William Griffel

BEAM FORMULAS

BEAM FORMULAS

WILLIAM GRIFFEL

Mechanical Engineering Scientist
Department of the Army
Picatinny Arsenal

FREDERICK UNGAR PUBLISHING CO.
NEW YORK

Printed in the United States of America

Library of Congress Catalog Card Number: 68-31449

Standard Book Number: 8044-4338-6

BEAM FORMULAS

Other books by the author:

HANDBOOK OF FORMULAS FOR STRESS AND STRAIN

PLATE FORMULAS

SHELL FORMULAS (*in preparation*)

For my grandson

SCOTT WILLIAM GRIFFEL

PREFACE

The fast tempo of the competitive industrial scene today imposes many restrictions on the engineer. One prime restriction is time.

A valuable aid—particularly for the design engineer—when working against a project deadline, is the use of the most simplified, accurate, and least time-consuming beam formulas. The order of simplification should be such that a complex beam analysis can be reduced merely to an exercise in algebra. This is precisely the purpose of this volume.

Tabulated equations for beams are presented to supply the practicing engineer with ready-to-be-used formulas in their most simplified form. Examples are provided to show how to solve a beam with any combination of load and support using the formulas in this book.

My knowledge of the French, German, and Polish languages has allowed me to survey the field of the strength of materials—with the startling conclusion that the presentation of beam formulas has not changed for the last quarter century. Textbook equations for the deformation of beams are still cumbersome and, even more important, quite time consuming.

With simplicity as the predominant factor, I have rewritten the standard beam formulas using a unique nondimensional form which facilitates the evaluation of equations by slide rule. In addition, a table is provided in which nondimensional quantities are given numerically.

This volume, third in a series of four, contains 101 loading conditions for beams and rigid frames with various supports, each complete with load, shear, moment, and deflection diagrams. Using the formulas for fixed-end moments in this volume, the tabulated data could also be used for continuous beams or frames. In the case of an indeterminate analysis, data for end slopes and deflections can also be taken from the book.

To expose the nondimensional beam formulas to criticism, I have published them in *Product Design and Value Engineering*. The readers' reactions were most encouraging and some of their constructive suggestions were incorporated in the book.

A book on beam formulas would not be complete without chapters on simultaneous loading and beams on elastic foundations. Tabulated, simplified equations for beams under simultaneous axial and transverse loading appear in my first book, *Handbook of Formulas*. In this volume, the formulas are extended to include cases where the axial load is near zero.

The proper arrangement of supports of beams (if required) is of prime importance from the viewpoint of economy of material. This relates both to statically determinate and statically indeterminate beams. Section 5 deals with this subject.

In collating this material, I have had to rely on the work and contributions of others. It is my sincere hope that due acknowledgment has been made to the source of all material. To the publishers and others who have generously permitted the use of the material, I wish to express my appreciation. Most of the material of this volume is based on my 130 articles published from 1961 to 1969 in the following professional magazines: *Product Engineering*, *Design News*, *Machine Design*, *Engineering Materials and Design*, *Product Design and Value Engineering*, and *Design Engineering*. The opportunity thus afforded for criticism was of great advantage.

WILLIAM GRIFFEL

CONTENTS

Part I DEFINITIONS AND SYMBOLS	1
1 *Basic Principles and Definitions*	1
General	1
Stress	1
Tensile Properties	2
Compressive Properties	4
2 *Types of Failures*	5
General	5
Material Failures	5
3 *Symbols and Units*	8
Sign Convention	8
A Summary of Sign Conventions	10
Part II STRAIGHT BEAMS	13
4 *Tabulated Formulas*	13
Notations for Tables 1 and 2	14
Assumptions	16
Bending Moment and Shearing Force Diagrams	16
Fixed-End Moment	17
Continuous Beams	18
5 *Optimum Arrangement of Supports*	121
6 *Simultaneous Axial and Transverse Loads*	125
Notations for Table 3	126
7 *Beams on Elastic Foundation*	129
Introduction	129
Notations for Tables 4, 5, and 6	129
Classification of Beams According to Stiffness	130
Beams of Medium Length	131
Sign Convention	132
REFERENCES	165
INDEX	167

