Developments in Theoretical and Applied Mechanics

Volume 3

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Volume 3

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Conference on
Theoretical and Applied Mechanics

Sponsored by
University of South Carolina

Held in
Columbia, S. Carolina, March 31–April 1, 1966

Edited by

W. A. SHAW

Executive Chairman
J. David Waugh

THE QUEEN'S AWARD TO INDUSTRY 1966

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FOREWORD

THE Third Biennial Southeastern Conference on Theoretical and Applied Mechanics (SECTAM) furnished most gratifying evidence that the purposes for which it exists are being satisfied. An unprecedented number of excellent papers were submitted by both prominent and neophyte research men from throughout the United States—and, indeed, the world. The attendance was record breaking and represented a cross-section of mechanics activity.

These pages commit to the literature the formal program of the Third Conference, but the interchange of ideas, the stimulation of young research minds, and the friendships made or renewed, will exert a continuing influence for years to come.

To Professors Eric Reissner and Ian Sneddon, the guest speakers, I offer my personal thanks and that of the Conference for their participation. Their excellent papers and their attendance added appreciably to the Third Conference.

I should like to express my gratitude to Professor E. H. Harris of Tulane University and to Professor C. E. Stoneking, Georgia Institute of Technology, for their valued support in innumerable ways.

To the Editorial Committee, I extend my sincere appreciation for the outstanding accomplishment of a most difficult task.

The University of South Carolina was a most generous host to the Third Conference, both financially and with less tangible assistance. Auburn University is also due a thank you for their continuing support.

I alone shall know the very special thanks due to Professor Jan Boal, University of South Carolina, and particularly to Professor W. A. Shaw of Auburn University.

University of South Carolina
Columbia, South Carolina

J. DAVID WAUGH
Executive Chairman

PREFACE

THE Third Southeastern Conference on Theoretical and Applied Mechanics was held March 31–April 1, 1966, at the University of South Carolina in Columbia. The Executive Committee was composed of Chairman J. David Waugh, University of South Carolina; Vice-Chairman E. H. Harris, Tulane University; Secretary Jan L. Boal, University of South Carolina; Immediate-Past-Chairman C. E. Stoneking, Georgia Institute of Technology; and the Editor.

Forty papers were presented in the areas of continuum mechanics, elasticity, plates and shells, applied mechanics, experimental mechanics, wave propagation, dynamics, vibrations, and fluid mechanics, plus two guest papers. The invited lecturers were Dr. Eric Reissner of Massachusetts Institute of Technology and Dr. Ian Sneddon, University of Glasgow. These special lectures are included in the Proceedings.

There were approximately 200 registrants exclusive of graduate students and personnel from the University of South Carolina. These participants represented all areas of the United States, Canada, and several foreign countries.

The complete program follows:

THURSDAY, MARCH 31, 1966 MORNING SESSIONS

8.20–8.50 a.m. Opening Session

Continuum Mechanics

J. L. BOAL, University of South Carolina

" Time and Displacement Bound Theorems for Viscous and Rigid-Visco-Plastic Continua Subjected to Impulsive Loading "
JOHN B. MARTIN
Brown University

" Some Uniqueness and Extremum Principles for Rate-Type Materials "
ROBERT D. SNYDER
West Virginia University

" Approach to Inelasticity Through Dislocations and Extended Slab Analogy "
T. MURA, A. OTSUKA, W. S. FU and J. JAMES
Northwestern University

" Extended Theorems of Limit Analysis of Anisotropic Solids "
W. H. RIMAWI
University of Illinois
T. MURA and S. L. LEE
Northwestern University

Vibrations

GROVER ROGERS, Florida State University

" Solutions for the Optimization of Support Conditions of Hypercritical Shafts on Three Flexible Supports "
J. A. FRIEDERICY, Y. N. LIU and R. T. EPPINK
University of Virginia

" Similar Motion of *n*-Degree of Freedom Nonlinear Vibrating Systems "
KENNETH E. HAUGHTON
Systems Development Division, IBM

