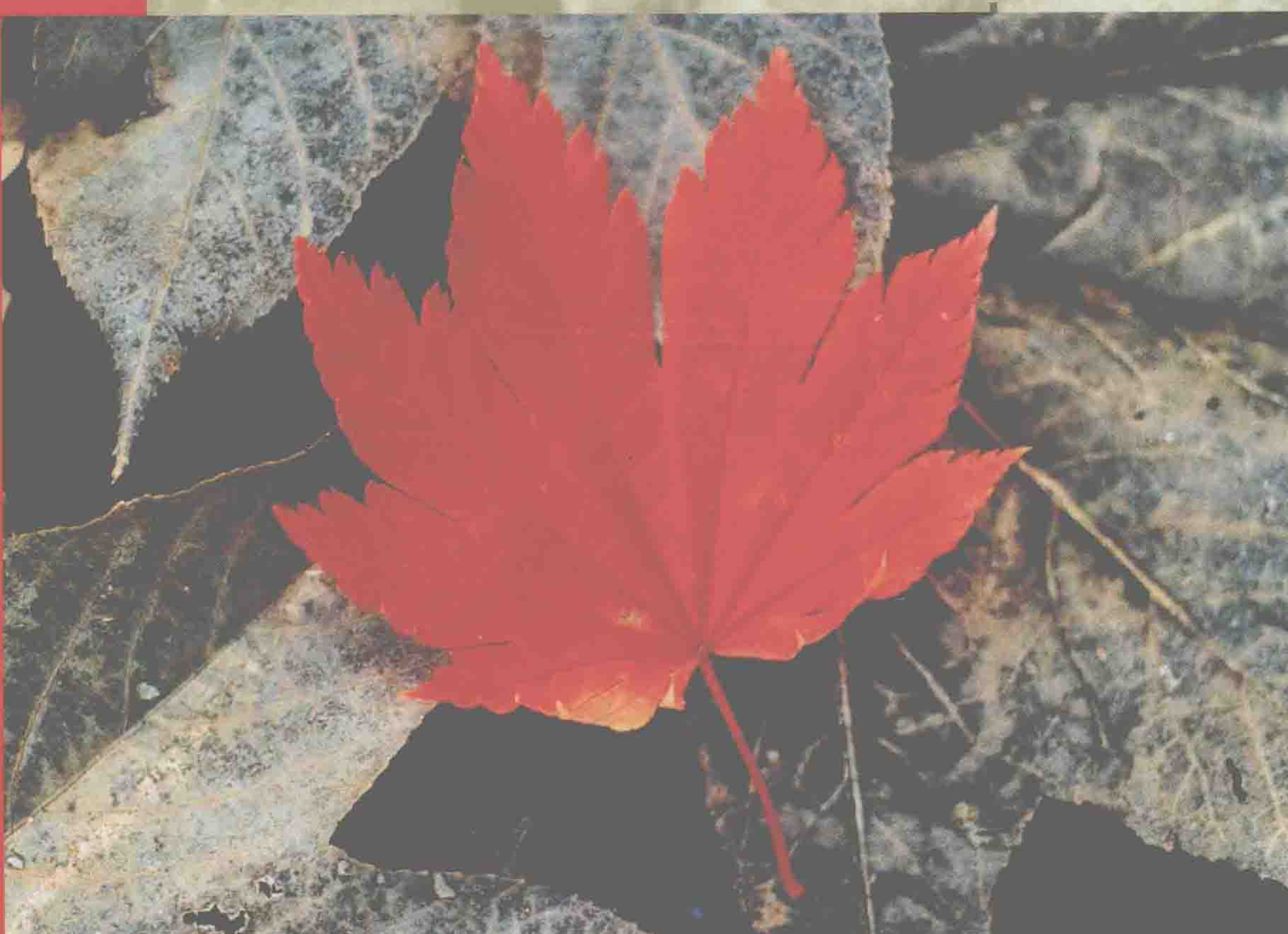
College Trigonometry

Aufmann | Barker | Nation



third edition

COLLEGE TRIGONOMETRY

THIRD EDITION

Richard N. Aufmann | Vernon C. Barker | Richard D. Nation

Palomar College

HOUGHTON MIFFLIN COMPANY Boston New York

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Printed in the U.S.A.

