PRECALCULUS

FIFTH EDITION

LARSON ◆ HOSTETLER

Precalculus

Fifth Edition

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A Word from the Authors

Welcome to *Precalculus*, Fifth Edition. In this revision we focus on student success, accessibility, and flexibility.

Student Success: During the past 30 years of teaching and writing, we have learned many things about the teaching and learning of mathematics. We have found that students are most successful when they know what they are expected to learn and why it is important to learn. With that in mind, we have restructured the Fifth Edition to include a thematic study thread in every chapter.

Each chapter begins with a study guide called *How to Study This Chapter*, which includes a comprehensive overview of the chapter concepts (*The Big Picture*), a list of *Important Vocabulary* that is integral to learning *The Big Picture* concepts, a list of study resources, and a general study tip. The study guide allows students to get organized and prepare for the chapter.

An old pedagogical recipe goes something like this: "First I'm going to tell you what I'm going to teach you, then I will teach it to you, and finally I will go over what I taught you." Following this recipe, we have also included a set of learning objectives in every section that outlines what students are expected to learn, followed by an interesting real-life application that illustrates why it is important to learn the concepts in that section. Finally, the chapter summary (*What did you learn?*), which reinforces the section objectives, and the chapter *Review Exercises*, which are correlated to the chapter summary, provide additional study support at the conclusion of each chapter.

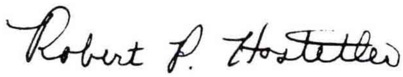
Our new *Student Success Organizer* supplement takes this study thread one step further, providing a content-based study aid.

Accessibility: Over the years we have taken care to write our texts for the student. We have paid careful attention to the presentation, using precise mathematical language and clear writing, to create an effective learning tool. We believe that every student can learn mathematics and we are committed to providing a text that makes the mathematics within it accessible to all students. In the Fifth Edition, we have revised and improved many text features designed for this purpose. The *Technology*, *Exploration*, and *Study Tip* features have been expanded. *Chapter Tests*, which give students an opportunity for self-assessment, now follow every chapter in the Fifth Edition. The exercise sets now include both *Synthesis* exercises, which check students' conceptual understanding, and *Review* exercises, which reinforce skills learned in previous sections and chapters. Also, students have access to several media resources that accompany this text—videotapes, *Interactive Precalculus* CD-ROM, and a *Precalculus* website—that provide additional text-specific support.

Flexibility: From the time we first began writing in the early 1970s, we have always viewed part of our authoring role as that of providing instructors with flexible teaching programs. The optional features within the text allow instructors with different pedagogical approaches to design their courses to meet both their instructional needs and the needs of their students. Instructors who stress applications and problem solving, or exploration and technology, or more traditional methods, will be able to use this text successfully. In addition, we provide several print and media resources to support instructors, including a new *Instructor Success Organizer*.

We hope you enjoy the Fifth Edition.

Ron Larson



Robert P. Hostetler

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Fifth Edition Reviewers

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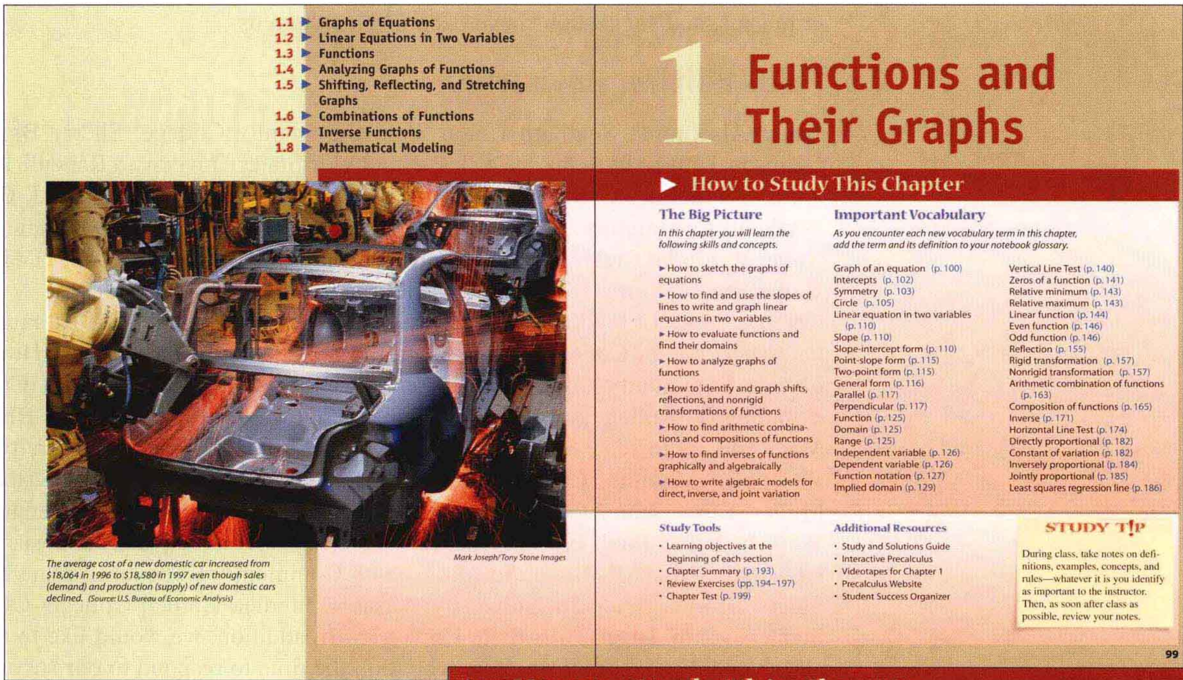
On a personal level, we are grateful to our wives, Deanna Gilbert Larson and Eloise Hostetler, for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to us. Over the past two decades we have received many useful comments from both instructors and students, and we value these comments very much.

Ron Larson
Robert P. Hostetler

Features Highlights

Student Success Tools



▶ "How to Study This Chapter"

The new chapter-opening study guide includes:

- *The Big Picture*—an objective-based overview of the main concepts of the chapter
- *Important Vocabulary*—mathematical terms integral to learning *The Big Picture* concepts
- *Study Tools*
- *Additional Resources*
- *Study Tip*

▶ How to Study This Chapter

The Big Picture

In this chapter you will learn the following skills and concepts.

- ▸ How to sketch the graphs of equations
- ▸ How to find and use the slopes of lines to write and graph linear equations in two variables
- ▸ How to evaluate functions and find their domains
- ▸ How to analyze graphs of functions
- ▸ How to identify and graph shifts, reflections, and nonrigid transformations of functions
- ▸ How to find arithmetic combinations and compositions of functions
- ▸ How to find inverses of functions graphically and algebraically
- ▸ How to write algebraic models for direct, inverse, and joint variation

Important Vocabulary

As you encounter each new vocabulary term in this chapter, add the term and its definition to your notebook glossary.