" An Engineer Attacks the Second-Order Linear Differential Equation "
FRANK M. WHITE
University of Rhode Island

" Longitudinal Vibrations of a Solid Propellant Rocket Motor "
PATRICIO A. LAURA and PAUL A. SHAHADY
The Catholic University of America

Experimental Mechanics

EDWARD BYARS, West Virginia University

" Stress Wave Propagation in a Half Plane due to a Transient Point Load "
J. W. DALLY and W. F. RILEY
Illinois Institute of Technology

" Theoretical and Experimental Analyses of Creep of Statically Indeterminate Portal Frames "
OMAR M. SIDEBOTTOM
University of Illinois
S. SOSRODININGRAT
Bandung Institute of Technology

" Objective Experimental Stress Analysis Using the Moiré Method "
BERNARD E. ROSS
University of South Florida

" An Experimental Method to Analyze Gravitational Stresses in Two-Dimensional Problems "
L. FERRER, V. J. PARKS and A. J. DURELLI
The Catholic University of America

THURSDAY, MARCH 31, 1966 AFTERNOON SESSIONS

Special Lecture. Dr. Ian Sneddon, University of Glasgow and North Carolina State University, " Crack Problems in the Theory of Elasticity "

Wave Propagation

RAY KINSLOW, Tennessee Technological University

" The Propagation and Reflection of Elastic Waves in Anisotropic Hollow Spheres and Cylinders "
W. B. BICKFORD
Arizona State University
W. E .WARREN
Sandia Laboratory

" Dynamic Response of an Infinite Cylinder to Asymmetric Pressure on its Lateral Surface "
C K. LIU
University of Alabama
T N. LEE
Northrop Space Laboratories

" Stress Wave Propagation in a Finite Viscoelastic Thin Rod w th a Constitutive Law of the Hereditary Type"
K. C VALANIS
Iowa State University of Science and Technology
S. CHANG
University of Alabama

" Initial Shear Stresses in the Viscoelastic Half-P ane"
M. ST PPES
University of Illinois
L. B. FREUND
Northwestern University

Plates and Shells

P. H. McDONALD, North Carolina State University

" Vibration and Buckling of Thermally Stressed Plates of Trapezoidal Planform "
CECIL D. BAILEY
Air Force Institute of Technology

" Axisymmetric Vibration of Hemispherical Shells "
JAMES TING-SHUN WANG
and
CHI-WEN LIN
Georgia Institute of Technology

" Plastic Buckling of Flat, Simply Supported, Rectangular Sandwich Panels of Orthotropic Core with Different Face Thicknesses "
C. C. CHANG
The Catholic University of America
I. K. EBCIOGLU
Universi y of Florida
J. J. BALTES
Minneapolis-Honeywell

" Limit Analysis of a Clamped Spherical Cap Using Linear Programming "
R. H. LANCE
Cornell University
DAVID W. RICKERT
IBM Development Laboratory

Fluid Mechanics I

T. S. CHANG, V.P.I.

" Further Investigation of Squeezing Flow between Parallel Plates "
J. F. THORPE
University of Kentucky

" The Laminar Boundary Layer on a Circular Cylinder in an Oscillatory Axial Flow "
KARL G. MAURER
North Dakota State University
YUN-SHENG YU
University of Kansas

" An Analysis of Axial Flow Through a Circular Channel Containing Rod Clusters "
T. C. MIN
F. N. PEEBLES
University of Tennessee
H. W. HOFFMAN
T. C. TUCKER
Oak Ridge National Laboratory

" Standing Gravity Waves of Finite Amplitude "
LAWRENCE R. MACK
BENNY E. JAY
University of Texas
DONALD F. SATTLER
Ling-Temco-Vought

FRIDAY, APRIL 1, 1966 MORNING SESSIONS

Shells

S. L. DELEEUW, University of Mississippi

" The Deformation of Thin Shells "
GERALD A. WEMPNER
University of Alabama

" An Exact Formulation of the Linear Equation for Thick, Orthotropic Shells with Arbitrary, Imposed Temperature and Force Fields and Temperature-Dependent Parameters "
J. F. SCHIPPER
Aerojet General Corporation

" Membrane Theory for a Hemispherical Dome Subjected to a Wind Load "
BERT H. GARCIA, JR.
North Carolina State University
DANIEL FREDERICK
V.P.I.