Library of Congress Catalog Card Number: 96-76858

ISBN: 0-395-78645-2

3456789-VH-00 99 98

College Trigonometry

PREFACE

Mathematics education continues to evolve at an ever-increasing rate. There is greater emphasis on *doing* mathematics rather than merely duplicating mathematics through extensive drill. Students are urged to investigate concepts, apply those concepts, and then present their findings. Technological advances in graphing calculators, computers, and software make it possible to explore the interdependence of mathematics and its application.

This third edition of *College Trigonometry* continues to build on the success of the second edition, enhancing the features of that edition and including new features that demonstrate the dynamic link between math concepts and math models. The special features include

- Topics for Discussion
- Projects
- Exploring Concepts with Technology

Topics for Discussion precede the exercise set in each section of the text. These topics can serve as group discussion or writing assignments.

Projects included at the end of each exercise set are designed to encourage students to research and write about mathematics and its applications. In the *Instructor's Resource Manual* there are additional projects that may be assigned. To ensure that projects are contemporary and in adequate supply, we have written additional projects that can be found on the Internet. Also included there are more modeling exercises arranged by subject area. These problems are updated once a semester and can be found at http://www.hmco.com.

A special end-of-chapter feature, **Exploring Concepts with Technology,** extends ideas introduced in the text by using technology to investigate applications or mathematical topics. These explorations can serve as group projects, class discussions, or extra-credit assignments.

Technology is introduced very naturally in the text to illustrate or enhance a concept. Our intention is to demonstrate appropriate technology when necessary to support a concept. We try to foster the idea that the concept motivates the use of technology, not to introduce technology for technology's sake. The technology icon (shown at this beginning of this paragraph) is used throughout the text to identify discussions of specific techniques for using graphing utilities, as well as examples and exercises that call for the use of technology. Optional exercises requiring the use of a graphing utility are designed to develop in students an appreciation for both the power and the limitations of technology. These exercises are supplemented by our new *Graphing Workbook,* which offers over 600 additional problems that may be worked using graphing utilities.

Despite the changes to this edition, we have retained our basic goal: to provide a comprehensive and mathematically sound treatment of the topics considered essential for a college algebra and trigonometry course. To help students master concepts, we have tried to maintain a balance among theory, application, modeling, and drill. Carefully developed mathematics is complemented by abundant, creative applications that are contemporary and represent a wide range of disciplines. Many application problems are accompanied by a diagram that helps the student visualize the mathematics involved.

The Features in Detail

Interactive Presentation *College Trigonometry* is written in a style that encourages the student to interact with the textbook. At various places throughout the text, for example, we pose a question to the student about the material being read. This question encourages the reader to pause and think about the current discussion and to frame an answer to the question. To make sure the student does not miss important information, the answer to the question is provided as a footnote on the same page.

Each section contains a variety of worked-out examples. Each example is given a descriptive title so the student can see at a glance what type of problem it illustrates. Many examples are accompanied by annotations that help the student follow the logic of the solution as it moves from step to step. After the worked-out example, we give the number of an exercise in that section's exercise set for the student to work. Such exercises are printed in red in the exercise set, and their *complete solutions* can be found in the Solutions to Selected Exercises section at the end of the text.

Worked-out examples include a descriptive title and annotations that help students follow the solution.

