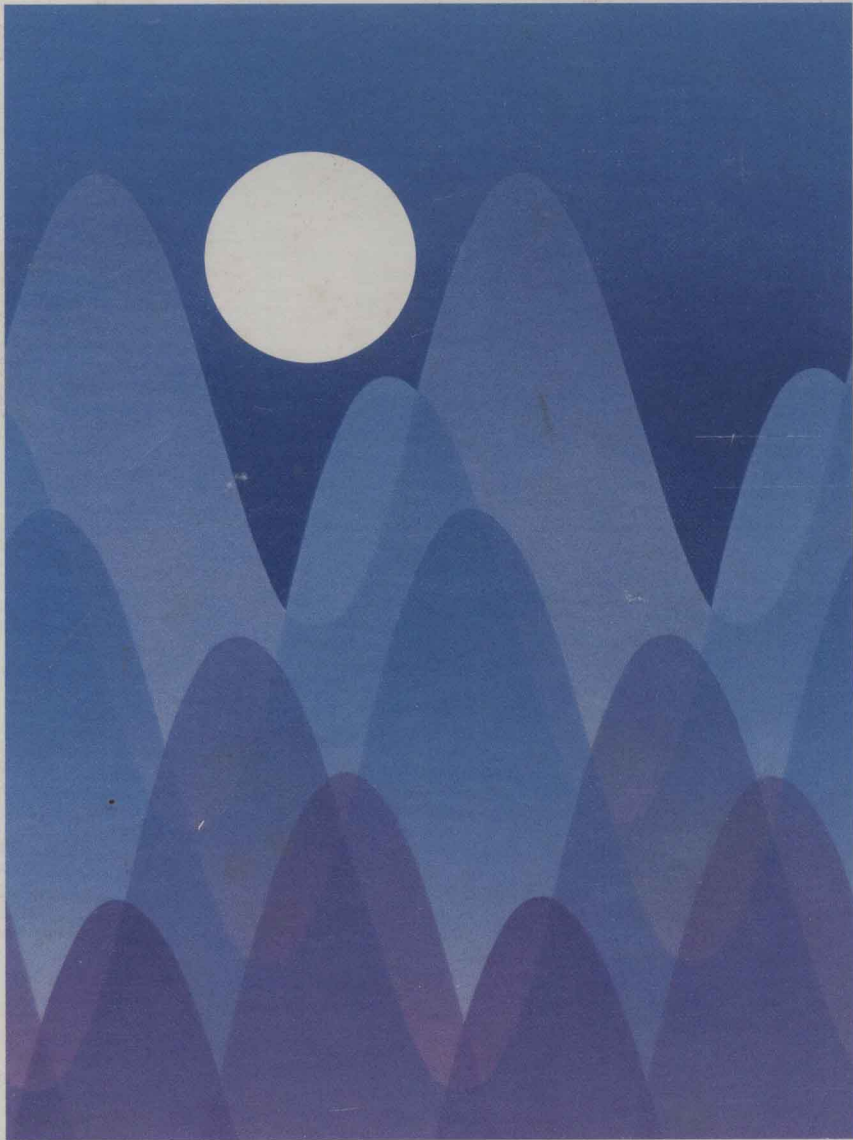


# INTERMEDIATE ALGEBRA

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McWaters  
Lin



# INTERMEDIATE ALGEBRA

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D. C. Heath and Company   Lexington, Massachusetts   Toronto

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*This book is dedicated to  
Marcus and Isola McWaters  
and to all of our students.*

# Preface

Our intention in writing is to provide students with a textbook for intermediate algebra that is accessible to them in a variety of ways. As the following list of text features indicates, we have made every effort to produce a pedagogically sound book filled with an enthusiasm for understanding algebraic concepts and their application to our everyday lives.