Graph of an equation (p. 100)
Intercepts (p. 102)
Symmetry (p. 103)
Circle (p. 105)
Linear equation in two variables (p. 110)
Slope (p. 110)
Slope-intercept form (p. 110)
Point-slope form (p. 115)
Two-point form (p. 115)
General form (p. 116)
Parallel (p. 117)
Perpendicular (p. 117)
Function (p. 125)
Domain (p. 125)
Range (p. 125)
Independent variable (p. 126)
Dependent variable (p. 126)
Function notation (p. 127)
Implied domain (p. 129)
Vertical Line Test (p. 140)
Zeros of a function (p. 141)
Relative minimum (p. 143)
Relative maximum (p. 143)
Linear function (p. 144)
Even function (p. 146)
Odd function (p. 146)
Reflection (p. 155)
Rigid transformation (p. 157)
Nonrigid transformation (p. 157)
Arithmetic combination of functions (p. 163)
Composition of functions (p. 165)
Inverse (p. 171)
Horizontal Line Test (p. 174)
Directly proportional (p. 182)
Constant of variation (p. 182)
Inversely proportional (p. 184)
Jointly proportional (p. 185)
Least squares regression line (p. 186)

Study Tools

- Learning objectives at the beginning of each section
- Chapter Summary (p. 193)
- Review Exercises (pp. 194–197)
- Chapter Test (p. 199)

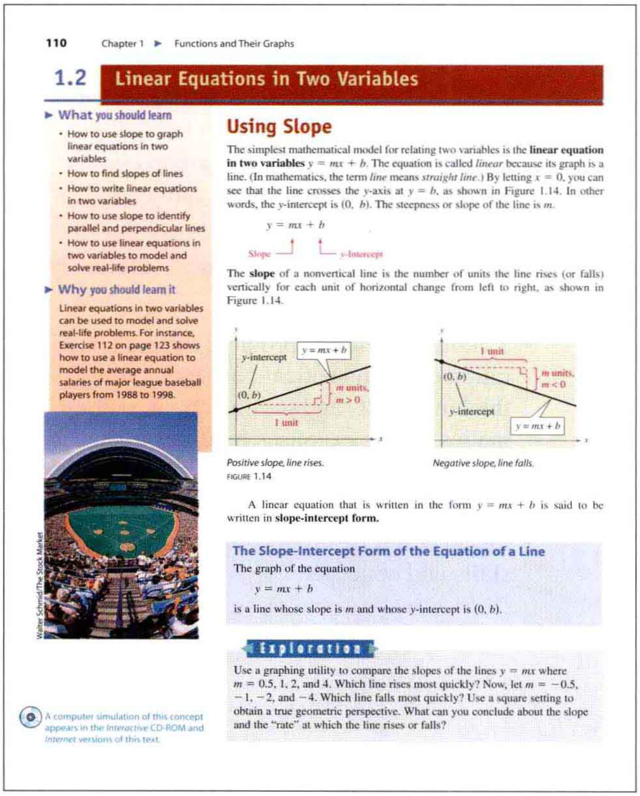
Additional Resources

- Study and Solutions Guide
- Interactive Precalculus
- Videotapes for Chapter 1
- Precalculus Website
- Student Success Organizer

STUDY T!P

During class, take notes on definitions, examples, concepts, and rules—whatever it is you identify as important to the instructor. Then, as soon after class as possible, review your notes.

New Section Openers include:



1.2 Linear Equations in Two Variables

▶ **What you should learn**

- How to use slope to graph linear equations in two variables
- How to find slopes of lines
- How to write linear equations in two variables
- How to use slope to identify parallel and perpendicular lines
- How to use linear equations in two variables to model and solve real-life problems

▶ **Why you should learn it**

Linear equations in two variables can be used to model and solve real-life problems. For instance, Exercise 112 on page 123 shows how to use a linear equation to model the average annual salaries of major league baseball players from 1988 to 1998.

Walter Schmid/The Stock Market

A computer simulation of this concept appears in the *Interactive* CD-ROM and *Internet* versions of this text.

Using Slope

The simplest mathematical model for relating two variables is the **linear equation in two variables** $y = mx + b$. The equation is called *linear* because its graph is a line. (In mathematics, the term *line* means *straight line*.) By letting $x = 0$, you can see that the line crosses the y-axis at $y = b$, as shown in Figure 1.14. In other words, the y-intercept is $(0, b)$. The steepness or slope of the line is m.

$$y = mx + b$$

Slope, y-Intercept

The **slope** of a nonvertical line is the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right, as shown in Figure 1.14.

Positive slope, line rises. *Negative slope, line falls.*

FIGURE 1.14

A linear equation that is written in the form $y = mx + b$ is said to be written in **slope-intercept form.**

The Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

Exploration

Use a graphing utility to compare the slopes of the lines $y = mx$ where $m = 0.5, 1, 2,$ and 4. Which line rises most quickly? Now, let $m = -0.5, -1, -2,$ and -4. Which line falls most quickly? Use a square setting to obtain a true geometric perspective. What can you conclude about the slope and the "rate" at which the line rises or falls?

▶ "What you should learn"

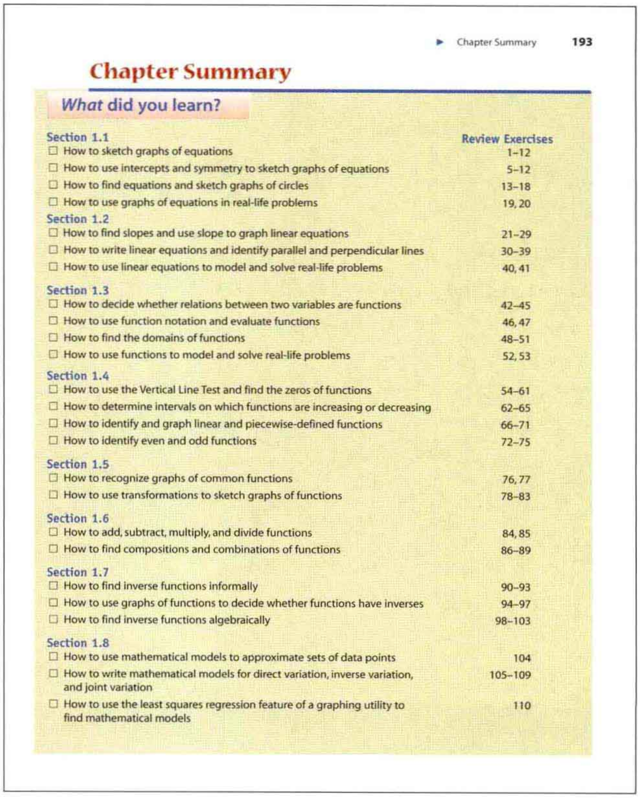
Objectives outline the main concepts and help keep students focused on *The Big Picture*.

