inequality in one variable.
To isolate the specified variable on just one side of a formula.
To find the values of both variables in a system of two equations.
To find the values of three variables in a system of three equations.
To draw on rectangular coordinate axes the solution set to a linear inequality in two variables.

# Index of Applications

## Agriculture & Animals



## Architecture & Construction



## Arts & Leisure



## Astronomy



## Biology



## Business & Economics



## Chemistry



## Engineering



*(continued)*

## Environmental Issues



## Food



## General & Physical Science



## General Interest



## Geometry



## Health & Medicine



## Household Maintenance



## Mathematics



*(continued on back endpaper)*

# Common Facts

## Time

60 seconds = 1 minute
24 hours = 1 day
365 days = 1 common year
366 days = 1 leap year
100 years = 1 century

60 minutes = 1 hour
30 days ≈ 1 month
$365\frac{1}{4}$ days ≈ 1 year
10 years = 1 decade
1000 years = 1 millennium

## Length

12 inches = 1 foot
16.5 feet = 1 rod
6 feet = 1 fathom
1 knot ≈ 6080 feet/hour

3 feet = 1 yard
660 feet = 1 furlong
5280 feet = 1 mile
1 knot = 1 nautical mile/hour

1000 millimeters = 1 meter
100 centimeters = 1 meter
1000 meters = 1 kilometer

$10^3$ millimeters = 1 meter
$10^2$ centimeters = 1 meter
$10^3$ meters = 1 kilometer

1 meter ≈ 39.37 inches

2.54 centimeters = 1 inch

## Area

1 square mile = 640 acres

1 hectare = 10,000 square meters

## Capacity (or Volume of Liquids)

16 tablespoons = 1 cup
2 cups = 1 pint
4 quarts = 1 gallon
8 fluid ounces = 1 cup

1 tablespoon = 3 teaspoons
2 pints = 1 quart
1 gallon = 231 cubic inches

1000 milliliters = 1 liter
1 liter ≈ 1.0567 quarts

$10^3$ milliliters = 1 liter

## Mass

16 ounces = 1 pound

2000 pounds = 1 ton

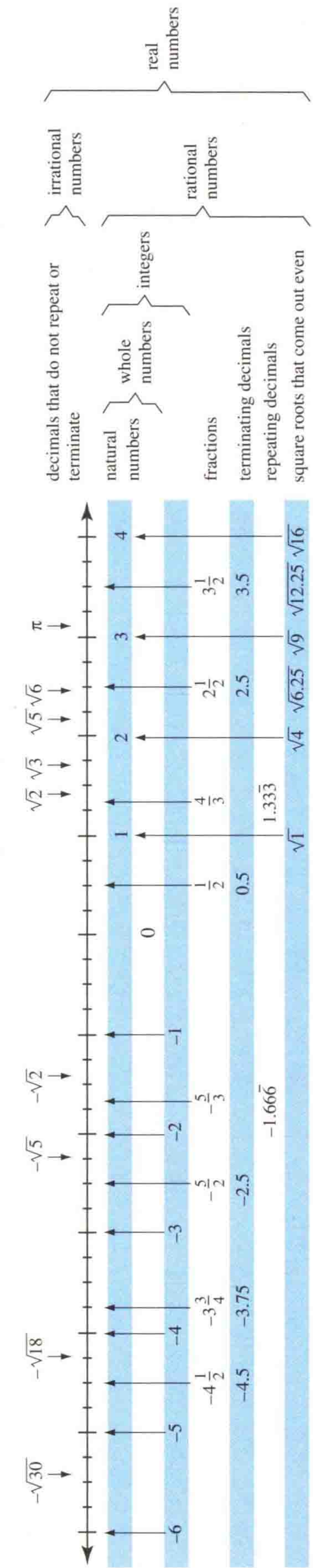
1000 grams = 1 kilogram
1000 milligrams = 1 gram
1000 kilograms = 1 metric ton

$10^3$ grams = 1 kilogram
$10^3$ milligrams = 1 gram

2.205 pounds = 1000 grams

# The Real Number Line

On the real number line, displayed to the right, it is not possible to show imaginary numbers *bi*.



