ENGINEERING MECHANICS

THIRD EDITION

FREE BODY DIAGRAM WORKBOOK
BY PETER SCHIAVONE
WORKING MODEL SIMULATION CD
BEDFORD AND FOWLER
PROBLEMS WEBSITE
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STATICS STUDY PACK

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Foreword

This Study Pack was designed to help students improve their study skills. It consists of three study components—a free body diagram workbook, a Visualization CD based on Working Model Software, and an access code to a website with over 500 sample Statics and Dynamics problems and solutions.

- Free Body Diagram Workbook—Prepared by Peter Schiavone of the University of Alberta. This workbook begins with a tutorial on free body diagrams and then includes 50 practice problems of progressing difficulty, with complete solutions. Further "strategies and tips" help students understand how to use the diagrams in solving the accompanying problems.
- Working Model CD—This CD contains pre-set simulations of text examples that include questions for further exploration. Simulations are powered by the Working Model Engine and were created with actual artwork from the text to enhance their correlation to the text. A set of questions associated with each simulation can be found in Appendix A of this workbook. The CD will install when inserted in your drive. Make sure to note and copy the case-sensitive serial number that appears. You will need it to complete the installation. During use, some systems may launch Microsoft Windows Media Player instead of Working Model. If this occurrs:

  1. Select "Start" and choose "Working Model 2D 5.0" from your program menu.
  2. After Working Model launches, select "File" and "Open" and explore your CD drive.
  3. Simulations for the text are located on the CD within the "WM Files" folder.

- Problems Website—Located at http://www.prenhall.com/bedford. This website contains 500 sample Statics and Dynamics problems for students to study. Problems are keyed to each chapter of the text and contain complete solutions. All problems are supplemental and do not appear in the third edition. Student access codes are printed on the inside cover of the Free Body Diagram Workbook. To access this site, go to http://www.prenhall.com/bedford, choose the link for the Problems Website, and follow the on-line directions to register. This site also contains an unprotected section with multiple choice and True/False check-up questions by Karim Nohra of the University of South Florida.
- On-Line Homework—http://www.prenhall.com/bedford also provides randomized homework problems. Your instructor may require you to use this feature of the site. The access code printed on the inside cover of this workbook provides access. Complete instructions are found at the site.

Preface

A thorough understanding of how to draw and use a free-body diagram is absolutely essential when solving problems in mechanics.

This workbook consists mainly of a collection of problems intended to give the student practice in drawing and using free-body diagrams when solving problems in Statics.

All the problems are presented as tutorial problems with the solution only partially complete. The student is then expected to complete the solution by "filling in the blanks" in the spaces provided. This gives the student the opportunity to build free-body diagrams in stages and extract the relevant information from them when formulating equilibrium equations. Earlier problems provide students with partially drawn free-body diagrams and lots of hints to complete the solution. Later problems are more advanced and are designed to challenge the student more. The complete solution to each problem can be found at the back of the page. The problems are chosen from two-dimensional theories of particle and rigid body mechanics. Once the ideas and concepts developed in these problems have been understood and practiced, the student will find that they can be extended in a relatively straightforward manner to accommodate the corresponding three-dimensional theories.

The book begins with a brief primer on free-body diagrams: where they fit into the general procedure of solving problems in mechanics and why they are so important. Next follows a few examples to illustrate ideas and then the workbook problems.

For best results, the student should read the primer and then, beginning with the simpler problems, try to complete and understand the solution to each of the subsequent problems. The student should avoid the temptation to immediately look at the completed solution over the page. This solution should be accessed only as a last resort (after the student has struggled to the point of giving up), or to check the student's own solution after the fact. The idea behind this is very simple:

We learn most when we **do** *the thing we are trying to learn.*

In other words, reading through someone else's solution is not the same as actually working through the problem. In the former, the student gains *information*, in the latter the student gains *knowledge*. For example, how many people learn to swim or drive a car by reading an instruction manual?

Consequently, since this book is based on **doing**, the student who persistently solves the problems in this book will ultimately gain a thorough, usable knowledge of how to draw and use free-body diagrams.

P. Schiavone

Contents

1

Basic Concepts in Statics

Statics is a branch of mechanics that deals with the study of objects in *equilibrium*. In everyday conversation, equilibrium means an *unchanging state* or a *state of balance*. Examples of objects in equilibrium include pieces of furniture sitting at rest in a room or a person standing stationary on the sidewalk. If a train travels at constant speed on a straight track, objects that are at rest relative to the train, such as a person standing in the aisle, are in equilibrium since they are not accelerating. If the train should start to increase or decrease its speed, however, the person standing in the aisle would no longer be in equilibrium and might lose his balance.