▶ "Why you should learn it"

A real-life application or a reference to other branches of mathematics illustrates the relevance of the section's content.

▶ "What did you learn?" Summary

The chapter summary provides a concise, section-by-section review of the section objectives. These objectives are correlated to the chapter Review Exercises.



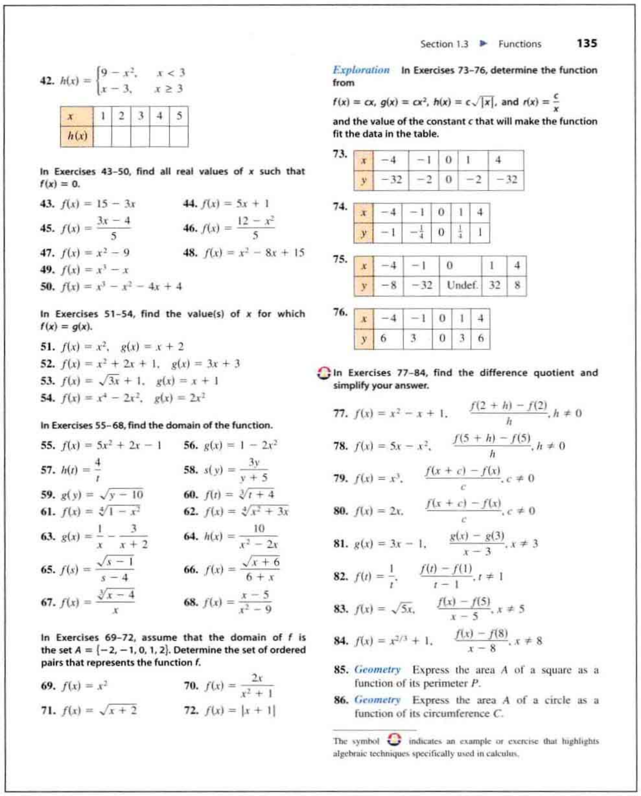
Chapter Summary

What did you learn?

	Review Exercises
Section 1.1	
☐ How to sketch graphs of equations	1–12
☐ How to use intercepts and symmetry to sketch graphs of equations	5–12
☐ How to find equations and sketch graphs of circles	13–18
☐ How to use graphs of equations in real-life problems	19, 20
Section 1.2	
☐ How to find slopes and use slope to graph linear equations	21–29
☐ How to write linear equations and identify parallel and perpendicular lines	30–39
☐ How to use linear equations to model and solve real-life problems	40, 41
Section 1.3	
☐ How to decide whether relations between two variables are functions	42–45
☐ How to use function notation and evaluate functions	46, 47
☐ How to find the domains of functions	48–51
☐ How to use functions to model and solve real-life problems	52, 53
Section 1.4	
☐ How to use the Vertical Line Test and find the zeros of functions	54–61
☐ How to determine intervals on which functions are increasing or decreasing	62–65
☐ How to identify and graph linear and piecewise-defined functions	66–71
☐ How to identify even and odd functions	72–75
Section 1.5	
☐ How to recognize graphs of common functions	76, 77
☐ How to use transformations to sketch graphs of functions	78–83
Section 1.6	
☐ How to add, subtract, multiply, and divide functions	84, 85
☐ How to find compositions and combinations of functions	86–89
Section 1.7	
☐ How to find inverse functions informally	90–93
☐ How to use graphs of functions to decide whether functions have inverses	94–97
☐ How to find inverse functions algebraically	98–103
Section 1.8	
☐ How to use mathematical models to approximate sets of data points	104
☐ How to write mathematical models for direct variation, inverse variation, and joint variation	105–109
☐ How to use the least squares regression feature of a graphing utility to find mathematical models	110

FEATURES

Revised Exercises and Applications



42. $h(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \geq 3 \end{cases}$

x	1	2	3	4	5
$h(x)$					

In Exercises 43–50, find all real values of x such that $f(x) = 0$.

43. $f(x) = 15 - 3x$ **44.** $f(x) = 5x + 1$

45. $f(x) = \frac{3x - 4}{5}$ **46.** $f(x) = \frac{12 - x^2}{5}$

47. $f(x) = x^2 - 9$ **48.** $f(x) = x^2 - 8x + 15$

49. $f(x) = x^3 - x$

50. $f(x) = x^3 - x^2 - 4x + 4$

In Exercises 51–54, find the value(s) of x for which $f(x) = g(x)$.

51. $f(x) = x^2, \quad g(x) = x + 2$

52. $f(x) = x^2 + 2x + 1, \quad g(x) = 3x + 3$

53. $f(x) = \sqrt{3x} + 1, \quad g(x) = x + 1$

54. $f(x) = x^4 - 2x^2, \quad g(x) = 2x^2$

In Exercises 55–68, find the domain of the function.

55. $f(x) = 5x^2 + 2x - 1$ **56.** $g(x) = 1 - 2x^2$

57. $h(t) = \frac{4}{t}$ **58.** $s(y) = \frac{3y}{y + 5}$

59. $g(y) = \sqrt{y - 10}$ **60.** $f(t) = \sqrt[3]{t + 4}$

61. $f(x) = \sqrt[4]{1 - x^2}$ **62.** $f(x) = \sqrt[4]{x^2 + 3x}$

63. $g(x) = \frac{1}{x} - \frac{3}{x + 2}$ **64.** $h(x) = \frac{10}{x^2 - 2x}$

65. $f(s) = \frac{\sqrt{s - 1}}{s - 4}$ **66.** $f(x) = \frac{\sqrt{x + 6}}{6 + x}$

67. $f(x) = \frac{\sqrt[3]{x - 4}}{x}$ **68.** $f(x) = \frac{x - 5}{x^2 - 9}$

In Exercises 69–72, assume that the domain of f is the set $A = \{-2, -1, 0, 1, 2\}$. Determine the set of ordered pairs that represents the function f.

69. $f(x) = x^2$ **70.** $f(x) = \frac{2x}{x^2 + 1}$

71. $f(x) = \sqrt{x + 2}$ **72.** $f(x) = |x + 1|$

Exploration **In Exercises 73–76, determine the function from**

$f(x) = cx$, $g(x) = cx^2$, $h(x) = c\sqrt{|x|}$, **and** $r(x) = \frac{c}{x}$

and the value of the constant c that will make the function fit the data in the table.

73.

x	−4	−1	0	1	4
y	−32	−2	0	−2	−32

74.

x	−4	−1	0	1	4
y	−1	$-\frac{1}{4}$	0	$\frac{1}{4}$	1

75.

x	−4	−1	0	1	4
y	−8	−32	Undef.	32	8

76.

x	−4	−1	0	1	4
y	6	3	0	3	6

In Exercises 77–84, find the difference quotient and simplify your answer.