Dynamics

FURMAN BARTON, Duke University

" First-Order Secular Perturbations of an Artificial Earth Satellite Due to the Sun and Moon "
C. C. DEARMAN, JR.
Marshall Flight Center, NASA

" Dynamics of an Annular Disk Rolling on its Inner Rim on a Circular Cylinder "
RONALD L. HUSTON
University of Cincinnati

" Kinematics of a Three-Axis Gimbal System "
KENNETH G. MCCONNELL
Iowa State University

" Dynamics of Elastically Connected Rigid Bodies "
WILLIAM WEAVER, JR.
Stanford University

Fluid Mechanics II

DICK LYON, Oak Ridge National Laboratory

" Generation and Propagation of Pressure Waves in Two-Phase Flow"
DON J. WOOD
T. Y. KAO
Duke University

" Nonlinear Propellant Sloshing in a Rectangular Container of Infinite Length "
HELMUT F. BAUER
Georgia Tech.

" The Effect of Permeability on Low Reynolds Number Flow Past a Circular Porous Cylinder "
YUN-YUAN SHI
Douglas Aircraft Company, Inc.
ROY E. BRADEN, JR.
Carnegie Institute of Technology

Special Lecture. Dr. Eric Reissner, MIT, " On the Nonlinear Theory of Thin Plates "

FRIDAY, APRIL 1, 1966 AFTERNOON SESSIONS

Elasticity

GABRIEL HORVAY, G.E. Labs.
Schenectady, New York

" Further Results on Center of Dilatation and Residual Stresses in Joined Elastic Half-Spaces "
DAVID L. GUELL
University of Missouri
J. DUNDURS
Northwestern University

" Three-Dimensional Thermoelastic Problems of Planes of Discontinuities or Cracks in Solids "
M. K. KASSIR
Lehigh University
GEORGE C. SIH
California Institute of Technology

" Plane Elastostatic Analysis of an Infinite Plate with a Doubly Periodic Array of Holes or Rigid Inclusions "
HOWARD B. WILSON, JR.
JAMES L. HILL
University of Alabama

Applied Mechanics

E. H. HARRIS, Tulane University

" A Numerical Method for the Conformal Mapping of Finite Doubly Connected Regions "
MELVIN K. RICHARDSON
Clemson University
H. B. WILSON, JR.
University of Alabama

" Determination of Elastic Compliances of Cylindrically Aeolotropic Plates "
W. H. HOPPMANN, II
and
I. A. MINKARAH
Rensselaer Polytechnic Institute

" Experiments on Large Amplitude Parametric Vibration on Rectangular Plates "
JAMES H. SOMERSET
and
ROSS M. EVAN-IWANOWSKI
Syracuse University

The rapid growth of the Conference in number and quality of papers submitted and in attendance is most gratifying to all of us who participated in the first three Conferences.

I am sure all who attended the Third meeting join me in expressing appreciation to the entire community associated with the University of South Carolina for being such a gracious host and sponsor. Also, I should like to express my appreciation to P. H. McDonald, Robert L. Maxwell, David G. Thomas, Grover L. Rogers and Charles H. Parr, who so ably and willingly served as members of the Editorial Committee; and, in particular, to Mrs. Helen Martin, Editorial Assistant at Auburn University, who assumed so much of the responsibility of the Editor. Finally, I express my regrets to the many authors who submitted truly significant papers which we were unable to accept because of limitations on the size of the Conference.

W. A. SHAW
Editor

CONTENTS

TIME AND DISPLACEMENT BOUND THEOREMS FOR VISCOUS AND RIGID-VISCO-PLASTIC CONTINUA SUBJECTED TO IMPULSIVE LOADING

J. B. MARTIN
Brown University

ABSTRACT

Earlier work which developed a method of computing deformation time and final displacement bounds for rigid-plastic structures and continua subjected to impulsive loading is extended to cover all materials in which stress and strain rate are related in a reversible manner. Particular emphasis is given to rigid-visco-plastic materials and illustrative examples are presented.