More precisely, we say that objects are in equilibrium only if they are at rest (if originally at rest) or move with constant velocity (if originally in motion). The velocity must be measured relative to a frame of reference in which Newton's laws are valid, which is called an **inertial reference frame**. In most engineering applications, the velocity can be measured relative to the earth.

In mechanics, real objects (e.g. planets, cars, planes, tables, crates, etc) are represented or *modeled* using certain idealizations which simplify application of the relevant theory. In this book we refer to only two such models:

- **Particle or Point in Space.** A *particle* has mass but no size/shape. When an object's size/shape can be neglected so that only its mass is relevant to the description of its motion, the object can be modeled as a particle. This is the same thing as saying that the motion of the object can be modeled as the motion of a *point in space* (the point itself representing the center of mass of the moving object). For example, the size of an aircraft is insignificant when compared to the size of the earth and therefore the aircraft can be modeled as a particle (or point in space) when studying its three-dimensional motion in space.
- **Rigid Body.** A *rigid body* represents the next level of modeling sophistication after the particle. That is, a rigid body is a collection of particles (which therefore has mass) which has a significant size/shape but this size/shape cannot change. In other words, when an object is modeled as a rigid body, we assume that any deformations (changes in shape) are relatively small and can be neglected. Although any object does deform as it moves, if its deformation is small, *you can approximate its motion by modeling it as a rigid body*. For example, the actual deformations occurring in most structures and machines are relatively small so that the rigid body assumption is suitable in these cases.

1.1 Equilibrium

1.1.1 Equilibrium of an Object Modeled as a Particle

An object is in equilibrium provided it is at rest if originally at rest or has a constant velocity if originally in motion. To maintain equilibrium of an object modeled as a particle, it is necessary and sufficient to satisfy Newton's first law of motion which requires the resultant force acting on the object (or, more precisely, the object's mass center) to be zero. In other words

$$\sum \mathbf{F} = \mathbf{0} \tag{1.1}$$

where $\sum \mathbf{F}$ is the vector sum of all the external forces acting on the object.

Successful application of the equilibrium equation (1.1) requires a complete specification of all the known and unknown external forces ($\sum \mathbf{F}$) that act on the object. The best way to account for these is to draw the object's *free-body diagram*.

1.1.2 Equilibrium of an Object Modeled as a Rigid Body

An object modeled as a particle is assumed to have no shape. Hence only external *forces* enter into the equilibrium equation (1.1).On the other hand, an object modeled as a rigid body is assumed to have mass *and* (unchanging) shape. Hence, both forces and moments need to be taken into account when writing down the corresponding equilibrium equations. In fact, an object modeled as a rigid body will be in equilibrium provided the sum of all the external forces acting on the object is equal to zero *and* the sum of the external moments taken about any point is equal to zero. In other words:

$$\sum \mathbf{F} = \mathbf{0} \tag{1.2}$$

$$\sum \mathbf{M}_O = \mathbf{0} \tag{1.3}$$

where $\sum \mathbf{F}$ is the vector sum of all the external forces acting on the rigid body and $\sum \mathbf{M}_O$ is the sum of the external moments about an arbitrary point O.

Successful application of the equations of equilibrium (1.2) and (1.3) requires a complete specification of all the known and unknown external forces ($\sum \mathbf{F}$) and moments ($\sum \mathbf{M}_O$) that act on the object. The best way to account for these is again to draw the object's *free-body diagram*.

2

Free-Body Diagrams: the Basics

2.1 Free-Body Diagram: Object Modeled as a Particle

The equilibrium equation (1.1) is used to determine unknown forces acting on an object (modeled as a particle) in equilibrium. The first step in doing this is to draw the *free-body diagram* of the object to identify the external forces acting on it. The object's free-body diagram is simply a sketch of the object *freed* from its surroundings showing *all* the (external) forces that *act* on it. The diagram focuses your attention on the object of interest and helps you identify *all* the external forces acting. For example:

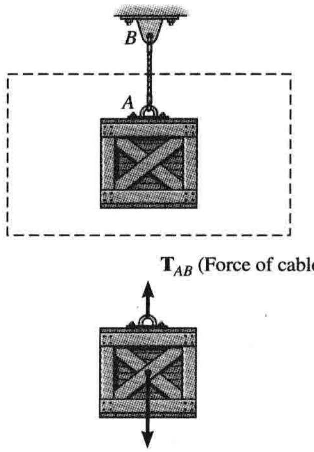


Figure 1

Note that once the crate is *separated* or *freed* from the system, forces which were previously internal to the system become external to the crate. For example, in Figure 1, such a force is the force of the cable AB *acting on the crate*.