# Symbols

| | |
|---|---|
| $a + b$ | addition of $a$ and $b$ |
| $\frac{a}{b}$ | division of $a$ by $b$ |
| $a \cdot b$, $a(b)$, $ab$, $(a)(b)$ | multiplication of $a$ and $b$ |
| $a - b$ | subtraction of $a$ and $b$ |
| $-3$ | negative 3 |
| $-b$ | opposite of $b$ |
| $\pm$ | plus or minus (add or subtract) |
| $+3$ | positive three |
| $\lvert\ \rvert$ | absolute value |
| { } | braces |
| [ ] | brackets |
| ( ) | parentheses |
| $\circ$ | circle on a graph: the point is excluded from the graph |
| $\bullet$ | dot or filled-in circle on a graph: the point is included in the graph |
| // | double slash on a graph: the spacing between the origin and the first number on the axis is different from the spacing between the other numbers |
| $-\infty$ | negative infinity |
| $+\infty$ | positive infinity |
| . . . | repeats or continues, as in a pattern of numbers |
| $\mathbb{R}$ | set of real numbers |
| $\varnothing$, { } | the empty set |
| $a \stackrel{?}{=} b$ | is $a$ equal to $b$? |
| $\approx$ | is approximately equal to |
| $=$ | is equal to |
| $>$ | is greater than |
| $\geq$ | is greater than or equal to |
| $<$ | is less than |
| $\leq$ | is less than or equal to |
| $\neq$ | is unequal to |
| $(a, b)$ | the open interval $a < x < b$ |
| $[a, b]$ | the closed interval $a \leq x \leq b$ |
| $(a, b]$ | the interval $a < x \leq b$ |
| $[a, b)$ | the interval $a \leq x < b$ |
| $(-\infty, b]$ | the interval $x \leq b$ |
| $(a, +\infty)$ | the interval $x > a$ |
| $b^n$ | base $b$ with exponent $n$ |
| $^2$ | square (exponent 2) |
| $^3$ | cube (exponent 3) |
| $\sqrt{\ }$ | principal square root, radical sign |
| $\sqrt[n]{a}$, $a^{1/n}$ | $n$th root of $a$, usually the principal $n$th root |
| $i$ $(j)$ | square root of $-1$, $\sqrt{-1}$ ($j$ is used in physics) |
| $e$ | base of the system of natural logarithms, approximately 2.71828 |
| $\ln a$ | natural logarithm of $a$, $\log_e a$ |
| $\log a$ | common logarithm of $a$, $\log_{10} a$ |
| $\log_b a$ | logarithm of $a$ with base $b$ |
| $^\circ$ | degree (temperature or angle) |
| $\Delta$ | delta: change |
| $f(x)$ | function of $x$ |
| % | percent |
| $\perp$ | perpendicular |
| $\pi$ | pi, approximately 3.14 |
| $a{:}b$ | ratio of $a$ to $b$ |
| ∟ | square corner at perpendicular lines |
| $x_1$ | variable $x$ with subscript 1 |
| $A^{-1}$ | inverse of the matrix $A$ |
| $I$ | an identity matrix |

# Functions and Sequences

Linear functions, $f(x) = ax + b$, correspond with arithmetic sequences and have common (constant) first differences.

Quadratic functions, $f(x) = ax^2 + bx + c$, correspond with quadratic sequences and have common (constant) second differences.

Exponential functions, $f(x) = ab^x$, correspond with geometric sequences and have both a multiple of the original sequence as a first difference and a common (constant) ratio of consecutive terms.

## Personal Finance



## Sports



## Transportation

## To my grandparents, for their belief in education

My paternal grandparents chose tuition over farm payments, sending their son to a university in the depth of the 1930s depression. My maternal grandfather, a widower and civil servant, sent all four daughters through that same university, where the youngest one met and married the farmers' son.

# Preface

## Why *Intermediate Algebra: Everyday Explorations*?

My purpose in writing Intermediate Algebra has been to present algebra with multiple representations within everyday applications using appropriate calculator operations.