EXAMPLE 7 ***Solve a Construction Application***

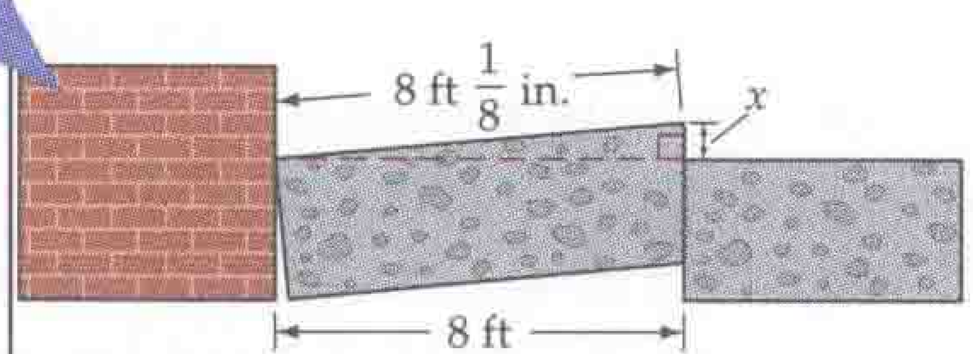


Figure 2.6

Concrete slabs often crack and buckle if proper expansion joints are not installed. Suppose a concrete slab expands as a result of an increase in temperature, as shown in **Figure 2.6.** Determine the height x, to the nearest inch, to which the concrete will rise as a consequence of this expansion.

Solution

Use the Pythagorean Theorem.

$$\left(8 \text{ feet} + \frac{1}{8} \text{ inch}\right)^2 = x^2 + (8 \text{ feet})^2$$

$$(96.125)^2 = x^2 + (96)^2 \quad \text{• Change units to inches.}$$

$$(96.125)^2 - (96)^2 = x^2$$

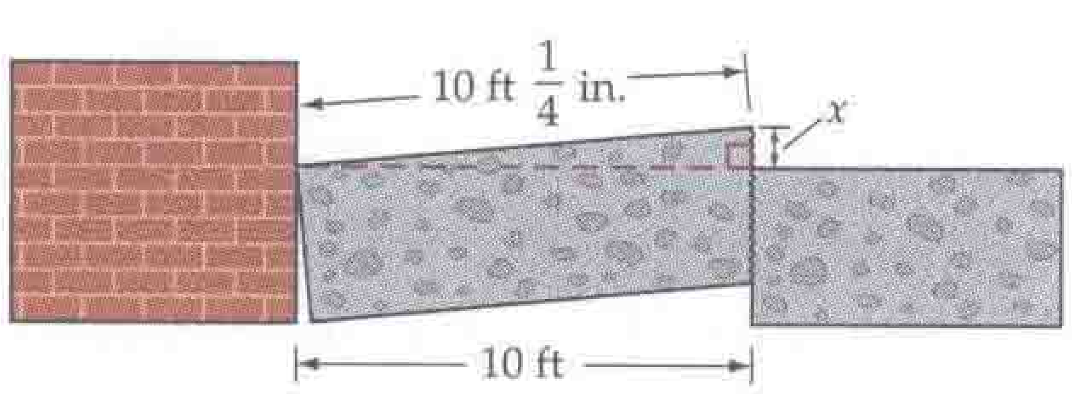
$$\sqrt{(96.125)^2 - (96)^2} = x \quad \text{• Only the positive root is taken because } x > 0.$$

$$4.9 \approx x$$

Thus, to the nearest inch, the concrete will rise 5 inches.

TRY EXERCISE 68, EXERCISE SET 2.3

68. CONSTRUCTION A concrete slab cracks and expands as a result of an increase in temperature, as shown in the following figure. Determine the height x, to the nearest inch, to which the concrete will rise as a consequence of this expansion.



Students are directed to try an exercise similar to the example.

68. $\left(10 \text{ feet} + \frac{1}{4} \text{ inch}\right)^2 = (10 \text{ feet})^2 + x^2$

$$\sqrt{(120.25)^2 - (120)^2} = x \quad \text{• Change feet to inches.}$$

$$7.75 = x$$

To the nearest inch, the concrete will rise 8 inches.

A complete solution to the exercise is given in the Solutions to Selected Exercises section at the end of the text.