77. $f(x) = x^2 - x + 1, \quad \frac{f(2 + h) - f(2)}{h}, h \neq 0$

78. $f(x) = 5x - x^2, \quad \frac{f(5 + h) - f(5)}{h}, h \neq 0$

79. $f(x) = x^3, \quad \frac{f(x + c) - f(x)}{c}, c \neq 0$

80. $f(x) = 2x, \quad \frac{f(x + c) - f(x)}{c}, c \neq 0$

81. $g(x) = 3x - 1, \quad \frac{g(x) - g(3)}{x - 3}, x \neq 3$

82. $f(t) = \frac{1}{t}, \quad \frac{f(t) - f(1)}{t - 1}, t \neq 1$

83. $f(x) = \sqrt{5x}, \quad \frac{f(x) - f(5)}{x - 5}, x \neq 5$

84. $f(x) = x^{2/3} + 1, \quad \frac{f(x) - f(8)}{x - 8}, x \neq 8$

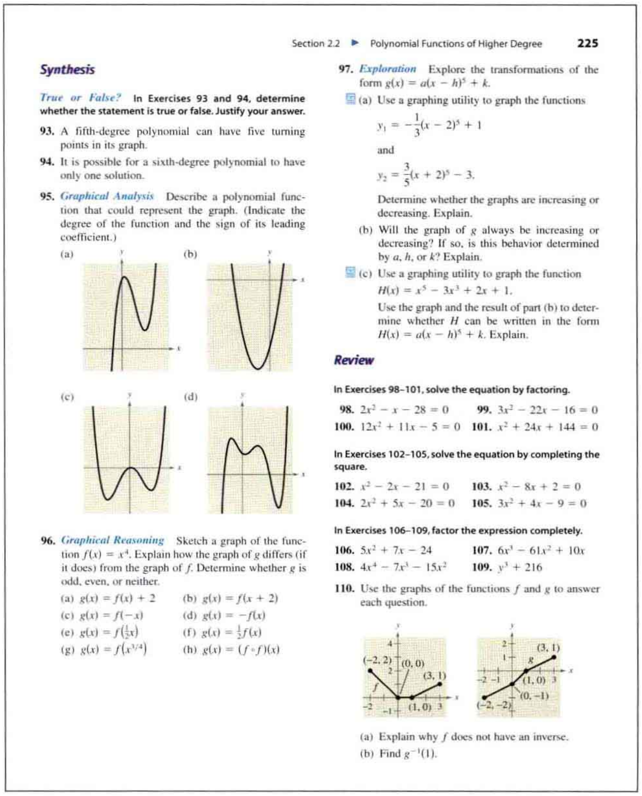
85. *Geometry* Express the area A of a square as a function of its perimeter P.

86. *Geometry* Express the area A of a circle as a function of its circumference C.

The symbol indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

▶ Exercises

- Each exercise set contains a variety of computational, conceptual, and applied problems.
- Each exercise set is carefully graded in difficulty to allow students to gain confidence as they progress.
- Each exercise set now concludes with two new types of exercises:
  - *Synthesis* exercises promote further exploration of mathematical concepts, critical thinking skills, and writing about mathematics. These exercises require students to synthesize the main concepts presented in the section and chapter.
  - *Review* exercises reinforce previously learned skills and concepts.



Synthesis

True or False? **In Exercises 93 and 94, determine whether the statement is true or false. Justify your answer.**

93. A fifth-degree polynomial can have five turning points in its graph.

94. It is possible for a sixth-degree polynomial to have only one solution.

95. *Graphical Analysis* Describe a polynomial function that could represent the graph. (Indicate the degree of the function and the sign of its leading coefficient.)

(a) (b) (c) (d)

96. *Graphical Reasoning* Sketch a graph of the function $f(x) = x^4$. Explain how the graph of g differs (if it does) from the graph of f. Determine whether g is odd, even, or neither.

(a) $g(x) = f(x) + 2$ (b) $g(x) = f(x + 2)$

(c) $g(x) = f(-x)$ (d) $g(x) = -f(x)$

(e) $g(x) = f\left(\frac{1}{2}x\right)$ (f) $g(x) = \frac{1}{2}f(x)$

(g) $g(x) = f\left(x^{3/4}\right)$ (h) $g(x) = (f \circ f)(x)$

97. *Exploration* Explore the transformations of the form $g(x) = a(x - h)^5 + k$.

(a) Use a graphing utility to graph the functions

$$y_1 = -\frac{1}{3}(x - 2)^5 + 1$$

and

$$y_2 = \frac{3}{5}(x + 2)^5 - 3.$$

Determine whether the graphs are increasing or decreasing. Explain.

(b) Will the graph of g always be increasing or decreasing? If so, is this behavior determined by a, h, or k? Explain.

(c) Use a graphing utility to graph the function $H(x) = x^5 - 3x^3 + 2x + 1$.

Use the graph and the result of part (b) to determine whether H can be written in the form $H(x) = a(x - h)^5 + k$. Explain.

Review

In Exercises 98–101, solve the equation by factoring.

98. $2x^2 - x - 28 = 0$ **99.** $3x^2 - 22x - 16 = 0$

100. $12x^2 + 11x - 5 = 0$ **101.** $x^2 + 24x + 144 = 0$

In Exercises 102–105, solve the equation by completing the square.

102. $x^2 - 2x - 21 = 0$ **103.** $x^2 - 8x + 2 = 0$

104. $2x^2 + 5x - 20 = 0$ **105.** $3x^2 + 4x - 9 = 0$

In Exercises 106–109, factor the expression completely.

106. $5x^2 + 7x - 24$ **107.** $6x^3 - 61x^2 + 10x$

108. $4x^4 - 7x^3 - 15x^2$ **109.** $y^3 + 216$

110. Use the graphs of the functions f and g to answer each question.

(a) Explain why f does not have an inverse.

(b) Find $g^{-1}(1)$.

▶ Real-Life Applications

- A wide variety of real-life applications, many using current, real data, are integrated throughout examples and exercises.
- The icon indicates an example that involves a real-life application.

In Exercises 69–74, use the functions $f(x) = \frac{1}{8}x - 3$ and $g(x) = x^3$ to find the indicated value or function.

69. $(f^{-1} \circ g^{-1})(1)$ **70.** $(g^{-1} \circ f^{-1})(-3)$

71. $(f^{-1} \circ f^{-1})(6)$ **72.** $(g^{-1} \circ g^{-1})(-4)$

73. $(f \circ g)^{-1}$ **74.** $g^{-1} \circ f^{-1}$

In Exercises 75–78, use the functions $f(x) = x + 4$ and $g(x) = 2x - 5$ to find the specified function.