NOTATION

σ_{ij}	Stress	
$\dot{\varepsilon}_{ij}$	Strain rate	
T_i	Surface tractions	
ν_i	Unit normal	
u_i, $\dot{u}_i$, $\ddot{u}_i$	Displacements, velocities and accelerations	
A	Area	
V	Volume	
t	Time	
L	Length	first example
P	Force	first example
M	Mass	first example
$\dot{y}$, v	Velocities	first example
δ	Displacement	first example
$\dot{\kappa}$	Curvature rate	second example
M	Bending moment	second example
R	Force	second example
r	Dimensionless force	second example
l, x	Length	second example
ϕ, ξ	Dimensionless length	second example
G	Concentrated mass	second example
$\dot{z}$	Velocity	second example
δ	Displacement	second example
M	Mass per unit length	second example
β	Mass ratio	second example

1. INTRODUCTION

In an earlier paper, Martin [1], theorems are developed which give deformation time and final displacement bounds for rigid-plastic structures or continua subjected to impulsive loading. Subsequently the displacement bound theorem is extended to cover elastic, elastic/ideally plastic and elastic/work hardening materials, Martin [2, 3]. In the development of these theorems the material is treated essentially as one in which stress and strain are related together either in a time independent one-to-one relation (elastic) or according to a time independent, path dependent law (plastic or work hardening).

It may be noted, however, that a rigid-plastic material may be regarded either as one in which there is a path dependent relation between stress and strain, or as one in which there is a reversible relation between stress and strain rate. This is illustrated in Fig. 1. Figure 1(a) shows the stress–strain

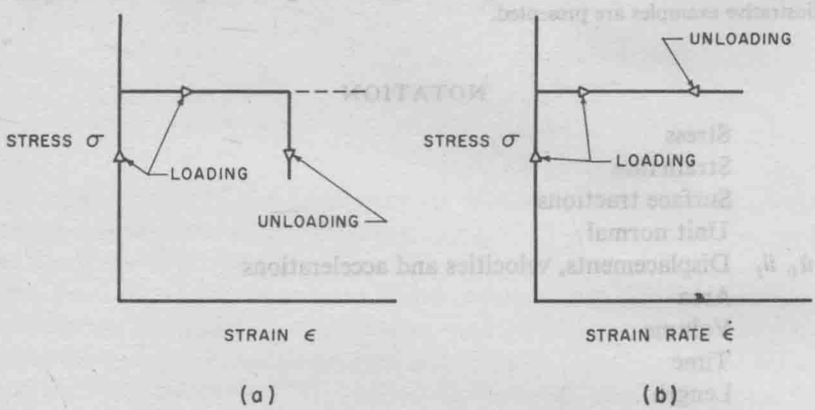

FIG. 1. Rigid-plastic material in simple tension.

relation for simple tension, and Fig. 1(b) shows that corresponding stress–strain rate relation. This phenomenon has stimulated interest in viscous materials and the behavior of viscous structures and continua subjected to impulsive loading. The term viscous is intended here to cover all reversible, path independent relations between stress and strain rate: a possible viscous stress–strain rate curve for simple tension is shown in Fig. 2. Loading and unloading proceeds along the same curve.

The purpose of this paper is to show that theorems given in [1] may be generalized to cover all viscous materials. Examples will be given to demonstrate their application. These theorems will have some technological application for rigid-visco-plastic (or rigid-viscous) materials. The stress–strain rate relation for such a material, which is a generalization of the

Bingham solid, is shown in Fig. 3. Rigid-visco-plastic constitutive relations are used by various authors (e.g. Bodner and Symonds [4], Ting and Symonds [5], Ting [6]) to simulate the dependence of yield stress on strain rate in steel and aluminium.

