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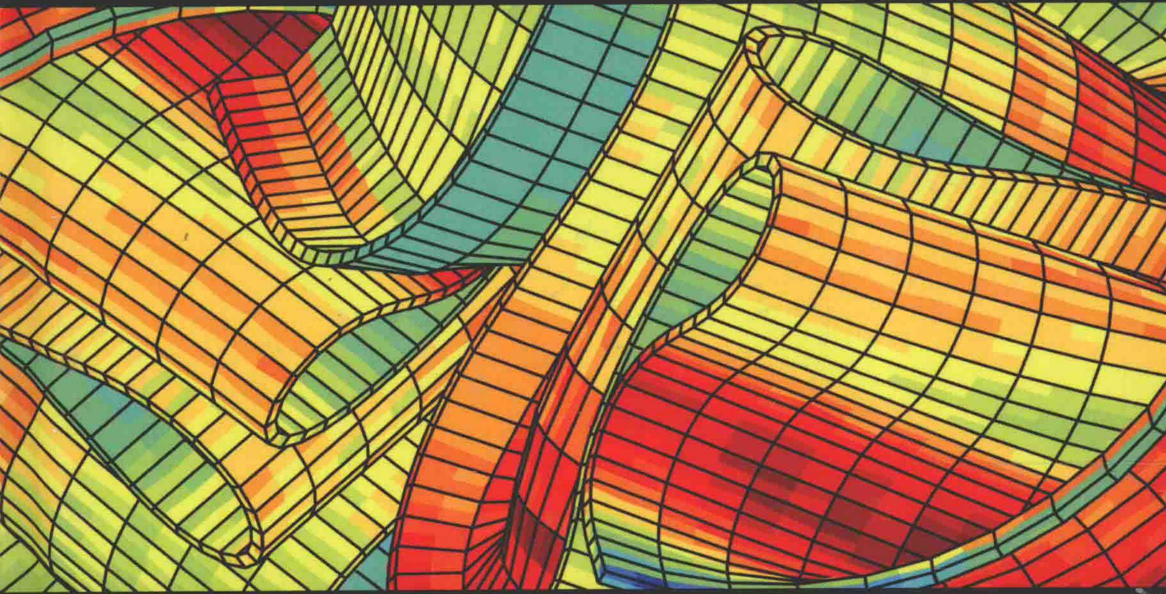

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Numerical Methods in Contact Mechanics

To Alexandra, Andrey and Daniel

Foreword

The area of contact mechanics has become a vivid research field since the modeling of engineering problems has become a lot more sophisticated. This is due to the available computing power that has led to more refined models including contact constraints. This book by Vladislav Yastrebov is related to this still emerging area of contact mechanics. It starts from the basic principles and geometrical relations of contact mechanics and then moves to the essential issue, the detection of contact constraints. Here, the book includes valuable help for those who want to implement contact algorithms since the detection procedures are described in detail, including many exceptions. The formulation of contact problems, again, provides many insights into the complex contact behavior and gives a complete overview with respect to frictionless and frictional contact. This is also true for the chapter that describes the numerical procedures for contact problems. These are discussed in detail and well presented such that the reader will understand the different approaches that can be applied to solve nonlinear contact.

The book will be useful as an introduction to contact mechanics and related algorithms for graduate students who have the necessary background in mathematics and continuum mechanics. However, the book is also a reliable and comprehensive source for researchers who are interested in implementing algorithms and discretization schemes for the solution of nonlinear contact problems. Last but not least, design engineers from industry can use this book as background information for contact analyses related to, e.g. forming, forging and other problems that involve contact and friction.

Prof. Dr. Ing. Peter WRIGGERS

December 2012
Institute of Continuum Mechanics
Leibniz Universität
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Preface

Nowadays, contact and friction are particularly important for our civilization. Think, for example, about car brakes, wheel–rail contact, assembled pieces in engines and turbines, bearings and gears in mechanical devices and electromechanical contacts. Numerical simulations permit us to study and improve these complex systems involving contact, friction and wear. This book answers the question of what is behind these simulations and uncovers for readers the underlying machinery of the finite element analysis in contact mechanics.

Regardless of the prevalence of contact and friction, these phenomena are hard to study experimentally because of their multiscale/multiphysical nature and the inaccessibility of contact interfaces to direct observations. Likewise, these problems are challenging for numerical treatment due to the particularity of contact and friction conditions and complexity of involved algorithms. Moreover, the mathematics and non-trivial notions introduced in this branch of computational mechanics are hard to comprehend for beginners. Thus, the first motivation of this book is to introduce new people to this field. The second motivation is to expose all involved components of computational contact in its integrity and interconnection: geometry, detection and resolution. And finally, I would like to expose some original developments in computational contact mechanics.