*Intermediate Algebra: Everyday Explorations* is based on

- the premise that concept development and understanding of mathematical thinking are facilitated by number patterns, problem solving, exploration, and discovery,
- agreement with the standards advocated by organizations such as the American Mathematical Association of Two Year Colleges (AMATYC), the Mathematical Association of America (MAA), and the National Council of Teachers of Mathematics (NCTM),
- the motivation of everyday applications,
- the availability of technology and its considered use, and
- the mastery of certain basic skills.

## What Is the Everyday Exploration Approach?

Students order transcripts, use a photocopy machine, rent moving trucks, experience wind chill and humid days, and adjust thermostats in their homes. This is everyday mathematics. By exploring these and settings such as flower arrangements, roadbed transition curves, and planets discovered with Doppler spectroscopy, Intermediate Algebra encourages observation of mathematics outside the classroom while identifying the rules and equations that form the mathematical basis inside the classroom.

From one edition to the next, what never changes is the four-fold representation (numeric, visual, verbal, and symbolic) with problem solving. In the first chapter, the input-output tables for the everyday settings lay the ground work for multiple representations and, at the same time, lay a solid foundation on which to build the function concept.

## Why Number Patterns?

Because numbers are usually students' most comfortable part of mathematics, Intermediate Algebra has an emphasis on relating number patterns to visual, symbolic, and verbal representations. Sometimes we need to spend time with numbers to see patterns. My mother had little or no interest in mathematics until she made an embroidery multiplication table for my classroom. She discovered that the digits in the multiples of nine added up to nine: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108. She was a very bright woman but had just never looked.

## What Has Changed with the New Edition?

With each edition comes the challenge of what can be done to improve the book. As in the past reviewers and current users have contributed ideas. These include:

- Open the text with a problem-solving setting.
- Include additional data sets for linear, quadratic, and exponential regression.
- Format quadratic function material from three chapters to two chapters. Place optional material toward the end of each chapter.
- Expand the number of short historical comments about people, mathematical history, or mathematical notation within the reading or as margin boxes.
- Solve absolute value inequalities stressing three basic forms (equality, less than, greater than).
- Include cumulative reviews after each chapter.
- Write a graphing calculator appendix.

In addition, the fourth edition

- Includes 170 Instructor Extras in the margin to provide additional in-class group practice or a source for quizzes, review, or tests.
- References websites so students can learn more or check for most current data.
- Re-evaluated problem-solving and project exercises. Placed summary projects into the Chapter Review Exercises to encourage alternative assessment.
- Core exercises are clearly identified by blue numbers throughout the text, making them easy for instructors to assign to students (see Exercises 5–8 on page 5 for examples of core exercises). These exercises allow students new to algebra to practice skills as well as core material, and encourage students reviewing algebra to extend their knowledge with more challenging exercises.

## Effective Instruction

Think about a task or activity you fear or hate to do. How do you feel when thinking about it? Instructors have been successful at mathematics whereas community college students, as a rule, have not had a successful mathematics experience.

When I've presented a topic differently than how you learned it, I've tried to explain why to the student (and, of course, to instructors who read the text). Remember, we all learn in different ways. I've tried to include approaches that reach the visual or kinesthetic learner who cannot memorize procedure. I stress to all students the importance of learning multiple approaches. It is empowering.

## What if I Am a New Instructor?

Because many new instructors have little teaching experience and adjunct teachers have minimal time to prepare, the explorations in many sections suggest how to introduce a topic.

Whatever you do, share why you are excited about mathematics. You may be one of the many instructors inspired by fractals or finding the largest prime number of the Mersenne type. You may be inspired by mathematics and music or mathematics and art.

Here are the topics in the first 15 pages of the Instructor's Resource Manual:

**Preface**

Thriving with Change
Get Your Colleagues on Board
Expectations: You and Your Students