**TOPIC TREATMENT** Chapter 1 treats only topics that should be familiar to most students from their beginning algebra course. Chapter 1 can be skimmed, assigned as outside reading, or omitted, depending on the background of the students in the class. Interval notation is introduced in Section 2.5 for use in graphing linear inequalities in Section 2.6. There is considerable emphasis on graphing linear and quadratic functions and relations (see Chapters 7 and 8). Chapter 10 provides separate sections for finding logarithm and antilog values using tables (Section 10.4) and using calculators (Section 10.5). Either section may be omitted or both may be covered, as the instructor prefers. The last three chapters are completely independent and may be taught in any order. Any of these may be omitted for a short course.

**CHAPTER FLOWCHARTS** Each chapter opens with a diagram that shows the various possibilities for ordering the coverage of sections within the chapter.


**APPLICATIONS** A large number of simple applications designed to be accessible to most students is included in every chapter. More substantial applications are available to challenge the better students. A step-by-step procedure for solving word problems is first introduced in Chapter 2. Thereafter every chapter provides word problems to further develop the problem-solving skills learned in Chapter 2.

**EXAMPLES** More than 500 completely worked-out examples are provided to illustrate concepts and demonstrate successful problem-solving techniques.

**EXERCISES** More than 4300 exercises and 450 word problems keyed to examples are provided. These exercises are divided into A and B groupings; the A exercises are routine problems emphasizing fundamental skills and the B exercises contain both theoretical problems and problems requiring more difficult mechanics.

**STANDARD ASSIGNMENT** A list of exercises designated as a standard assignment appears in the beginning of each exercise set. The exercises in the

list are always from the A group and the answers to these exercises appear in the answer section of the text. The standard assignment can be used by graduate students or new instructors as trial assignments until the needs of their particular classes can be determined. They also serve as a guide for students who miss class or are unable to get the assignment given by their instructor.

**CALCULATOR EXERCISES** Exercises intended for calculator solutions are included throughout the book. They are optional exercises and are identified by the symbol .

**WARNINGS** A warning symbol clearly marks the many Warnings intended to call attention to errors and misconceptions common among students.

**CHAPTER SUMMARY** Key words and phrases are identified and keyed to the section in which they are defined. Key concepts and rules are summarized to provide an overview of the chapter.

**REVIEW EXERCISES** Numerous exercises, keyed to the appropriate section of the text, provide additional practice on all topics covered in the chapter.

**PRACTICE TESTS** Sample exams, comparable in length to that of actual classroom tests, are provided at the end of each chapter.

**EXTENDED APPLICATIONS** Each chapter ends with an application that uses topics covered in the chapter. Interested students can be assigned these applications for presentation in class and/or to work the accompanying exercises for extra credit.

**SECOND COLOR** A second color is used to indicate key steps in the examples and for the explanatory side comments accompanying the examples. Key formulas, definitions, and procedures are also highlighted using color. A vertical colored bar runs along each example, making the example stand apart from the rest of the text's material.

**SUPPLEMENTS** An *Instructor's Guide with Tests* contains five chapter tests for each chapter. Two of the tests for each chapter are in multiple choice format. A *Solutions to Even-Numbered Exercises*, featuring complete solutions to all even-numbered exercises, is available. A *Student Study Guide*, containing problems in a semiprogrammed format, with solutions, provides additional guidance for students requiring extra help. A *Student Solutions Guide* contains solutions to all the odd-numbered exercises in the text. Students can use this as an additional source of examples.

We are grateful to the people who reviewed all or part of the manuscript for their many insightful suggestions: Ann Anderson, Broward Community

College; Arthur Dull, Diablo Valley College; Barbara Juister, Elgin Community College; Theodore Laetsch, University of Arizona; Pamela E. Matthews, Chemeketa Community College; Mary McCammon, Pennsylvania State University; Raymond McGivney, University of Hartford; Bruce Partner, Ball State University; Mark Phillips, Cypress College; B. Louise Whisler, San Bernardino Valley College.