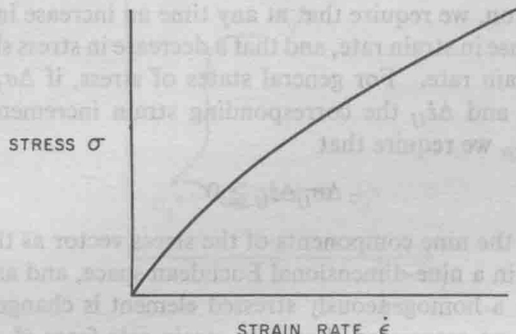

FIG. 2. Typical viscous material.

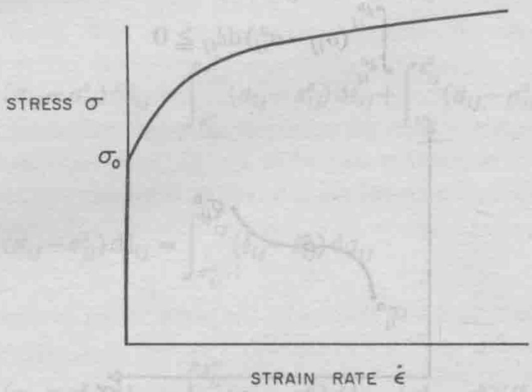

FIG. 3. Typical rigid-visco-plastic material.

In addition, viscous stress–strain rate relations which have no obvious practical application will be discussed. Such examples are presented as part of an attempt to understand more fully the mechanical behavior of impulsively loaded structures and continua of materials which obey simple idealized constitutive relations. The study of simple systems, even when the results are of doubtful practical value, is prerequisite to the study of more complex and more realistic materials.

Throughout this paper it is assumed that displacements are sufficiently small that the effects of changes of geometry on the equilibrium equations are negligible for the body under discussion. The limitations imposed by this assumption are discussed in the conclusions.

2. STABLE VISCOUS MATERIALS

The theorems given in [1], [2] and [3] follow from a stability restriction placed on the constitutive equations. An analogous restriction will be employed in this paper.

In simple tension, we require that at any time an increase in stress should produce an increase in strain rate, and that a decrease in stress should produce a decrease in strain rate. For general states of stress, if $\Delta\sigma_{ij}$ represents a stress increment and $\Delta\dot{\varepsilon}_{ij}$ the corresponding strain increment imposed on some state σ_{ij}, $\dot{\varepsilon}_{ij}$, we require that

$$\Delta\sigma_{ij}\Delta\dot{\varepsilon}_{ij} \geqq 0 \tag{1}$$

If we consider the nine components of the stress vector as the coordinates of a stress point in a nine-dimensional Euclidean space, and assume that the state of stress in a homogeneously stressed element is changed from σ_{ij}^a to σ_{ij}^b (Fig. 4), with an associated change in strain rate from $\dot{\varepsilon}_{ij}^a$ to $\dot{\varepsilon}_{ij}^b$, Eq. (1) may be written (Drucker [7]) as

$$\int_{\dot{\varepsilon}_{ij}^a}^{\dot{\varepsilon}_{ij}^b} (\sigma_{ij} - \sigma_{ij}^a)\,\mathrm{d}\dot{\varepsilon}_{ij} \geqq 0 \tag{2}$$

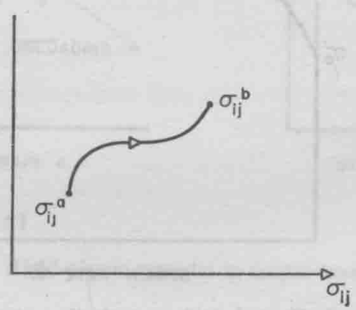

FIG. 4.

The reversibility and time independence of the material is given by the requirement that the integral on the left-hand side of Eq. (2) is independent of both the path (i.e. the locus of the stress point) from σ_{ij}^a to σ_{ij}^b and the stress rate or time taken to traverse the path. σ_{ij}^a in the integrand remains constant during integration along the path.