**Developmental Algebra**

Course Planning
Week-by-Week Course Plans

**Strategies for Teaching and Learning**

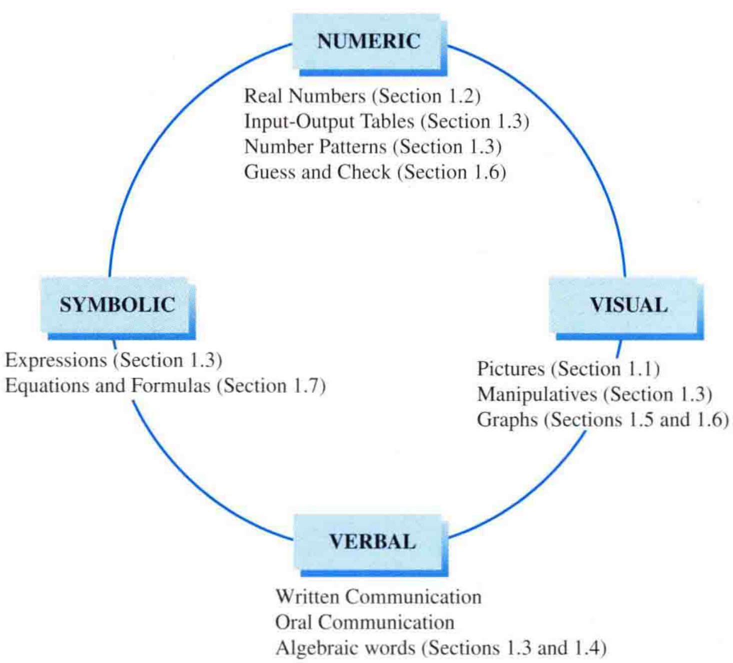
- Problem Solving
- Discovery Learning
- Four-fold Approach
- Vocabulary
- Planning Assignments
- Guided Reading
- Applications
- Projects
- Tests
- Testing and Assessment Strategies
- Learning from Tests
- Quizzes
- Portfolios
- Graphing Calculators

**Tips on Classroom Management**

- Be Prepared!
- Start Each Class with a Warm-up
- The First Class
- Taking Attendance and Returning Homework
- In Class
- Paying Attention to Your Timing
- Maintain a Positive Atmosphere
- Organizing Groups
- Getting Students Acquainted
- Monitoring Groups: Listen; Don't Interrupt
- Calculator Teaching Aids
- Questions on Homework
- Coaching on Homework

The remaining IRM is Section by Section Lessons, Extra Activities, and Tips by Topic. For a copy of the Instructor's Resource Manual, contact your local Houghton Mifflin sales representative.

Within the remainder of this Preface and in To the Student are additional elements for effective teaching and learning. Encourage your students to read their section at the times suggested within the material.



# Pedagogy

## Objectives

The learning outcomes for each section are listed at the beginning of the section. They serve as a summary for both students and instructors and coordinate with the titles on the examples.

> **Objectives**
> - *Identify and apply Polya's four steps for problem solving.*
> - *Identify conditions and assumptions in problem solving.*
> - *Apply strategies within problem-solving steps.*

## Warm-ups

The Warm-up at the beginning of each section is designed to serve as a class opener, reviewing important concepts, beginning exploration, and linking prior and upcoming topics. Warm-ups tend to be skill-oriented; they generally connect to the algebra needed to solve text examples. The answers to the Warm-up appear in the Answer Box at the end of the section.

> WARM-UP
>
> Explain why each of these multiplications can be done by reasoning. Do them.
>
> **1.** $1\frac{1}{2} \times 10$ **2.** $1\frac{1}{2} \times 16$ **3.** $\frac{2}{3} \times 9$ **4.** $\frac{3}{4} \times 24$
> **5.** $\frac{2}{3} \times 18$ **6.** $\frac{1}{2} \times 15$ **7.** $\frac{1}{2} \times 11$ **8.** $\frac{1}{2} \times 14\frac{1}{2}$

## Small-Group Work

Some sections contain introductory questions or activities. These are intended to be done in class in small groups. In Section 1.1, for example, the Example 1 Exploration leads into problem solving. The examples demonstrate how each student may contribute to the class and, in turn, learn from others. It is important to emphasize that students improve their own understanding by helping others.

> **EXAMPLE 1** **Exploration with Ikebana** Koshi has a flower-arranging business. In order to teach both efficient use of materials and the art of the arrangement, Koshi gives you, his new apprentice, a challenge: Choose the height of a "miniature" tall vase and find the lengths of the stems required to create a three-stem tall-vase Ikebana arrangement using a total of 12 inches of chenille stem.