I address this book to students, engineers and researchers who solve contact problems by means of the finite element method. Also, I am aiming at developers wishing to implement or improve contact algorithms in their commercial or in-house finite element software. To make the book accessible to people unfamiliar with basics of the computational contact, I shall introduce all terms and notions and give many examples. For all developments in contact geometry I used a new tensor algebra, so some effort are needed from the reader to “get used to it”. But I believe that for readers familiar with programming, this novelty should not present a difficulty, because the main concept is transparent – array of arrays.

Contact algorithms are rich in details that are seemingly negligible but are crucial for the robustness and accuracy of the code. So, based on our experience, I made an attempt to expose most of them. Furthermore, as the implementation of contact algorithms is delicate (both for contact detection and resolution steps), it requires an extended validation and testing. For that purpose, I standardized and exposed many tests from the literature and suggested some new ones.

I hope that this book will introduce new people to the field of computational contact mechanics and that the ideas expressed here will engender the development of new methods and approaches to make the simulation of contact more reliable and accurate.

This book would not be possible without the help and encouragement of Georges Cailletaud, Frédéric Feyel and the financial support of CNRS and SNECMA, which I gratefully acknowledge. I express my thanks to my dear wife Alexandra, my sons, Andrey and Daniel, my parents and my brother for their love, patience and comprehension. I am grateful to my colleagues Djamel Missoum-Benziane and Nikolay Osipov for their constant support, help and friendship. I also acknowledge André Pineau, Jacques Besson and Samuel Forest for creating a stimulating scientific atmosphere. I thank also Liliane Locicero, Konaly Sar, Odile Adam, and Anne Piant for their permanent administrative help, empathy and friendly attention.

Vladislav A. YASTREBOV

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December 2012

Notations

Vectors and tensors

– *Scalar* (zero-order tensor) – small Latin and Greek letters:

$$a, \alpha, b, \dots$$

– *Vector* (first-order tensor) – underlined small bold Latin and Greek letters:

$$\underline{\boldsymbol{c}}, \underline{\boldsymbol{\beta}}, \underline{\boldsymbol{d}}, \dots$$

– *Second-order tensor* – capital bold Latin letters underlined twice:

$$\underline{\underline{\boldsymbol{E}}}, \underline{\underline{\boldsymbol{F}}}, \dots$$

– *Higher order tensor* – capital bold Latin letters underlined twice with upper left index of order:

$${}^{3}\underline{\underline{\boldsymbol{G}}}, {}^{4}\underline{\underline{\boldsymbol{H}}}, \dots$$

V-Vectors and V-tensors

– *V-scalar* ("vector of scalars") – small Latin and Greek letters underlined by a wave:

$$\underset{\sim}{\mathrm{i}}, \underset{\sim}{\gamma}, \dots \in {}_{1}^{m}\mathbb{S}_{0}^{n}$$

– *V-vector* ("vector of vectors") – small Latin and Greek letters underlined by a line and a wave:

$$\underset{\sim}{\underline{\boldsymbol{j}}}, \underset{\sim}{\underline{\boldsymbol{\varepsilon}}}, \dots \in {}_{1}^{m}\mathbb{S}_{1}^{n}$$

– *V-tensor* (“vector of tensors”) – capital bold Latin letters underlined by a double line and a wave:

$$\underset{\sim}{\underline{\underline{\boldsymbol{K}}}}, \underset{\sim}{\underline{\underline{\boldsymbol{L}}}}, \cdots \in {}_{1}^{m}\mathbb{S}_{2}^{n}$$

T-Vectors and T-tensors

– *T-scalar* (“tensor of scalars”) – capital bold Latin letter underlined by a double wave:

$$\underset{\approx}{\mathrm{M}}, \underset{\approx}{\mathrm{N}}, \cdots \in {}_{2}^{m}\mathbb{S}_{0}^{n}$$

– *T-vector* (“tensor of vectors”) – small Latin and Greek letters underlined by a line and a double wave:

$$\underset{\approx}{\underline{\boldsymbol{o}}}, \underset{\approx}{\underline{\boldsymbol{\eta}}}, \cdots \in {}_{2}^{m}\mathbb{S}_{1}^{n}$$

– *T-tensor* (“tensor of tensors”) – capital bold Latin letters underlined by a double line and a double wave:

$$\underset{\approx}{\underline{\underline{\boldsymbol{P}}}}, \underset{\approx}{\underline{\underline{\boldsymbol{Q}}}}, \cdots \in {}_{2}^{m}\mathbb{S}_{2}^{n}$$

Vector and tensor operations

– $\| \underline{\boldsymbol{a}} \|$: Euclidean norm of a vector;

– $\det \underline{\underline{\boldsymbol{A}}}$: determinant of a tensor;

– $\underline{\underline{\boldsymbol{I}}}$: unit tensor;

– $\underset{\approx}{\mathrm{I}}$: unit t-scalar;

– $\mathrm{tr}\underline{\underline{\boldsymbol{A}}}$: trace of a tensor;