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Marcus McWaters  
You-Feng Lin

# To the Student

This is your book. Its purpose is to help you learn algebra. To use the book to your best advantage, read with pencil in hand and keep a good supply of paper handy. Every new concept or technique is illustrated by completely worked-out examples. As you work through the examples, pay attention to the comments printed in color on the right-hand side of the examples. Read ahead and be prepared to ask your instructor any questions you may have that are not cleared up after hearing the class lecture.

Always work the homework problems assigned by your instructor. Work alone in order to develop confidence in your ability to understand the topics covered in class. If you have trouble working an exercise, refer to the examples cited in the instructions for that exercise. Review these examples and try again. If you are still unsuccessful, see your instructor for help. If you miss class and cannot find out what your homework assignment is, work the Standard Assignment given for the section or sections you miss.

The Warnings, which appear throughout the book, will point out common errors that students make. Be aware of these common mistakes and make note of the proper technique given in the Warnings.

The review material at the end of each chapter is an important part of the book. Read the Key Words and Phrases in the Chapter Summary and be sure you know what they mean. If you are not clear on the meaning of any listed word or phrase, refer to the section in which that word or phrase is defined (the numbers in brackets give the section number). Review the Key Concepts and Rules and then work through the Review Exercises. These exercises are keyed to an appropriate section so that you may refer to this section if you have any difficulty solving a problem.

Once you have finished the Review Exercises you are ready to take the Practice Test. The answers to these tests are in the answer section at the back of the text. This exam should tip you off to any existing weakness in your skills. Review once again the sections you are having difficulty with. You should now be ready for your class test. Good luck!

Marcus McWaters  
You-Feng Lin



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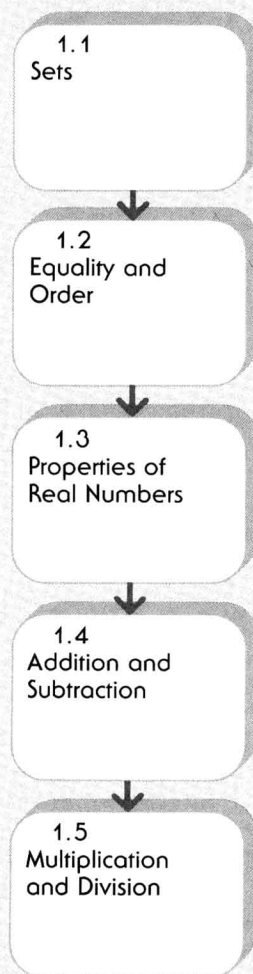
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# 1

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## The Real Numbers



## 1.1 Sets

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The familiar word “set” is used in both of the following sentences: “A complete set of baseball cards is worth a fortune,” and “Mom wants a set of fine china.” A **set** is a collection of objects. The objects in a set are called the **elements** or **members** of the set. In the study of algebra we are primarily interested in sets of numbers. The numbers used in counting, 1, 2, 3, 4, 5, 6, 7, 8, . . . are the elements of the set we call the **natural numbers**.

In order to specify or define a set we either

1. List the elements of the set, or
2. Describe the elements of the set.

When listing the elements of a set, it is customary to enclose the listed elements in braces,  $\{ \}$ . We say that two sets  $A$  and  $B$  are **equal**, or  $A = B$ , if they contain exactly the same members.

EXAMPLE 1

- (a) The set described by the phrase “the first five natural numbers” can be listed as  $\{1, 2, 3, 4, 5\}$ .
- (b) The set described by the phrase “the natural numbers less than 6 and larger than 3” can be listed as  $\{4, 5\}$ .

To indicate that an object,  $a$ , is an element of the set  $A$  we use the symbol  $\in$ . Specifically,

$a \in A$  means “ $a$  is an element of  $A$ ” so  $5 \in \{3, 5, 7\}$  means “5 is an element of  $\{3, 5, 7\}$ .”