Next, we present a formal procedure for drawing free-body diagrams for an object modeled as a particle.

2.1.1 Procedure for Drawing a Free-Body Diagram

1. *Identify the object you wish to isolate.* This choice is often dictated by the particular forces you wish to determine.
2. *Draw a sketch of the object isolated from its surroundings and show any relevant dimensions and angles.* Imagine the object to be isolated or cut free from the system of which it is a part. Your drawing should be reasonably accurate but it can omit irrelevant details.
3. *Show all external forces acting on the isolated object.* Indicate on this sketch *all* the external forces that act on the object. These forces can be *active forces*, which tend to set the object in motion, or they can be *reactive forces* which are the result of the constraints or supports that prevent motion. This stage is crucial: it may help to trace around the object's boundary, carefully noting each external force acting on it. Don't forget to include the weight of the object (unless it is being intentionally neglected).
4. *Identify and label each external force acting on the (isolated) object.* The forces that are known should be labeled with their known magnitudes and directions. Use letters to represent the magnitudes and arrows to represent the directions of forces that are unknown.
5. *The direction of a force having an unknown magnitude can be assumed.*

EXAMPLE 2.1

The crate in Figure 2 weighs 20lb. Draw free-body diagrams of the crate, the cord BD and the ring at B. Assume that the cords and the ring at B have negligible mass.

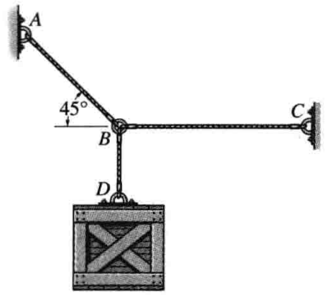


Figure 2

Solution

Free-Body Diagram for the Crate. Imagine the crate to be isolated from its surroundings, then, by inspection, there are only two external forces acting on the crate, namely, the weight with magnitude 20lb and the force of the cord BD.

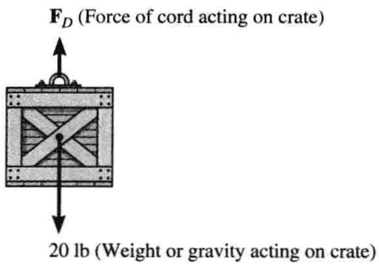


Figure 3

Free-Body Diagram for the Cord BD. Imagine the cord to be isolated from its surroundings, then, by inspection, there are only two external forces *acting on the cord*, namely, the force of the crate $\mathbf{F_D}$ and the force $\mathbf{F_B}$ caused by the ring. These forces both tend to pull on the cord so that the cord is in *tension*. Notice that $\mathbf{F_D}$ shown in this free-body diagram (Figure 4) is equal and opposite to that shown in Figure 3, a consequence of Newton's third law.

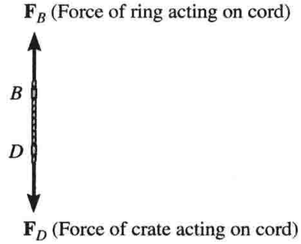


Figure 4

Free-Body Diagram for the ring at B Imagine the ring to be isolated from its surroundings, then, by inspection, there are actually three external forces acting on the ring, all caused by the attached cords. Notice that $\mathbf{F_B}$ shown in this free-body diagram (Figure 5) is equal and opposite to that shown in Figure 4, a consequence of Newton's third law. ◀

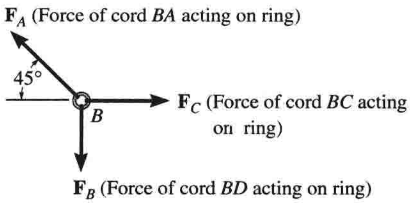


Figure 5

2.1.2 Using the Free-Body Diagram: Solving Equilibrium Problems

The free-body diagram is used to identify the unknown forces acting on the object when applying the equilibrium equation (1.1) to the object. The procedure for solving equilibrium problems is therefore as follows:

1. *Draw a free-body diagram*—you must choose an object to isolate that results in a free-body diagram including both known forces and forces you want to determine.
2. *Introduce a coordinate system* and establish the x, y-axes in any suitable orientation. Apply the equilibrium equation (1.1) in *component form* in each direction:

$$\sum F_x = 0 \text{ and } \sum F_y = 0 \tag{2.1}$$

   to obtain equations relating the known and unknown forces.
3. Components are positive if they are directed along a positive axis and negative if they are directed along a negative axis.
4. If more than two unknowns exist and the problem involves a spring, apply $F = ks$ to relate the magnitude F of the spring force to the deformation s of the spring (here, k is the spring constant).
5. If the solution yields a negative result, this indicates the sense of the force is the reverse of that shown/assumed on the free-body diagram.

EXAMPLE 2.2

In Example 2.1, the free-body diagrams established in Figures 3–5 give us a picture of all the information we need to apply the equilibrium equations (2.1) to find the various unknown forces. In fact, taking the positive x-direction to be horizontal ($\rightarrow +$) and the positive y-direction to be vertical ($\uparrow +$), the equilibrium equations (2.1) when applied to each of the objects (regarded as particles) are:

For the Crate: $\uparrow + \sum F_y = 0: \quad F_D - 20 = 0$ (See Figure 3)

$$F_D = 20 \text{ lb} \tag{2.2}$$

For the Cord BD: $\uparrow + \sum F_y = 0: \quad F_B - F_D = 0$ (See Figure 4)

$$F_B = F_D \tag{2.3}$$

For the Ring:

$$\uparrow + \sum F_y = 0: \quad F_A \sin 45^\circ - F_B = 0 \quad \text{(See Figure 5)} \tag{2.4}$$

$$\rightarrow + \sum F_x = 0: \quad F_C - F_A \cos 45^\circ = 0 \quad \text{(See Figure 5)} \tag{2.5}$$

Equations (2.2) - (2.5) are now 4 equations which can be solved for the 4 unknowns F_A, F_B, F_C and F_D. That is: $F_B = 20$ lb; $F_D = 20$ lb, $F_A = 28.28$ lb, $F_C = 20$ lb. These are the magnitudes of each of the forces $\mathbf{F_B}$, $\mathbf{F_D}$, $\mathbf{F_A}$ and $\mathbf{F_C}$, respectively. The corresponding directions of each of these forces is shown in the free-body diagrams above (Figures 3–5). ◄

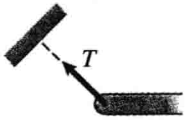
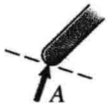
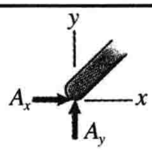
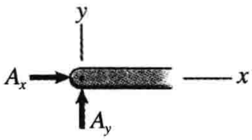
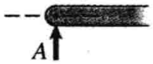
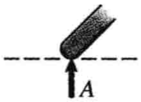
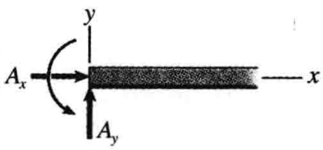
2.2 Free-Body Diagram: Object Modeled as a Rigid Body

The equilibrium equations (1.2) and (1.3) are used to determine unknown forces and moments acting on an object (modeled as a rigid body) in equilibrium. The first step in doing this is again to draw the *free-body diagram* of the object to identify *all of* the external forces and moments acting on it. The procedure for drawing a free-body diagram in this case is much the same as that for an object modeled as a particle with the main exception that now, because the object's "size/shape" is taken into account, it can support also external couple moments and moments of external forces.

2.2.1 Procedure for Drawing a Free-Body Diagram: Rigid Body

1. Imagine the body to be isolated or "cut free" from its constraints and connections and sketch its outlined shape.
2. Identify all the external forces and couple moments that act on the body. Those generally encountered are:
   (a) Applied loadings
   (b) Reactions occurring at the supports or at points of contact with other bodies (See Table 2.1)
   (c) The weight of the body (applied at the body's center of gravity G)
3. The forces and couple moments that are known should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and direction angles of forces and couple moments that are *unknown*. Establish an x, y-coordinate system so that these unknowns, for example, A_x, B_y etc can be identified. Indicate the dimensions of the body necessary for computing the moments of external forces. In particular, if a force or couple moment has a known line of action but unknown magnitude, the arrowhead which defines the sense of the vector can be assumed. The correctness of the assumed sense will become apparent after solving the equilibrium equations for the unknown magnitude. By definition, the magnitude of a vector is *always positive*, so that if the solution yields a *negative* scalar, the *minus sign* indicates that the vector's sense is *opposite* to that which was originally assumed.