LIST OF TABLES

Table 1	Summary of Beams and Rigid Frames	21
	Straight Beams	21
	Rigid Frames	29
Table 2	Tabulation of Beam and Rigid-Frame Formulas	31
	Beam Formulas	31
	Shear, Moment, and Deflection (Cases 1-70)	31
	Rigid-Frame Formulas	101
	Frame Constants and Notations	101
	Rectangular Frames with Hinged Supports (Cases 71-80)	101
	Rectangular Frames with Fixed Supports (Cases 81-86)	107
	Single-Leg Frames (Cases 87-92)	113
	Trapezoidal Frames (Cases 93-98)	116
	Triangular Frames (Cases 99-101)	120
Table 3	Beams under Simultaneous Axial and Transverse Loads	127
Table 4	Equations for Medium-Length Beams	133
Table 5	Equations for Unlimited Length Beams	137
Table 6	Beams on Elastic Foundation (Cases 1-14)	139

Part I

DEFINITIONS AND SYMBOLS

1 BASIC PRINCIPLES AND DEFINITIONS*

General

It is assumed that engineers using this volume are thoroughly familiar with the basic principles of strength of materials, such as can be found in any standard textbook on this subject. A brief summary of such material is presented here for the sake of uniformity and to emphasize certain principles of special importance.

Stress

The term stress as used here always implies a force per unit area and is a measure of the intensity of the force acting on a definite plane passing through a given point. The stress distribution may or may not be uniform, depending on the nature of the loading condition. For example, tensile stresses as found from the equation $S = P/A$ are considered to be uniform, whereas the bending stress determined from the equation $S = Mc/I$ refers to the stress at a point located at a distance c from the neutral axis. Obviously the stress over the cross section of a member subjected to bending is not uniform.

Normal and shear stresses. The stresses acting at a point in any stressed member can be resolved into components acting on planes through the point.

The normal and shear stresses acting on any particular plane are the stress components perpendicular and parallel, respectively, to the plane. A simple conception of these stresses is that normal stresses tend to pull apart (or press together) adjacent particles of the material, whereas shear stresses tend to cause such particles to slide on each other.

Axial strain. This term refers to the elongation per unit length in a member or portion of a member in the axial direction. There are usually

* Reference 19.

strains present in other directions also. To determine the stress state of a member, therefore, strains in three directions must be considered, and the principle stresses can then be calculated.

Poisson's ratio. Uniaxial strain in a metal is always accompanied by lateral strains of opposite sign in the two directions mutually perpendicular to the uniaxial strain. Under uniaxial conditions, the absolute value of the ratio of either of the lateral strains to the uniaxial strains is called Poisson's ratio. This ratio is usually between 0.25 and 0.33 for steel and aluminium alloys. In multiaxially stressed members, the lateral strain will affect strain readings and must be considered in strain measurements under these conditions. The formulas for principle stresses and principle strains in terms of the other principle stresses are given in standard texts on the theory of elasticity. For materials stressed beyond the elastic limit Poisson's ratio is not a constant but is a function of the axial strain.

Shearing strain. If a square element of uniform thickness is subjected to pure shear there will be a displacement of each side of the element relative to the opposite side. The shearing strain is obtained by dividing this displacement by the distance between the sides of the element. It should be noted that shearing strain is obtained by dividing a displacement by a distance at right angles to the displacement, whereas axial strain is obtained by dividing the deformation by a length measured in the same direction as the deformation.

Tensile Properties

When a specimen of a certain material is tested in tension it is customary to plot the results of such a test as a "stress-strain diagram." This diagram forms the basis for most strength specifications and should be thoroughly understood and frequently applied. Typical tensile diagrams, not to scale, are shown in Fig. 1. It should be noted that the strain scale is nondimensional, whereas the stress scale is in pounds per square inch. The important physical properties which can be shown on the stress-strain diagram are discussed in the following sections.

Modulus of elasticity (E). Referring to Fig. 1, it will be noted that the first part of the diagram is substantially a straight line. This indicates a constant ratio between stress and strain over that range. The numerical value of the ratio is called the modulus of elasticity, denoted by E. It will be noted that E is the slope of the straight portion of the stress-strain diagram and is determined by dividing the stress (in pounds per square inch) by the strain (which is nondimensional). Therefore, E has the same dimensions as a stress. A useful conception of E is the stress at which the member would have elongated a distance equal to its original length (assuming no departure from the straight portion of the stress-strain diagram).