75. $g^{-1} \circ f^{-1}$ **76.** $f^{-1} \circ g^{-1}$

77. $(f \circ g)^{-1}$ **78.** $(g \circ f)^{-1}$

79. *Hourly Wage* Your wage is \$8.00 per hour plus \$0.75 for each unit produced per hour. So, your hourly wage y in terms of the number of units produced is

$y = 8 + 0.75x.$

(a) Find the inverse of the function.

(b) What does each variable represent in the inverse function?

(c) Determine the number of units produced when your hourly wage is \$22.25.

80. *Cost* Suppose you need a total of 50 pounds of two commodities costing \$1.25 and \$1.60 per pound, respectively.

(a) Verify that the total cost is

$y = 1.25x + 1.60(50 - x)$

where x is the number of pounds of the less expensive commodity.

(b) Find the inverse of the cost function. What does each variable represent in the inverse function?

(c) Use the context of the problem to determine the domain of the inverse function.

(d) Determine the number of pounds of the less expensive commodity purchased if the total cost is \$73.

81. *Diesel Mechanics* The function

$y = 0.03x^2 + 245.50, \quad 0 < x < 100$

approximates the exhaust temperature y in degrees Fahrenheit where x is the percent load for a diesel engine.

(a) Find the inverse of the function. What does each variable represent in the inverse function?

(b) Use a graphing utility to graph the inverse function.

(c) Determine the percent load interval if the exhaust temperature of the engine must not exceed 500 degrees Fahrenheit.

82. *New Car Sales* The total value of new car sales f (in billions of dollars) in the United States from 1992 through 1997 is shown in the table. The time (in years) is given by t, with $t = 2$ corresponding to 1992. (Source: National Automobile Dealers Association)

t	2	3	4	5	6	7
$f(t)$	333.8	377.3	430.6	456.2	490.0	507.5

(a) Does f^{-1} exist?

(b) If f^{-1} exists, what does it mean in the context of the problem?

(c) If f^{-1} exists, find $f^{-1}(456.2)$.

83. If the table in Exercise 82 were extended to 1998 and if the total value of new car sales for that year was \$430.6 billion, would f^{-1} exist? Explain.

84. *Cellular Phones* The average local bill (in dollars) for cellular phones in the United States from 1990 to 1997 is shown in the table. The time (in years) is given by t, with $t = 0$ corresponding to 1990. (Source: Cellular Telecommunications Industry Association)

t	0	1	2	3
$f(t)$	80.90	72.74	68.68	61.48

t	4	5	6	7
$f(t)$	56.21	51.00	47.70	42.78

(a) Find $f^{-1}(51)$.

(b) What does f^{-1} mean in the context of the problem?

(c) Use the regression feature of a graphing utility to find a linear model for the data, $y = mx + b$. Round m and b to two decimal places.

(d) Algebraically find the inverse of the linear model in part (c).

(e) Use the inverse of the linear model you found in part (d) to approximate $f^{-1}(11)$.

▶ Algebra of Calculus

- Special emphasis is given to the algebraic techniques used in calculus.
- Algebra of Calculus examples and exercises are integrated throughout the text.
- The symbol indicates an example or exercise in which the Algebra of Calculus is featured.

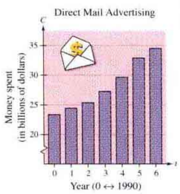


FIGURE 1.30

Example 8 ▶ Direct Mail Advertising

The money C (in billions of dollars) spent for direct mail advertising in the United States increased in a linear pattern from 1990 to 1992, as shown in Figure 1.30. Then, in 1993, the money spent took a jump and, until 1996, increased in a *different* linear pattern. These two patterns can be approximated by the function

$$C(t) = \begin{cases} 23.40 + 1.01t, & 0 \le t \le 2 \\ 19.85 + 2.50t, & 3 \le t \le 6 \end{cases}$$

where $t = 0$ represents 1990. Use this function to approximate the total amount spent for direct mail advertising between 1990 and 1996. (Source: McCann-Erickson)

Solution

From 1990 to 1992, use the formula $C(t) = 23.40 + 1.01t$.

$$\underbrace{\$23.40}_{1990},\ \underbrace{\$24.41}_{1991},\ \underbrace{\$25.42}_{1992}$$

From 1993 to 1996, use the formula $C(t) = 19.85 + 2.50t$.

$$\underbrace{\$27.35}_{1993},\ \underbrace{\$29.85}_{1994},\ \underbrace{\$32.35}_{1995},\ \underbrace{\$34.85}_{1996}$$

The total of these seven amounts is \$197.63, which implies that the total amount spent was approximately \$197,630,000,000.

One of the basic definitions in calculus employs the ratio

$$\frac{f(x+h) - f(x)}{h}, \quad h \ne 0.$$

This ratio is called a **difference quotient,** as illustrated in Example 9.

Example 9 ▶ Evaluating a Difference Quotient

For $f(x) = x^2 - 4x + 7$, find $\frac{f(x+h) - f(x)}{h}$.

Solution

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{[(x+h)^2 - 4(x+h) + 7] - (x^2 - 4x + 7)}{h} \\ &= \frac{x^2 + 2xh + h^2 - 4x - 4h + 7 - x^2 + 4x - 7}{h} \\ &= \frac{2xh + h^2 - 4h}{h} \\ &= \frac{h(2x + h - 4)}{h} \\ &= 2x + h - 4, \quad h \ne 0 \end{aligned}$$

The symbol indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

FEATURES

Flexibility and Accessibility

Exponential Growth and Decay

Example 1 ▶ Population Increase

Estimates of the world population (in millions) from 1992 through 2000 are shown in the table. The scatter plot of the data is shown in Figure 3.23. (Source: U.S. Bureau of the Census, International Data Base)

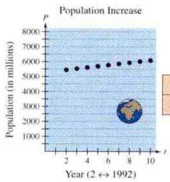


FIGURE 3.23

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Population	5445	5527	5607	5688	5767	5847	5926	6005	6083

An exponential growth model that approximates this data is

$$P = 5304e^{0.013819t}, \quad 2 \le t \le 10$$

where P is the population (in millions) and $t = 2$ represents 1992. Compare the values given by the model with the estimates given by the U.S. Bureau of the Census. According to this model, when will the world population reach 6.5 billion?

Solution

The following table compares the two sets of population figures. The graph of the model is shown in Figure 3.24.

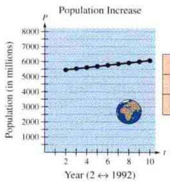


FIGURE 3.24

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Population	5445	5527	5607	5688	5767	5847	5926	6005	6083
Model	5453	5529	5605	5683	5763	5843	5924	6006	6090

To find when the world population will reach 6.5 billion, let $P = 6500$ in the model and solve for t.

$5304e^{0.013819t} = P$ Write original model.

$5304e^{0.013819t} = 6500$ Let $P = 6500$.