Table 2.1. Supports used in Two-Dimensional Applications

Supports	**Reactions**
 Rope or Cable Spring	 One Collinear Force
 Contact with a Smooth Surface	 One Force Normal to the Supporting Surface
 Contact with a Rough Surface	 Two Force Components
 Pin Support	 Two Force Components
 Roller Support  Equivalents	 One Force Normal to the Supporting Surface
 Constrained Pin or Slider	 One Normal Force
 Built-in (Fixed) Support	 Two Force Components and One Couple

Important Points

- No equilibrium problem should be solved without first drawing the free-body diagram, so as to account for all the external forces and moments that act on the body.
- If a support *prevents translation* of a body in a particular direction, then the support exerts a force on the body to prevent translation in that direction.
- If *rotation is prevented* then the support exerts a couple moment on the body.
- Internal forces are never shown on the free-body diagram since they occur in equal but opposite collinear pairs and therefore cancel each other out.
- The weight of a body is an external force and its effect is shown as a single resultant force acting through the body's center of gravity G.
- *Couple moments* can be placed anywhere on the free-body diagram since they are *free vectors*. Forces can act at any point along their lines of action since they are *sliding vectors*.

EXAMPLE 2.3

Draw the free-body diagram of the beam of mass 10 kg. The beam is pin-connected at A and rocker-supported at B.

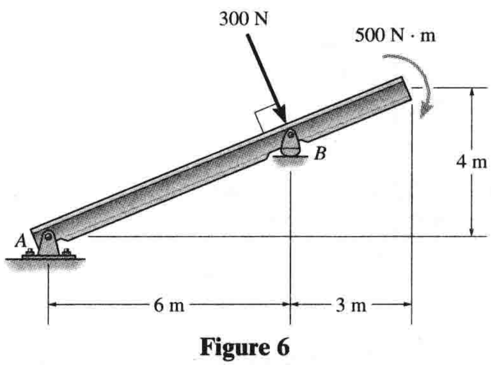


Figure 6

Solution

The free-body diagram of the beam is shown in Figure 7. From Table 2.1, since the support at A is a pin-connection, there are two reactions acting *on the beam at A* denoted by $\mathbf{A}_x$ and $\mathbf{A}_y$. In addition, there is one reaction *acting on the beam* at the rocker support at B. We denote this reaction by the force **F** which acts perpendicular to the surface at B, the point of contact (see Table 2.1). The magnitudes of these vectors are *unknown* and their sense has been *assumed* (the correctness of the assumed sense will become apparent after solving the equilibrium equations for the unknown magnitude i.e. if application of the equilibrium equations to the beam yields a negative result for the magnitude F, this indicates the sense of the force is the reverse of that shown/assumed on the free-body diagram). The weight of the beam acts through the beam's mass center. ◀

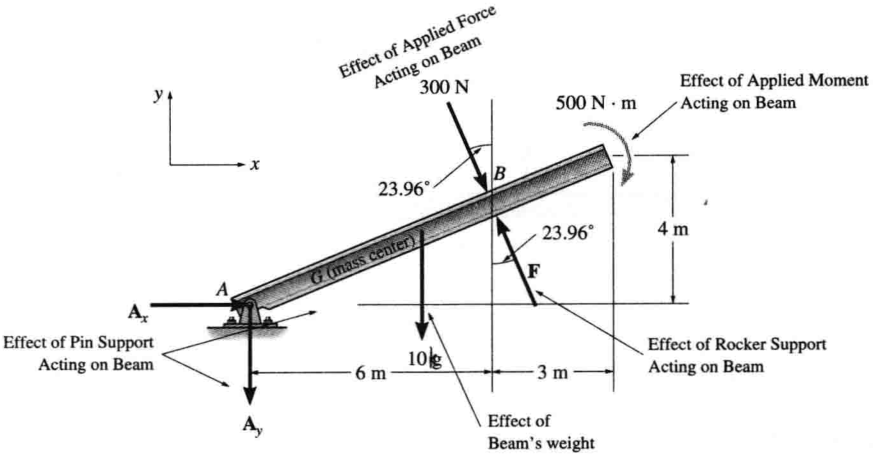


Figure 7

2.2.2 Using the Free-Body Diagram: Equilibrium

The equilibrium equations (1.2) and (1.3) can be written in component form as:

$$\sum F_x = 0, \tag{2.6}$$
$$\sum F_y = 0, \tag{2.7}$$
$$\sum M_O = 0, \tag{2.8}$$