Extensive Exercise Sets The exercise sets of *College Trigonometry* were carefully developed to provide the student with a variety of exercises. The exercises range from drill and practice to interesting challenges and were chosen to illustrate many facets of topics discussed in the text. Each exercise set emphasizes skill building, skill maintenance, and (as appropriate) applications. Included in each exercise set are Supplemental Exercises that incorporate material from previous chapters, offer extensions of topics, require data analysis, and present challenge problems or problems in the "prove or disprove" format. Four types of exercises are identified by icons:

Writing exercises
Graphing Utility exercises
Group exercises
Data Analysis exercises

Applications One way to stimulate student interest in mathematics is through applications. The applications in *College Trigonometry* have been taken from many disciplines, including agriculture, architecture, biology, business, chemistry, earth science, economics, engineering, medicine, and physics. Besides providing motivation to study mathematics, the applications help students develop good problem-solving skills.

Projects One of our goals in writing this text has been to involve the student with the text. As mentioned earlier, we do this through various pedagogical features, such as pausing in the course of developing a concept to ask the student to answer a pertinent question. *Projects* are another feature designed to engage the student in mathematics, this time through writing. The projects at the end of each section's exercise set provide guidelines for further investigations. Some projects ask the student to solve a more complex application problem. Others ask the student to write a proof of some statement. Still others invite the student to chronicle the procedure used to solve a problem and to suggest extensions to that problem. Many of these projects are ideal candidates for small-group assignments.

Exploring Concepts with Technology Calculators and computers have expanded the limits of the types of problems that students can solve. To take advantage of the new technologies, we have incorporated in each chapter some optional extensions of ideas presented in that chapter. These problems are not so much conceptually difficult as they are computationally messy. For each of these problems, we encourage the student to use a calculator or computer to investigate solutions. As the student progresses through a solution, we challenge the student to think about the pitfalls of computational solutions.

Changes for the Third Edition

Thanks to the helpful suggestions of our users and reviewers, we have made some changes to this edition that strengthen the text. We sincerely appreciate all suggestions we receive.

Essays and Projects from the previous edition have been renamed *Projects* and are now found at the end of every section rather than at the end of each chapter. We have included additional projects in the *Instructor's Resource Manual.* These projects and others can be found on the Internet at http://www.hmco.com. Supplemental application problems can also be found through this home page. These problems are updated once each semester.

Graphing technology is introduced and used as a natural outgrowth of content. Rather than present technology for its own sake, we introduce technology as it is needed and always to illustrate a concept. In this way, we hope to encourage students to see technology as an aid to better *understanding* rather than just a tool for obtaining answers. All graphing technology material is optional.

Topics for Discussion precede the exercise set in each section. These questions, which are conceptual rather than computational, can be used for group discussions or as writing exercises.

We have included two types of margin notes: *Take Note* and *Point of Interest. Point of Interest* notes contain interesting sidelights about mathematics, its history, or its application. Each *Take Note* alerts the student to a point that requires special attention.

Besides adding many contemporary application problems, we have made the following organizational and topical changes.

- Chapter 1 has been completely reorganized.
- Section 2.8 of Chapter 2 now includes damped harmonic motion.
- Chapter 6 has been extended and includes new sections on rotation of axes, polar equations of the conics, and parametric equations.
- The number and variety of applications in Chapter 7 have been increased.
- A Glossary that defines key terms has been added at the end of the text.

Supplements for the Instructor

College Trigonometry has an unusually comprehensive set of teaching aids for the instructor.

Solutions Manual The *Solutions Manual* contains worked-out solutions for all end-of-section, supplemental, and end-of-chapter exercises.

Projects on the Internet Projects like those at the end of each exercise set in the text can also be found on the Internet at http://www.hmco.com. Contemporary and motivating, these projects are updated every semester to reflect the relevance of mathematics to world events occurring right now. Furthermore, additional modeling exercises are added to take advantage of the latest web technology. These modeling exercises enable users to see the results immediately when they interact with the data and graphics.