$e^{0.013819t} \approx 1.22549$ Divide each side by 5304.

$\ln e^{0.013819t} \approx \ln 1.22549$ Take natural log of each side.

$0.013819t \approx 0.203341$ Inverse Property

$t \approx 14.71$ Divide each side by 0.013819.

According to the model, the world population will reach 6.5 billion in 2004.

Technology

Some graphing utilities have curve-fitting capabilities that can be used to find models that represent data. If you have such a graphing utility, try using it to find a model for the data given in Example 1. How does your model compare with the model given in Example 1?

An exponential model increases (or decreases) by the same percent each year. What is the annual percent increase for the model in Example 1?

Exploration

- Before introduction of selected topics, *Exploration* engages students in active discovery of mathematical concepts and relationships, often through the power of technology.
- *Exploration* strengthens students' critical thinking skills and helps them develop an intuitive understanding of theoretical concepts.
- *Exploration* is an optional feature and can be omitted without loss of continuity in coverage.

▶ Additional Features

Carefully crafted learning tools designed to create a rich learning environment can be found throughout the text. These learning tools include Study Tips, Historical Notes, Writing About Mathematics, Chapter Projects, Chapter Review Exercises, Chapter Tests, Cumulative Tests, and an extensive art program.

Technology

- Point-of-use instructions for using graphing utilities appear in the margins, encouraging the use of graphing technology as a tool for visualization of mathematical concepts, for verification of other solution methods, and for facilitation of computations.
- The use of technology is optional in this text. This feature and related exercises can easily be omitted without loss of continuity in coverage. Exercises that require the use of a graphing utility are identified by the symbol ▦.

▶ Examples

- Each example was carefully chosen to illustrate a particular mathematical concept or problem-solving skill.
- All examples contain step-by-step solutions, most with side-by-side explanations that lead students through the solution process.

A computer animation of this concept appears in the *Interactive* CD-ROM and *Internet* versions of this text.

Even and Odd Functions

In Section 1.1, you studied different types of symmetry of a graph. In the terminology of functions, a function is said to be **even** if its graph is symmetric with respect to the y-axis and to be **odd** if its graph is symmetric with respect to the origin. The symmetry tests in Section 1.1 yield the following tests for even and odd functions.

Tests for Even and Odd Functions

A function $y = f(x)$ is **even** if, for each x in the domain of f,

$f(-x) = f(x)$.

A function $y = f(x)$ is **odd** if, for each x in the domain of f,

$f(-x) = -f(x)$.

Exploration

Graph each of the following functions with a graphing utility. Determine whether the function is *even, odd,* or *neither.*

$f(x) = x^2 - x^4$

$g(x) = 2x^3 + 1$

$h(x) = x^5 - 2x^3 + x$

$j(x) = 2 - x^6 - x^8$

$k(x) = x^5 - 2x^4 + x - 2$

$p(x) = x^9 + 3x^5 - x^3 + x$

What do you notice about the equations of functions that are odd? What do you notice about the equations of functions that are even? Can you describe a way to identify a function as odd or even by inspecting the equation? Can you describe a way to identify a function as neither odd nor even by inspecting the equation?

Example 9 ▶ Even and Odd Functions

a. The function $g(x) = x^3 - x$ is odd because $g(-x) = -g(x)$, as follows.

$g(-x) = (-x)^3 - (-x)$ Substitute $-x$ for x.

$= -x^3 + x$ Simplify.

$= -(x^3 - x)$ Distributive Property

$= -g(x)$ Test for odd function.

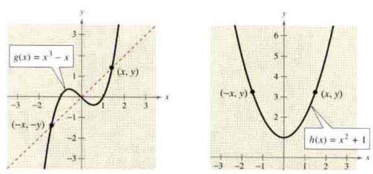
b. The function $h(x) = x^2 + 1$ is even because $h(-x) = h(x)$, as follows.

$h(-x) = (-x)^2 + 1$ Substitute $-x$ for x.

$= x^2 + 1$ Simplify.

$= h(x)$ Test for even function.

The graphs of these two functions are shown in Figure 1.42.



(a) Symmetric to Origin: Odd Function (b) Symmetric to y-Axis: Even Function

FIGURE 1.42

Supplements

Resources

Website *(college.hmco.com)*
Many additional text-specific study and interactive features for students and instructors can be found at the Houghton Mifflin website. These features include, but are not limited to, the following.

- Glossary
- Video clips
- Graphing calculator emulator
- Sample chapters
- Presentation slides

For the Student

Student Success Organizer

Study and Solutions Guide by Dianna L. Zook (Indiana University/Purdue University–Fort Wayne)

Graphing Technology Guide by Benjamin N. Levy and Laurel Technical Services

Instructional Videotapes by Dana Mosely

Instructional Videotapes for Graphing Calculators by Dana Mosely

For the Instructor

Instructor's Annotated Edition

Instructor Success Organizer

Complete Solutions Guide by Dianna L. Zook (Indiana University/Purdue University–Fort Wayne), Laurel Technical Services, and Mike Jones

Test Item File

Problem Solving, Modeling, and Data Analysis Labs by Wendy Metzger (Palomar College)

Computerized Testing (Windows, Macintosh)

Instructor's CD-ROM

An Introduction to Graphing Utilities

Graphing utilities such as graphing calculators and computers with graphing software are very valuable tools for visualizing mathematical principles, verifying solutions to equations, exploring mathematical ideas, and developing mathematical models. Although graphing utilities are extremely helpful in learning mathematics, their use does not mean that learning algebra is any less important. In fact, the combination of knowledge of mathematics and the use of graphing utilities allows you to explore mathematics more easily and to a greater depth. If you are using a graphing utility in this course, it is up to you to learn its capabilities and to practice using this tool to enhance your mathematical learning.

In this text there are many opportunities to use a graphing utility, some of which are described below.

Some Uses of a Graphing Utility

A graphing utility can be used to

- check or validate answers to problems obtained using algebraic methods.
- discover and explore algebraic properties, rules, and concepts.
- graph functions, and approximate solutions to equations involving functions.
- efficiently perform complicated mathematical procedures such as those found in many real-life applications.
- find mathematical models for sets of data.

In this introduction, the features of graphing utilities are discussed from a generic perspective. To learn how to use the features of a specific graphing utility, consult your user's manual or the website for this text found at *college.hmco.com*. Additionally, keystroke guides are available for most graphing utilities, and your college library may have a videotape on how to use your graphing utility.

The Equation Editor

Many graphing utilities are designed to act as "function graphers." In this course, you will study functions and their graphs in detail. You may recall from previous courses that a function can be thought of as a rule that describes the relationship between two variables. These rules are frequently written in terms of x and y. For example, the equation $y = 3x + 5$ represents y as a function of x.