## Problem Solving

George Polya's four-step approach to problem solving—understanding the problem, making a plan, carrying out the plan, and checking the solution—is introduced in Section 1.1 and revisited where appropriate. The text then focuses on planning strategies: *looking for a number pattern, making a table of inputs and outputs,* and *using manipulatives, finding number patterns, breaking mind set, making a graph, working backwards, choosing a test number or ordered pair and checking, guessing and checking,* and *making a systematic list.*

**HISTORICAL NOTE**
Polya earned his Ph.D. in mathematics in 1912, taught for 26 years at the Swiss Federal Institute of Technology, and left Europe in 1940 because "Hitler was too close." He retired in 1953, at age 65, after teaching at Stanford. Polya was a prolific writer and an active educator and speaker well into his 90s. *How to Solve It* has been published in 15 languages and is still in print.

### Problem Solving

The exploration suggests problem solving. As defined by the National Council of Teachers of Mathematics, **problem solving** is *engaging in a task for which the solution method is not known in advance.* We use problem-solving techniques thoughout this course. We start by dividing the exploration into four steps. This four-step approach was first published by George Polya in *How to Solve It* in 1945. Polya suggested that the elements of problem solving can be summarized as understanding the problem, making a plan, carrying out the plan, and then checking the solution.

PROBLEM-SOLVING STEP 1: UNDERSTAND THE PROBLEM. The first step in solving a problem is to understand the problem. Understanding a problem requires finding the conditions and then stating our assumptions. A **condition** is a *requirement or restriction stated within a problem setting.* An **assumption** is *something not stated but taken as a fact.*

## Explorations

Some examples are intended to be used in class for individual or group exploration. The solutions to these exploratory examples are included in the Answer Box at the end of the section.

**EXAMPLE 1** Exploring wind chill: reading a table and finding temperature change

**a.** Which two rows of Table 3 are shown in Table 4?
**b.** From Table 4, what is the wind-chill apparent temperature at 25°F with a wind of 30 miles per hour (mph)?

**TABLE 4** For $T = 25$°F Current Temperature

| Wind (mph), $S$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| Apparent Temp. $A$ (°F) | 25 | 22 | 10 | 2 | −3 | −7 | −10 |

**c.** By how much did the wind in part b change the temperature?
**d.** Which two rows of Table 3 are shown in Table 5?
**e.** From Table 5, what is the wind-chill apparent temperature at −5°F with a wind of 10 mph?

**TABLE 5** For $T = -5$°F Current Temperature

| Wind (mph), $S$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| Apparent Temp. $A$ (°F) | −5 | −10 | −27 | −38 | −46 | −52 | −56 |

**f.** By how much did the wind in part e change the temperature?

SOLUTION *Hint:* To find the change in temperature, subtract the starting temperature from the ending temperature. See the Answer Box.

## Hands-On Materials

The text supports use of an assortment of hands-on materials: algebra tiles for adding like terms; paper folding; and explorations with coin tossing. Many of the projects in the Exercises involve hands-on materials.

### Adding Like Terms

Many steps in algebra require that we see how expressions are alike and how they are different. To distinguish expressions such as $x + 2$, $2x$, and $x^2$, it may be helpful to associate them with physical objects such as algebra tiles. In Example 9, let $x$ be the side of the large square, let 1 be the side of the small square, and let $x$ and 1 be the sides of the rectangle.

**EXAMPLE 9** Finding area with algebra tiles

**a.** What is the area of each rectangle or square shown in Figure 9?
**b.** What is the total area of the figures?

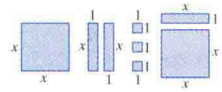



**FIGURE 9** Algebra tiles

## Examples

Each example begins with a title, which states the purpose of the example. Usually these titles relate back to the objectives for the section.

**EXAMPLE 3** Completing the square in an equation Find what must be added to each side of the quadratic equation $ax^2 + bx + c = 0$ to create a binomial square on the left. Draw a binomial square figure.