The set of people in your algebra class who are over eight feet tall is a peculiar set; it has no members. A set with no members is called the **empty**, or **null, set** and is represented by a special symbol.

$\emptyset$  denotes “the empty set”

In algebra we often use a symbol, which is usually a letter, to denote an unspecified element in a given set. When the given set contains more than one element, such a symbol is called a **variable**. A symbol used to represent a specific element in a set is called a **constant**. Physicists use the letter  $c$  as a constant to represent the speed of light ( $c = 300,000,000$  m/sec).

Variables are helpful in describing sets through **set-builder notation**. In this notation,

$$\{x \mid x \text{ is a natural number less than six}\}$$

describes the set  $\{1, 2, 3, 4, 5\}$ . The vertical bar following  $x$  is read “such that.” The variable  $x$  represents an unspecified natural number.



The first and last braces are read together as “the set of all.” Thus,  $\{x \mid x \text{ is a natural number less than six}\}$  is read: “The set of all  $x$  such that  $x$  is a natural number less than six.” Generally,  $\{x \mid x \text{ has property } P\}$  designates the set of all  $x$  such that  $x$  has the given property  $P$ .

#### EXAMPLE 2

List the elements of  $\{x \mid \text{twice } x \text{ is } 8 \text{ and } x \text{ is a natural number}\}$ .

*Solution:* Since twice  $x$  is 8 exactly when  $x = 4$ , the desired set is  $\{4\}$ .

If each element of a set  $A$  is also an element of a set  $B$ , we say that  $A$  is a **subset** of  $B$ . The symbol  $\subset$  is used to denote the subset relation, so that  $A \subset B$  is read: “ $A$  is a subset of  $B$ .”

$A$  is a subset of  $B$  or  $A \subset B$  means every element of  $A$  is an element of  $B$ .

For example,  $\{1, 3\} \subset \{1, 2, 3, 4, 5\}$  since 1 and 3 are both in the set  $\{1, 2, 3, 4, 5\}$ . Similarly,  $\{\text{Russia, Germany, France}\} \subset \{x \mid x \text{ is a country currently using the metric system}\}$  since Russia, Germany, and France all use the metric system. The slash,  $/$ , together with the symbols  $\subset$ ,  $=$ , and  $\in$  is read “not . . .,” and thus

$A \not\subset B$  indicates that  $A$  is *not* a subset of  $B$ ,

$A \neq B$  indicates that  $A$  is *not* equal to  $B$ , and

$x \notin A$  indicates that  $x$  is *not* an element of  $A$ .

For instance,  $\{1, 3, 6\} \not\subset \{1, 2, 3, 4, 5\}$  since 6 is not a member of  $\{1, 2, 3, 4, 5\}$ . Similarly,  $\{1, 3, 6\} \neq \{1, 2, 3, 4, 5\}$ , and  $7 \notin \{1, 2, 3, 4, 5\}$ .

## Some Important Sets of Numbers

The natural numbers are adequate for representing the profit in dollars that a successful business earns in a given year. However, if we want to represent the loss a business may show in a given year or a “break-even” year, we need to introduce zero and negative numbers. This leads to our first three sets of numbers: natural numbers, whole numbers, and integers.

*The Natural Numbers:*  $\mathbf{N}$ , whose elements are 1, 2, 3, 4, 5, 6, . . . (The three dots, . . . , may be read as “and so on.”)

*The Whole Numbers:*  $\mathbf{W}$ , whose elements are 0, 1, 2, 3, 4, 5, . . .

*The Integers:*  $\mathbf{I}$ , whose elements are 0,  $\pm 1$ ,  $\pm 2$ ,  $\pm 3$ ,  $\pm 4$ , . . . (Here we use  $\pm 1$  to indicate the two numbers 1 and  $-1$ .)

If two people equally split a candy bar, however, none of these numbers will properly represent the portion of the bar (one-half) that each person gets. This requires us to introduce the rational numbers.