Instructor's Resource Manual with Chapter Tests The *Instructor's Resource Manual* contains the printed testing program, which is the first of three sources of testing material available to the user. Six printed tests (in two formats: free response and multiple choice) are provided for each chapter. The *Instructor's Resource Manual* also includes additional projects that can be assigned as group activities or for extra credit. Moreover, there are suggestions for course sequencing, suggestions for incorporating graphing utilities, and outlines for solutions of the projects.

Computerized Test Generator with On-Line Testing The Computerized Test Generator is the second source of testing material. These questions are unique to the test generator and do not repeat items provided in the *Instructor's Resource Manual* testing program. The Test Generator is designed to produce an unlimited number of tests for each chapter of the text, including cumulative tests and final exams. It is available for the IBM PC and compatible computers and for the Macintosh. DOS and Windows versions also offer algorithms that can produce an unlimited number of some types of test questions and provide new **on-line testing** and **gradebook** functions.

Printed Test Bank The Printed Test Bank, the third component of the testing material, is a printout of all items in the Computerized Test Generator. Instructors employing the Test Generator can use the Test Bank to select specific items from the database. Instructors who do not have access to a computer can use the Test Bank to select items to include on tests prepared by hand.

Supplements for the Student

In addition to the *Student Study Guide*, the *Graphing Workbook*, the Math Assistant software, and the Houghton Mifflin Video Library, two computerized study aids accompany this text: The Review Tutor covers prerequisite material, and the Computer Tutor covers material in the text.

Student Study Guide with Internet Guide The *Student Study Guide* contains complete solutions to all odd-numbered exercises in the text as well as study tips and a practice test for each chapter. The *Internet Guide* explains the basics of using the Internet—such as how to gain access, how to search, and how to download information—and also provides a list of useful and interesting math sites.

Graphing Workbook The *Graphing Workbook* offers over 600 exercises that can be solved using a graphing utility. These exercises are designed to extend and explore such concepts as approximating roots of equations, translating graphs, and solving inequalities. Students may complete the exercises individually or in small groups.

Math Assistant This software is instructional and allows students to practice a skill, such as finding the inverse of a matrix, as well as to perform numerical calculations. In addition, the software includes a function grapher that graphs elementary functions and polar equations. The Math Assistant is available for the IBM PC, compatible microcomputers, and the Macintosh.

Houghton Mifflin Video Library These review videos contain 32 segments that cover the essential topics in this series. The videos, professionally produced specifically for the text, offer a valuable resource for further instruction and review.

Review Tutor The Review Tutor is a self-paced, interactive computer tutorial covering all necessary prerequisite material, such as would be found in an intermediate algebra course, that the student will need to know to progress successfully through this course. It is algorithmically based and includes color, animation, and free response. The algorithmic feature allows the Tutor to provide an unlimited number of practice problems. This tutorial is available for the Macintosh and for IBM PC and compatible computers running Windows.

Computer Tutor The *Computer Tutor*, an interactive computer tutorial covering every college algebra topic, has been completely revised. It is now an algorithmically based tutor that includes color and animation. The algorithmic feature allows the Tutor to provide an unlimited number of practice problems. A management system is also available to help instructors track student progress. The

Computer Tutor can be used in several ways: (1) to cover material the student missed because of absence from class, (2) to reinforce instruction on a concept the student has not yet mastered, or (3) to review material in preparation for examinations. This tutorial is available for the Macintosh and for the IBM PC and compatible computers running Windows. There is also a DOS tutorial available that covers every college algebra and college trigonometry topic.

Acknowledgments

The authors would like to thank the people who have reviewed this manuscript and provided many valuable suggestions:

Art Bukowski, *University of Alaska at Anchorage*
Richard Eells, *Roxbury Community College, MA*
Sudhir Goel, *Valdosta State University, GA*
JoAnne Kennedy, *LaGuardia Community College, NY*
Michael Longfritz, *Rensselaer Polytechnic Institute, NY*
Frank Mauz, *Honolulu Community College*
Susan Pfeifer, *Butler County Community College, KS*
Beverly Reed, *Kent State University, OH*
Mary Rice, *University of Nevada–Las Vegas*
Helen Salzberg, *Rhode Island College*
Ming Xue, *Massachusetts Institute of Technology*

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CHAPTER 1

Functions and Graphs

Stockbrokers display market trends using Cartesian graphs.