Other moduli that are often of interest are the tangent modulus E_t and

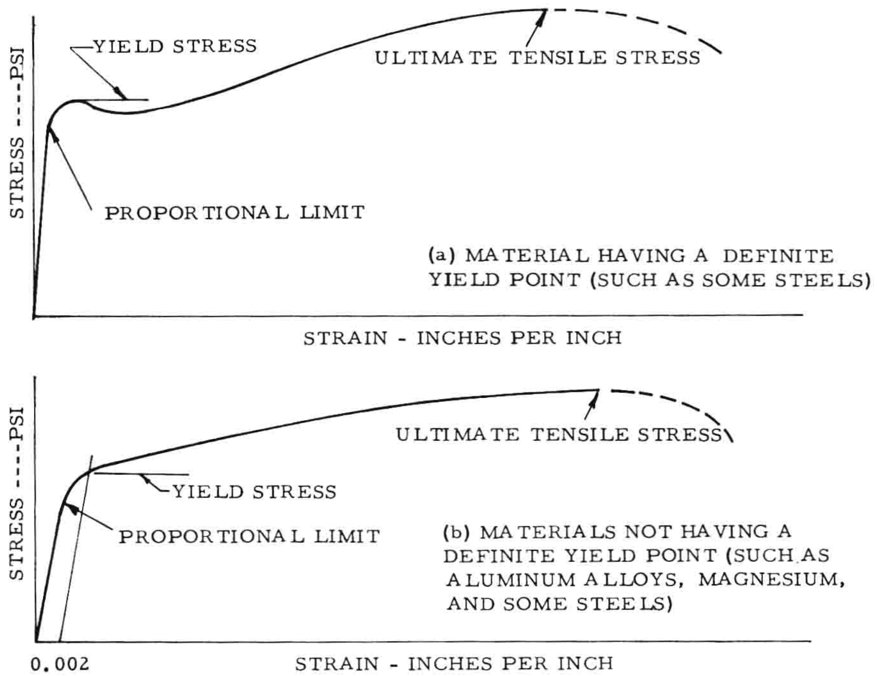

Figure 1.

the secant modulus E_s. The tangent modulus is the slope of the stress-strain diagram at a point corresponding to a given stress, whereas the secant modulus is the slope of a line drawn through the same point and the origin.

Tensile proportional limit. Since it is practically impossible to determine the stress at which the stress-strain diagram begins to depart from a straight line, it is customary to assign a small value of permanent strain for this purpose. As a rule, the limit of proportionality will be taken as the stress at which the stress-strain diagram departs from a straight line by a strain of 0.0001. This property or characteristic of a material gives an indication of the type of stress-strain diagram which applies in the working range. It also indicates the stress beyond which the standard value of E cannot be accurately applied. This is of special interest in the analysis of redundant structures.

Tensile yield stress. The stress-strain diagrams for some steels show a sharp break at a stress below the ultimate tensile stress. At this critical stress the material elongates considerably with little or no increase in stress. (Refer to Fig. 1.) The stress at which this takes place is referred to as the yield point. Nonferrous metals and some steels do not show this sharp break but yield more gradually so that there is no definite yield point. This condition is illustrated in Fig. 1. Since permanent deformations of any appreciable amount are undesirable in most structures, it is customary to adopt an arbitrary amount of permanent strain that is considered admissible for general purposes. The value of this strain has been established by material testing engineers as 0.002, and the corresponding stress is called the *yield stress.* For practical purposes, this may be determined from the stress-strain diagram by drawing a line parallel to the straight or elastic portion of the curve through a point representing zero stress and 0.002 strain. The yield stress is taken as the stress at the intersection of this straight line with the stress-strain curve.

Ultimate tensile stress. Figure 1 shows how the ultimate tensile stress is determined from the stress-strain diagram. It is simply the stress at the maximum load reached in the test. It should be noted that all stresses are based on the original cross-sectional area of the test specimen, without regard to the lateral contraction of the specimen which actually occurs during the test. The ultimate tensile stress is commonly used as a criterion of the strength of the material of aircraft structures.

Compressive Properties

The results of compression tests can be plotted as stress-strain diagrams similar to those shown in Fig. 1 for tension. The preceding remarks (with the exception of those pertaining to ultimate stress) concerning the specific tensile properties of the material apply in a similar manner to the compressive properties. It should be noted that the moduli of elasticity in tension and compression are approximately equal for most of the commonly used structural materials.