– $\underline{\underline{\boldsymbol{A}}}^{-1}$: inverse of a tensor;

– $\underline{\underline{\boldsymbol{A}}}^{T}$: transpose of a tensor;

– ${}^{i}\underline{\underline{\boldsymbol{A}}} \cdot {}^{j}\underline{\underline{\boldsymbol{B}}} = {}^{i+j-2}\underline{\underline{\boldsymbol{C}}}$: scalar or dot product;

– ${}^{i}\underline{\underline{\boldsymbol{A}}} \times {}^{j}\underline{\underline{\boldsymbol{B}}} = {}^{i+j-1}\underline{\underline{\boldsymbol{C}}}$: vector or cross product;

– ${}^{i}\underline{\underline{\boldsymbol{A}}} \otimes {}^{j}\underline{\underline{\boldsymbol{B}}} = {}^{i}\underline{\underline{\boldsymbol{A}}}\,{}^{j}\underline{\underline{\boldsymbol{B}}} = {}^{i+j}\underline{\underline{\boldsymbol{C}}}$: tensor product;

– ${}^{i}\underline{\underline{\boldsymbol{A}}} \cdot\cdot\, {}^{j}\underline{\underline{\boldsymbol{B}}} = {}^{i+j-4}\underline{\underline{\boldsymbol{C}}}$: tensor contraction.

Other operations

– $(\bullet)^{\cdot} = \frac{d\bullet}{dt}$: full time derivative;

– $\delta(\bullet),\ \Delta(\bullet)$: first variations;

– $\bar{\delta}(\bullet),\ \bar{\Delta}(\bullet)$: full first variations;

– $\Delta\delta(\bullet)$: second variation;

– $\bar{\Delta}\bar{\delta}(\bullet)$: full second variation;

– $\nabla \otimes (\bullet)$: gradient;

– $\nabla \cdot (\bullet)$: divergence;

– $\nabla \times (\bullet)$: rotor.

Miscellaneous

– δ_i^j: Kronecker's delta $\delta_i^j = 1$, if $i = j$ else $\delta_i^j = 0$;

– $\langle x \rangle = \frac{1}{2}(x + |x|)$: Macaulay brackets;

– $[\bullet,\ \bullet]$; $(\bullet,\ \bullet)$; $(\bullet,\ \bullet]$: closed, open, open-closed intervals;

– $\forall$, $\exists$, $\exists!$, $\exists!!$, $\nexists$: for all, exists, exists only one, exists infinitely many, does not exist;

– $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$: sufficient, necessary, sufficient and necessary conditions;

– $\min$, $\max$, ext, $\sup$, $\inf$: minimum, maximum, extremum, supremum, infimum;

– $\widetilde{\min}$, $\widetilde{\max}$: global minimum, global maximum;

– $i = \overline{1, n}$: i changes from 1 to n.

Abbreviations

– PM: penalty method;

– LMM: Lagrange multiplier method;

– ALM: augmented Lagrangian method;

– FEM: finite element method;

– FEA: finite element analysis;

– CAD: computer-aided design;

– NTN: node-to-node;

– NTS: node-to-segment discretizations;

– MPC: multi-point constraints;

– PDN: partial dirichlet–neumann;

– SDMR: single detection multiple resolution;

– MDMR: multiple detection multiple resolution.

Remark. Macaulay brackets, dist$(.,.)$ and $\theta(.)$ functions.

Throughout the book, we use the notation of Macaulay brackets.

$$\langle x\rangle = \begin{cases} x, & x \geq 0, \\ 0, & x < 0 \end{cases}; \qquad \langle -x\rangle = \begin{cases} -x, & x \leq 0, \\ 0, & x > 0 \end{cases}$$

The θ function is a similar notation widely used in both engineering and mathematical literature:

$$\theta(x) = \max(x,0) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0 \end{cases}; \quad \theta(-x) = \min(x,0) = \begin{cases} -x, & x \leq 0, \\ 0, & x > 0 \end{cases}$$

or a more general dist$(,)$ function:

$$dist(x,\Omega) = \begin{cases} dist(x,\partial\Omega), & x \notin \Omega \\ 0, & x \in \Omega, \end{cases}$$

where $dist(x,\partial\Omega)$ is a somehow defined distance from point x to the closure of the set Ω. For example, in the simplest case $\Omega = \mathbb{R}_-$, $x \in \mathbb{R}$, then $\partial\mathbb{R}_- = 0$:

$$dist(x,\mathbb{R}_-) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0 \end{cases}; \qquad dist(x,\mathbb{R}_+) = \begin{cases} -x, & x \leq 0, \\ 0, & x > 0. \end{cases}$$

All these functions are equivalent for the considered case and interchangeable, so the reader is invited to interpret the Macaulay brackets as one of the above-mentioned functions to which he/she is more accustomed.

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