A line graph is a concise way to translate a large amount of raw data.

At a business meeting, different types of graphs prove useful in displaying a range of information.

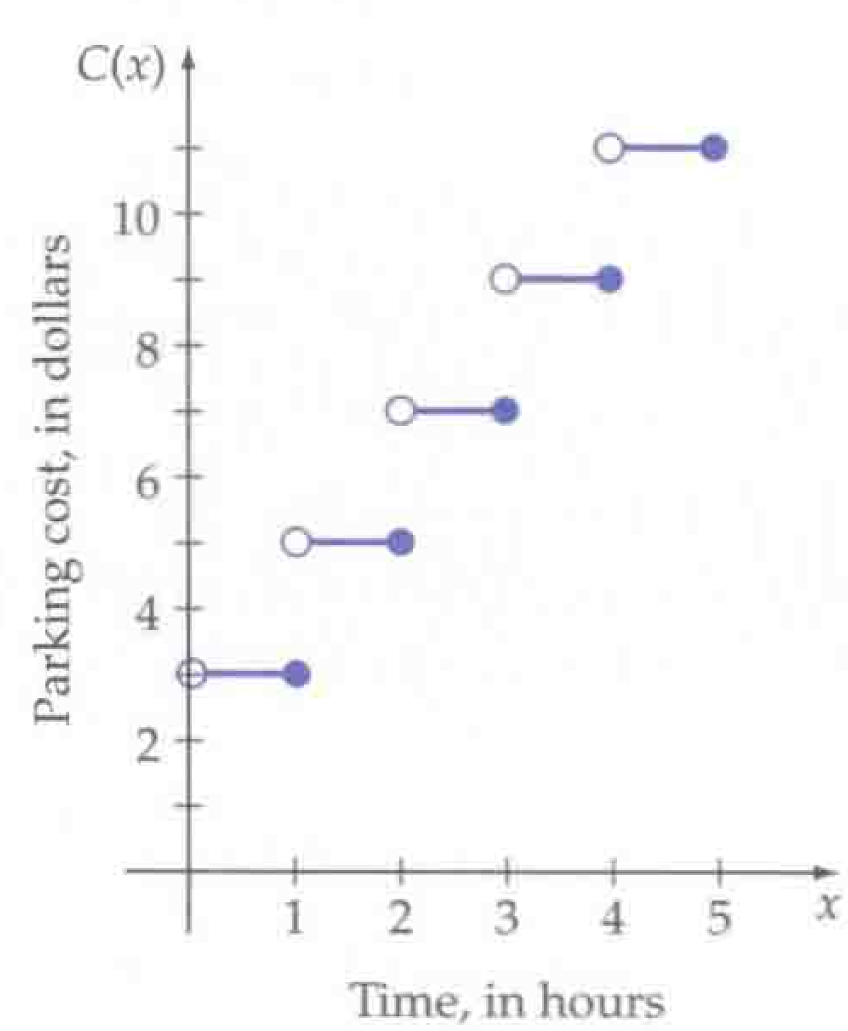
A Simple but Powerful Concept

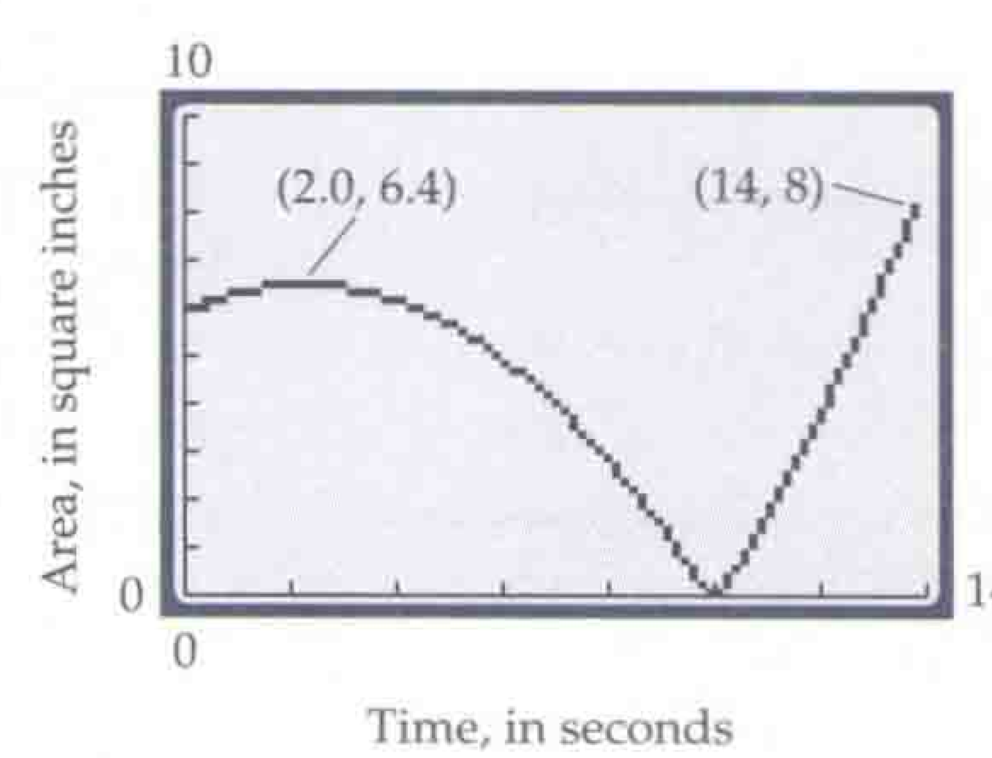
This chapter is concerned with the concepts of a function, the graph of a function, and elementary analytic geometry. The following two quotes lend support to the power of these ideas.

> *As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited. But when the sciences joined company, they drew from each other's vitality and thence forward marched on at a rapid pace toward perfection.*
>
> —*Joseph Louis Lagrange*

> *[Analytic geometry], far more than any of his metaphysical speculations, immortalized the name of Descartes, and constitutes the greatest single step ever made in the progress of the exact sciences.*
>
> —*John Stuart Mill*

Two types of graphs that are used in this chapter are shown below.





SECTION

1.1 EQUATIONS AND INEQUALITIES

THE REAL NUMBERS

POINT OF INTEREST

Archimedes (c. 287–212 B.C.) was the first to calculate π with any degree of precision. He was able to show that

$$3\frac{10}{71} < \pi < 3\frac{1}{7}$$

from which we get the approximation $3\frac{1}{7} \approx \pi$. The use of the symbol π for this quantity was introduced by Leonhard Euler (1707–1783) in 1739, approximately 2000 years after Archimedes.

The real numbers are used extensively in mathematics. The set of real numbers is quite comprehensive and contains several unique sets of numbers.

The **integers** are the set of numbers

$$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

Recall that the brace symbols, { }, are used to identify a set. The positive integers are called **natural numbers.**

The **rational numbers** are the set of numbers of the form a/b, where a and b are integers and $b \neq 0$. Thus the rational numbers include $-3/4$ and $5/2$. Because each integer can be expressed in the form a/b with denominator $b = 1$, the integers are included in the set of rational numbers. Every rational number can be written as either a terminating or a repeating decimal.

A number written in decimal form that does not repeat or terminate is called an **irrational number.** Some examples of irrational numbers are $0.141141114\ldots$, $\sqrt{2}$, and π. These numbers cannot be expressed as quotients of integers. The set of **real numbers** is the union of the sets of rational and irrational numbers.