Here, $\sum F_x$ and $\sum F_y$ represent, respectively, the algebraic sums of the x and y components of all the external forces acting on the body and $\sum M_O$ represents the algebraic sum of the couple moments and the moments of all the external force components about an axis perpendicular to the x–y plane and passing through the arbitrary point O, which may lie either on or off the body. The procedure for solving equilibrium problems for a rigid body once the free-body diagram for the body is established, is as follows:

- Apply the moment equation of equilibrium (2.8), about a point (O) that lies at the intersection of the lines of action of two unknown forces. In this way, the moments of these unknowns are zero about O and a direct solution for the third unknown can be determined.
- When applying the force equilibrium equations (2.6) and (2.7), orient the x and y-axes along lines that will provide the simplest resolution of the forces into their x and y components.
- If the solution of the equilibrium equations yields a negative scalar for a force or couple moment magnitude, this indicates that the sense is opposite to that which was assumed on the free-body diagram.

Example 2.4

The pipe assembly has a built-in support and is subjected to two forces and a couple moment as shown. Find the reactions at A.

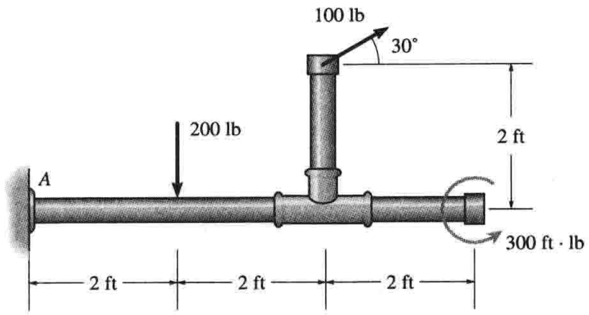


Figure 8

Solution

Free-Body Diagram The first thing to do is to draw the free-body diagram of the assembly in order to identify all the external forces and moments acting. We isolate the assembly from its built-in support at A (that way the reactions at A become external forces acting on the assembly). There are three unknown reactions at A: two force components A_x and A_y and a couple M_A (see Table 2.1). It might also be useful to resolve the applied 100 lb force into its components in anticipation of the application of the equilibrium equations (2.6)–(2.8). ◄

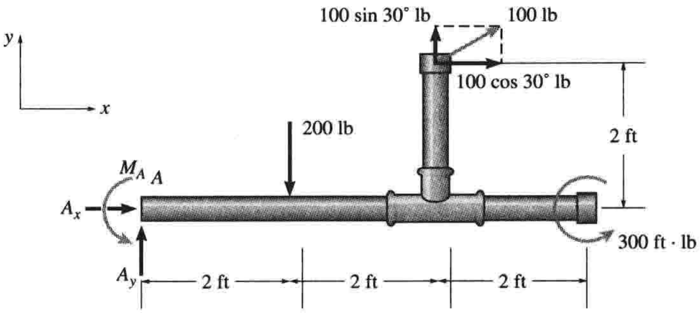


Figure 9

Equilibrium Equations The free-body diagram of the assembly suggests we can sum moments about the point A to eliminate the moment contribution of the reaction forces $\mathbf{A}_x$ and $\mathbf{A}_y$ acting on the beam. The equilibrium equations (2.6)–(2.8) are then:

$$\rightarrow +\sum F_x = 0: \quad A_x + 100\cos 30^\circ = 0$$
$$\uparrow +\sum F_y = 0: \quad A_y - 200 + 100\sin 30^\circ = 0$$

Taking counterclockwise as positive when computing moments, we have:

$$\sum M_A = 0: \quad M_A + 300 - (200)(2) - (100\cos 30^\circ)(2) + (100\sin 30^\circ)(4) = 0$$

(Notice that since the moment due to a couple is the same about any point, the moment about point A due to the 300 ft-lb counterclockwise couple is 300 ft-lb counterclockwise.) Solving these three equations we obtain the reaction components:

$A_x = -86.6$ lb,	$A_y = 150.0$ lb.

(Note that we have obtained a negative value for A_x which means that the sense or direction of the force $\mathbf{A}_x$ is opposite to that which was assumed on the free-body diagram.

3

Problems