2 TYPES OF FAILURES

General

In the following discussion the term "failure" will usually denote actual rupture of the member, or the condition of the member when it has just attained its maximum load.

Material Failures

Fracture of a material may occur by either a *separation* of adjacent particles across a section perpendicular to the direction of loading or by a *sliding* of adjacent particles along other sections. In some cases the mechanism of failure includes both of these actions. For instance, in a simple tension test sliding action along inclined sections may occur first with a consequent reduction in the cross-sectional area of the specimen. This may result in strain hardening of the material so that the resistance to sliding is increased, and the final failure may occur by separation of the material across a section perpendicular to the direction of the loading.

Direct tension or compression. This type of failure is associated with the ultimate tensile or compressive stress of the material. For compression it can apply only to members having large cross-sectional dimensions as compared to the length in the direction of the load.

Shear. Pure shear failures are usually obtained only when the shear load is transmitted over a very short length of the member. This condition is approached in the case of rivets and bolts. In cases where the ultimate shear stress is relatively low, a pure shear failure may result, but in general a member subjected to a shear load fails under the action of the resulting normal stresses usually the compressive stresses. The failure of a tube in torsion, for instance, is not usually caused by exceeding the allowable shear stress but by exceeding a certain allowable normal compressive stress which causes the tube to buckle. It is customary, for convenience, to determine the allowable stresses for members subjected to shear in the form of shear stresses. Such allowable shear stresses are therefore an indirect measure of the stresses actually causing failure.

Bearing. The failure of a material in bearing may consist of crushing, splitting, or progressively rapid yielding in the region where the load is applied.

Failure of this type will depend, to a large extent, on the relative size and shape of the two connecting parts. The allowable bearing stress will not always be applicable to cases in which one of the contacting members is relatively thin. It is also necessary, for practical reasons, to limit the working bearing stress to low values in such cases as joints subjected to reversals of load or in bearings between movable surfaces. These special cases are covered by specific rulings involving the use of higher factors of safety.

Bending. For compact sections not subject to instability, a bending failure can be classed as a tensile or compressive failure caused by exceeding a certain allowable stress in some portion of the specimen. It is customary to determine, experimentally, the *modulus of rupture in bending*, which is a stress derived from test results through the use of equation Mc/I in which case M is the value of bending moment which caused failure. If not determined experimentally, the value of the modulus of rupture in bending may be assumed equal to the ultimate tensile stress when instability is not critical. Since the above equation is based on assumptions which are not always fulfilled at failure, the modulus at failure cannot be considered as the actual stress at the point of rupture. This should be borne in mind in dealing with combined stresses, such as bending and compression, or bending and torsion.

Failure due to stress concentrations. The static strength properties are determined on machined specimens containing no notches, holes, or other avoidable stress raisers. In the design of machine structures such simplicity is unattainable, and stress distributions are not of the uniform quality obtained in the specimen tests. Consideration must be given to this condition since maximum stresses in a material, and not average stresses, are the critical factor in design. The effects of stress raisers vary, and references should be made to available data.

Failure due to fatigue. Although the component parts of structures are usually designed for static load conditions, they are subjected in service to repeated loads. It is well known that the strength of a material under repeated loads is less than that which would be obtained under static loading. This phenomenon of the decreased strength of a material under repeated loading is commonly called fatigue. Stress raisers, such as abrupt changes in cross section, holes, notches and re-entrant corners, cause a much greater effect on the fatigue strength than they do on static strength. The local high-stress concentrations caused by such stress raisers are often greatly in excess of the nominal calculated stress on the part, and consequently it is at such locations that fatigue fractures usually begin. Other factors of major importance in fatigue are the range of a repeated stress cycle, from maximum to minimum stress, and the mean stress in the stress cycle.

Failure from combined stresses. In combined stress conditions where failure is not caused by buckling or instability, it is necessary to refer to some theory of failure. The "maximum shear" theory has received wide acceptance as a simple working basis in the case of ductile materials. It should be noted that this theory interprets failure as the first yielding of the material, so that any extension of the theory to cover conditions of final rupture must be based on the experience of the designer. The failure of brittle materials under combined stresses can generally be treated by the "maximum stress" theory.