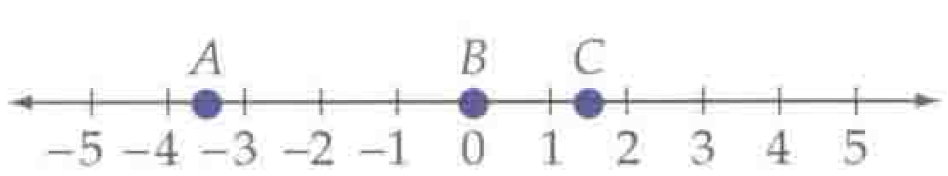


Figure 1.1

A real number can be represented geometrically on a **coordinate axis** called a **real number line.** Each point on this line is associated with a real number called the **coordinate** of the point. Conversely, each real number can be associated with a point on a real number line. In **Figure 1.1,** the coordinate of A is $-7/2$, the coordinate of B is 0, and the coordinate of C is $\sqrt{2}$.

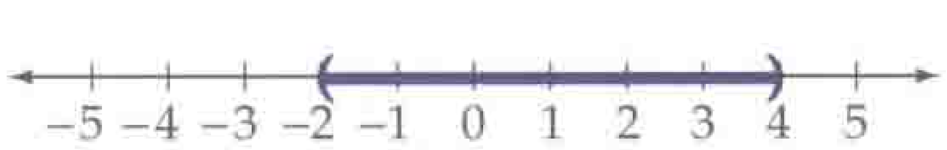
Given any two real numbers a and b, we say that a is **less than** b, denoted by $a < b$, if $a - b$ is a negative number. Similarly, we say that a is **greater than** b, denoted by $a > b$, if $a - b$ is a positive number. When a **equals** b, $a - b$ is zero. The symbols $<$ and $>$ are called **inequality symbols.** Two other inequality symbols, $\leq$ (less than or equal to) and $\geq$ (greater than or equal to) are also used.

The inequality symbols can be used to designate sets of real numbers. If $a < b$, the **interval notation** (a, b) is used to indicate the set of real numbers between a and b. This set of numbers can be described using **set-builder notation:**

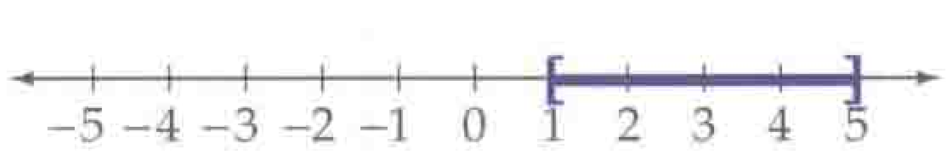
$$(a, b) = \{x \mid a < x < b\}$$

When reading a set written in set-builder notation, we read $\{x|$ as "the set of x such that." The expression that follows the vertical bar designates the elements in the set.

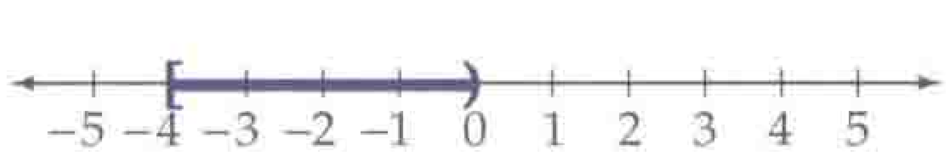
The set (a, b) is called an **open interval.** The graph of the open interval consists of all the points on the real number line between a and b, not including a and b. A **closed interval,** denoted by $[a, b]$, consists of all points between a and b, including a and b. We can also discuss **half-open intervals.** An example of each type of interval is shown in **Figure 1.2.**



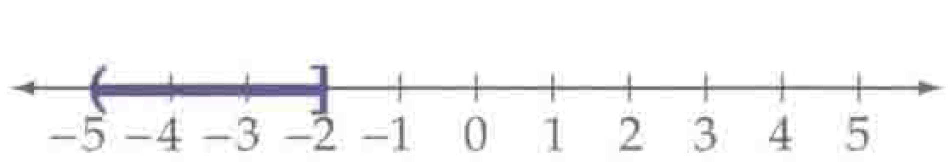
The open interval (−2, 4)



The closed interval [1, 5]



The half-open interval [−4, 0)



The half-open interval (−5, −2]

Figure 1.2