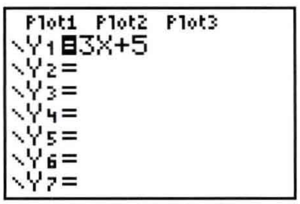


FIGURE 1

Many graphing utilities have an equation editor that requires an equation to be written in "$y =$" form in order to be entered, as shown in Figure 1. (You should note that your equation editor screen may not look like the screen shown in Figure 1.) To determine exactly how to enter an equation into your graphing utility, consult your user's manual.

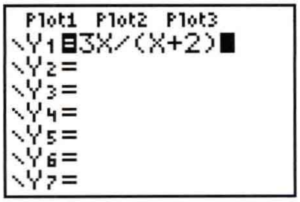


FIGURE 2

The Table Feature

Most graphing utilities are capable of displaying a table of values with x-values and one or more corresponding y-values. These tables can be used to check solutions of an equation and to generate ordered pairs to assist in graphing an equation.

To use the *table* feature, enter an equation into the equation editor in "$y =$" form. The table may have a setup screen, which allows you to select the starting x-value and the table step or x-increment. You may then have the option of automatically generating values for x and y or building your own table using the *ask* mode. In the *ask* mode, you enter a value for x and the graphing utility displays the y-value.

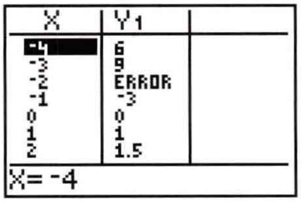


FIGURE 3

For example, enter the equation

$$y = \frac{3x}{x + 2}$$

into the equation editor, as shown in Figure 2. In the table setup screen, set the table to start at $x = -4$ and set the table step to 1. When you view the table, notice that the first x-value is -4 and each value after it increases by 1. Also notice that the Y_1 column gives the resulting y-value for each x-value, as shown in Figure 3. The table shows that the y-value when $x = -2$ is ERROR. This means that the variable x may not take on the value -2 in this equation.

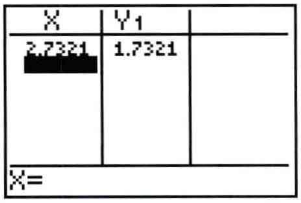


FIGURE 4

With the same equation in the equation editor, set the table to *ask* mode. In this mode you do not need to set the starting x-value or the table step, because you are entering any value you choose for x. You may enter any real value for x—an integer, fraction, decimal, irrational number, and so forth. If you enter $x = 1 + \sqrt{3}$, the graphing utility may rewrite the number as a decimal approximation, as shown in Figure 4. You can continue to build your own table by entering additional x-values in order to generate y-values.

If you have several equations in the equation editor, the table may generate y-values for each equation.

Creating a Viewing Window

A **viewing window** for a graph is a rectangular portion of the coordinate plane. A viewing window is determined by the following six values.

Xmin = the smallest value of x
Xmax = the largest value of x
Xscl = the number of units per tick mark on the x-axis
Ymin = the smallest value of y
Ymax = the largest value of y
Yscl = the number of units per tick mark on the y-axis

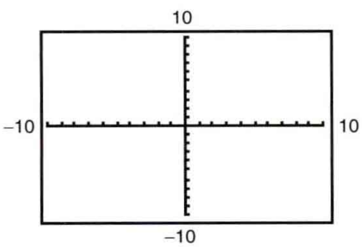


FIGURE 5

When you enter these six values into a graphing utility, you are setting the viewing window. Some graphing utilities have a standard viewing window, as shown in Figure 5.

By choosing different viewing windows for a graph, it is possible to obtain very different impressions of the graph's shape. For instance, Figure 6 shows four different viewing windows for the graph of

$$y = 0.1x^4 - x^3 + 2x^2.$$

Of these, the view shown in part (a) is the most complete.

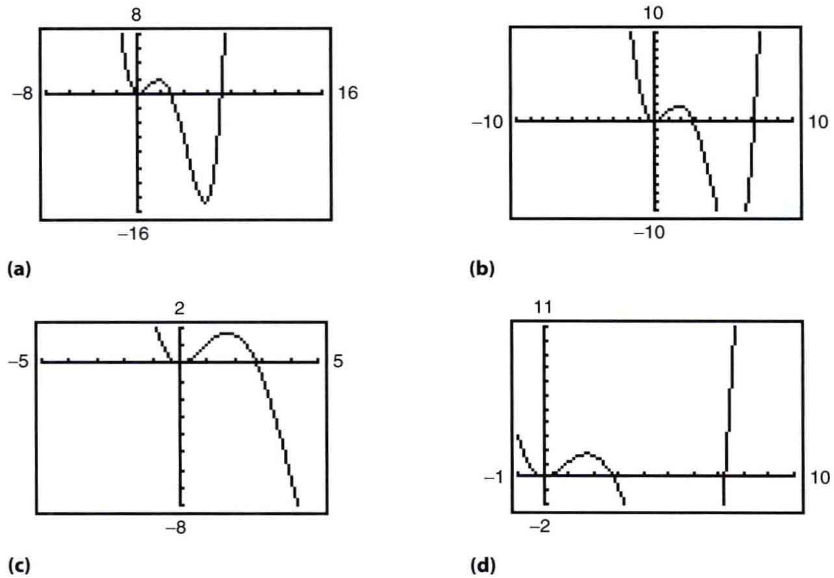


FIGURE 6

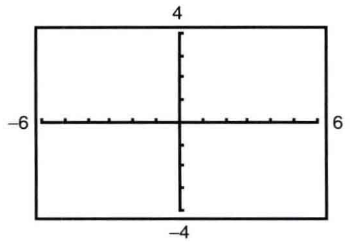


FIGURE 7

On most graphing utilities, the display screen is two-thirds as high as it is wide. On such screens, you can obtain a graph with a true geometric perspective by using a **square setting**—one in which

$$\frac{\text{Ymax} - \text{Ymin}}{\text{Xmax} - \text{Xmin}} = \frac{2}{3}.$$

One such setting is shown in Figure 7. Notice that the x and y tick marks are equally spaced on a square setting, but not on a standard setting.

To see how the viewing window affects the geometric perspective, graph the semicircles $y_1 = \sqrt{9 - x^2}$ and $y_2 = -\sqrt{9 - x^2}$ in a standard viewing window. Then graph y_1 and y_2 in a square window. Note the difference in the shapes of the circles.