SOLUTION

| | | |
|---|---|---|
| | $ax^2 + bx + c = 0$ | We need $x^2 + \frac{b}{a}x$ to complete the square. |
| *Step 1:* | $ax^2 + bx = -c$ | Subtract $c$ from both sides. |
| *Step 2:* | $x^2 + \frac{b}{a}x = -\frac{c}{a}$ | Divide both sides by $a$. |

The completion of the square is shown in Figure 19.

| | | |
|---|---|---|
| *Step 3:* | $x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$ | Complete the square. Add the square of half the coefficient of $x$, $\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$ or $\left(\frac{b}{2a}\right)^2$, to both sides. |
| *Step 4:* | $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$ | Write the left side of step 3 as a binomial square. |

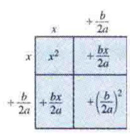



**FIGURE 19** Binomial square $(x + \frac{b}{2a})^2$

The dimensions of the square in Figure 19 appear on the left side of step 4.

## Think about it

"Think about it" questions are included within the reading material to encourage students to relate examples to prior material, to extend examples, and to practice verbalization skills. Answers to the questions are provided in the Answer Box.

LINEAR FUNCTION

A **linear function** can be written $f(x) = mx + b$. The constants $m$ and $b$ may be any real number. The variable, $x$, has 1 as its exponent.

## Sequences

Number patterns are used to identify and distinguish linear, quadratic, and exponential functions.

### 4.1 Exercises

What is the next number in each sequence in Exercises 1 and 2? Identify each as linear, quadratic, or neither.

1. a. 1, 3, 9, 27, 81, 243
   b. 11, 18, 25, 32, 39
   c. 56, 47, 38, 29, 20
   d. −1, 1, 7, 17, 31
2. a. 2, 3, 6, 11, 18
   b. −9, −7, −3, 3, 11
   c. −3, −8, −13, −18, −23
   d. 1, 2, 4, 8, 16

In Exercises 7 and 8, which equations are quadratic? Change to standard form with $a > 0$ and identify the coefficients $a$, $b$, and $c$.

7. a. $x^3 = 8$
   b. $x - 4 = x(x + 4)$
   c. $x - 2(x - 4) = x(x - 4)$
   d. $1 + x = x - 2(x + 1)$
8. a. $x(4 - x) = x + 4$
   b. $5 - 3(x - 3) = -x(x - 3)$
   c. $5 - x(x - 3) = -x(x + 3)$
   d. $3 - x^2(4 - x^2) = 3(4 - x)^2$

## Applications

To encourage creative thinking and depth in understanding, the text often poses a variety of questions about a single application setting. In addition, several applications, such as the cost of transcripts are repeated throughout the text so that students may observe the continuity and connections among topics.

**EXAMPLE 3** **Building a table and graph from an application** Suppose the first copy of a school transcript costs \$3 and each additional copy costs \$2.

a. Identify the independent (input) and dependent (output) variables.
b. Make a table and graph to show the total cost of ordering 1 to 5 transcripts.
c. Use the table and graph to find the cost of 3 transcripts.
d. Use the table and graph to solve for the number of transcripts that can be bought for \$9.

SOLUTION

a. The total cost depends on the number of transcripts ordered. The number of transcripts is the independent variable (on the horizontal axis). The total cost is the dependent variable (on the vertical axis).
b. The total costs are shown in Table 16 and Figure 12.

**TABLE 16** Transcript Cost

| Input: Transcripts Ordered | Output: Total Cost (dollars) |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |
| 5 | 11 |

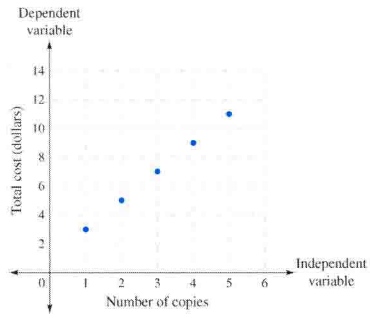



**FIGURE 12** Transcript cost

c. In Table 16, three transcripts match with \$7. In Figure 12, find 3 on the horizontal axis, and then look for the point on the graph where $x = 3$. Again, for 3 transcripts, the total cost is \$7.
d. In Table 16, the \$9 cost matches with 4 transcripts. In Figure 12, find \$9 on the vertical axis, and then look for the point on the graph where $y = 9$.