The same requirements are given by Hill [8] in a slightly different form. Hill requires that the functions

$$\int \sigma_{ij}\,\mathrm{d}\dot{\varepsilon}_{ij} \quad \text{and} \quad \int \dot{\varepsilon}_{ij}\,\mathrm{d}\sigma_{ij}$$

be convex. It may readily be shown that these postulates lead to uniqueness for standard boundary value problems, although the uniqueness may not be complete if Eq. (2) is equal to zero for $\dot{\varepsilon}^a_{ij}$ not identically equal to $\dot{\varepsilon}^b_{ij}$.

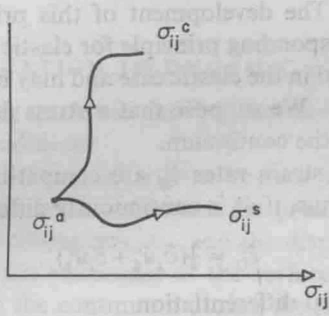

FIG. 5.

Consider now the path from σ^s_{ij} to σ^c_{ij} through σ^a_{ij} shown in Fig. 5. From Eq. (2),

$$\int_{\dot{\varepsilon}^s_{ij}}^{\dot{\varepsilon}^c_{ij}} (\sigma_{ij}-\sigma^s_{ij})\,\mathrm{d}\dot{\varepsilon}_{ij} = \int_{\dot{\varepsilon}^s_{ij}}^{\dot{\varepsilon}^a_{ij}} (\sigma_{ij}-\sigma^s_{ij})\,\mathrm{d}\dot{\varepsilon}_{ij} + \int_{\dot{\varepsilon}^a_{ij}}^{\dot{\varepsilon}^c_{ij}} (\sigma_{ij}-\sigma^s_{ij})\,\mathrm{d}\dot{\varepsilon}_{ij} \tag{3}$$

Now

$$\int_{\dot{\varepsilon}^s_{ij}}^{\dot{\varepsilon}^a_{ij}} (\sigma_{ij}-\sigma^s_{ij})\,\mathrm{d}\dot{\varepsilon}_{ij} = \int_{\sigma^a_{ij}}^{\sigma^s_{ij}} (\dot{\varepsilon}_{ij}-\dot{\varepsilon}^a_{ij})\,\mathrm{d}\sigma_{ij} \tag{4a}$$

and

$$\int_{\dot{\varepsilon}^a_{ij}}^{\dot{\varepsilon}^c_{ij}} (\sigma_{ij}-\sigma^s_{ij})\,\mathrm{d}\dot{\varepsilon}_{ij} = \int_{\dot{\varepsilon}^a_{ij}}^{\dot{\varepsilon}^c_{ij}} (\sigma_{ij}-\sigma^a_{ij})\,\mathrm{d}\dot{\varepsilon}_{ij} - (\sigma^s_{ij}-\sigma^a_{ij})(\dot{\varepsilon}^c_{ij}-\dot{\varepsilon}^a_{ij}) \tag{4b}$$

Substituting Eqs. (4a) and (4b) into (3),

$$\int_{\sigma^a_{ij}}^{\sigma^s_{ij}} (\dot{\varepsilon}_{ij}-\dot{\varepsilon}^a_{ij})\,\mathrm{d}\sigma_{ij} + \int_{\dot{\varepsilon}^a_{ij}}^{\dot{\varepsilon}^c_{ij}} (\sigma_{ij}-\sigma^a_{ij})\,\mathrm{d}\dot{\varepsilon}_{ij} \geqq (\sigma^s_{ij}-\sigma^a_{ij})(\dot{\varepsilon}^c_{ij}-\dot{\varepsilon}^a_{ij}) \tag{5}$$

The integrals in Eq. (5) are now in a form involving stress paths from σ^a_{ij} to σ^s_{ij} and σ^c_{ij} respectively. Further, the integrals for the two paths may be carried out independently of each other. Thus Fig. 5 may be interpreted as two independent stress paths from an initial state σ^a_{ij}. In the following section we shall use Eq. (5) to establish a minimum principle for viscous continua.