Zoom and Trace Features

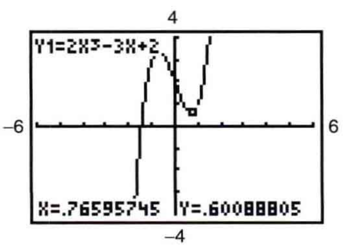
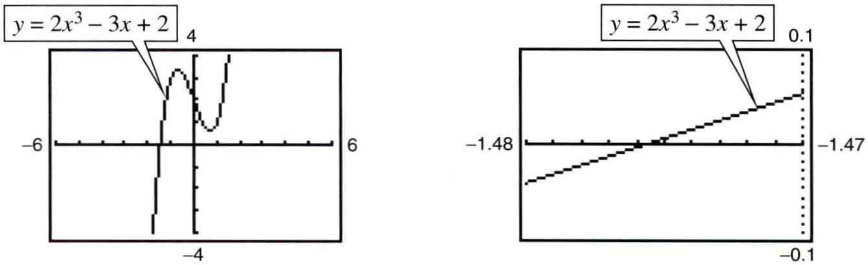


FIGURE 8

When you graph an equation, you can move from point to point along its graph using the *trace* feature. As you trace the graph, the coordinates of each point are displayed, as shown in Figure 8. The *trace* feature combined with the *zoom* feature allows you to obtain better and better approximations of desired points on a graph. For instance, you can use the *zoom* feature of a graphing utility to approximate the x-intercept(s) of a graph [the point(s) where the graph crosses the x-axis]. Suppose you want to approximate the x-intercept(s) of the graph of $y = 2x^3 - 3x + 2$.

Begin by graphing the equation, as shown in Figure 9(a). From the viewing window shown, the graph appears to have only one x-intercept. This intercept lies between -2 and -1. By zooming in on the intercept, you can improve the approximation, as shown in Figure 9(b). To three decimal places, the solution is $x \approx -1.476$.



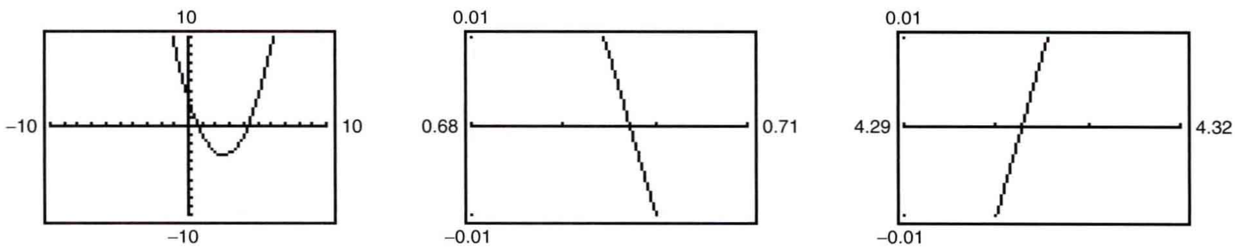
(a) **(b)**

FIGURE 9

Here are some suggestions for using the *zoom* feature.

1. With each successive zoom-in, adjust the x-scale so that the viewing window shows at least one tick mark on each side of the x-intercept.
2. The error in your approximation will be less than the distance between two scale marks.
3. The *trace* feature can usually be used to add one more decimal place of accuracy without changing the viewing window.

Figure 10(a) shows the graph of $y = x^2 - 5x + 3$. Figures 10(b) and 10(c) show "zoom-in views" of the two x-intercepts. From these views, you can approximate the x-intercepts to be $x \approx 0.697$ and $x \approx 4.303$.



(a) **(b)** **(c)**

FIGURE 10

Zero or Root Feature

Using the *zero* or *root* feature, you can find the real zeros of functions of the various types studied in this text—polynomial, exponential, logarithmic, and trigonometric functions. To find the zeros of a function such as $f(x) = \frac{3}{4}x - 2$, first enter the function as $y_1 = \frac{3}{4}x - 2$. Then use the *zero* or *root* feature, which may require entering lower and upper bound estimates of the root, as shown in Figure 11.

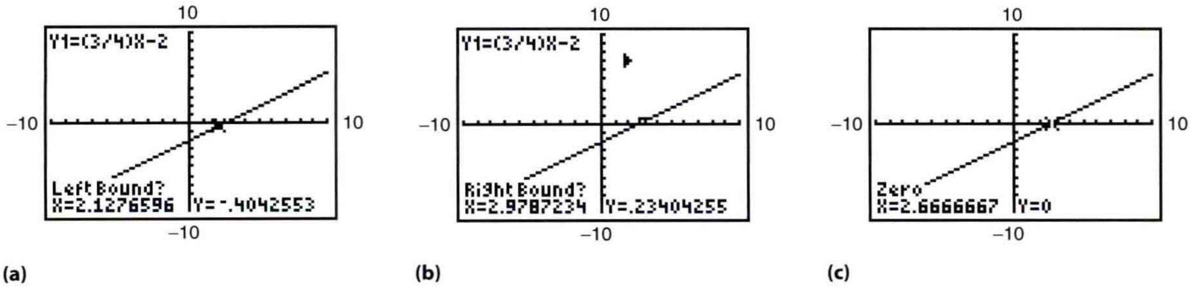


FIGURE 11

In Figure 11(c), you can see that the zero is $x = 2.6666667 \approx 2\frac{2}{3}$.

Intersect Feature

To find the points of intersection of two graphs, you can use the *intersect* feature. For instance, to find the points of intersection of the graphs of $y_1 = -x + 2$ and $y_2 = x + 4$, enter these two functions and use the *intersect* feature, as shown in Figure 12.

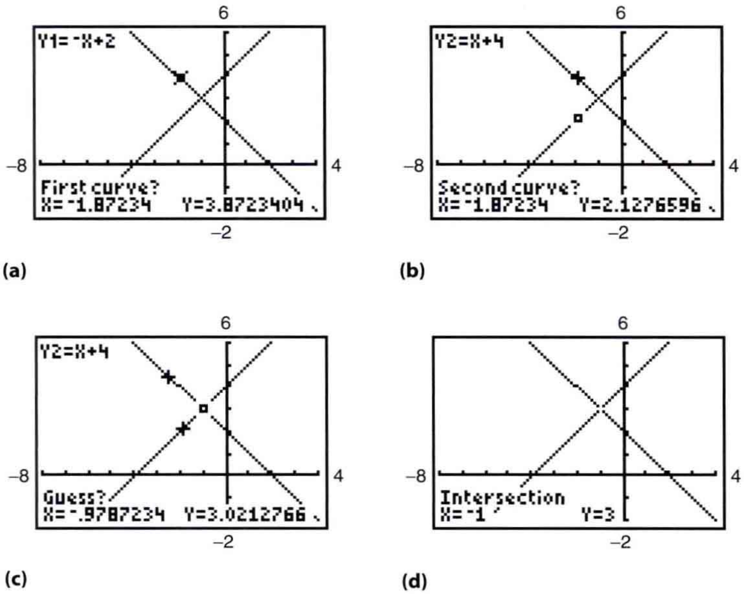


FIGURE 12

From Figure 12(d), you can see that the point of intersection is $(-1, 3)$.