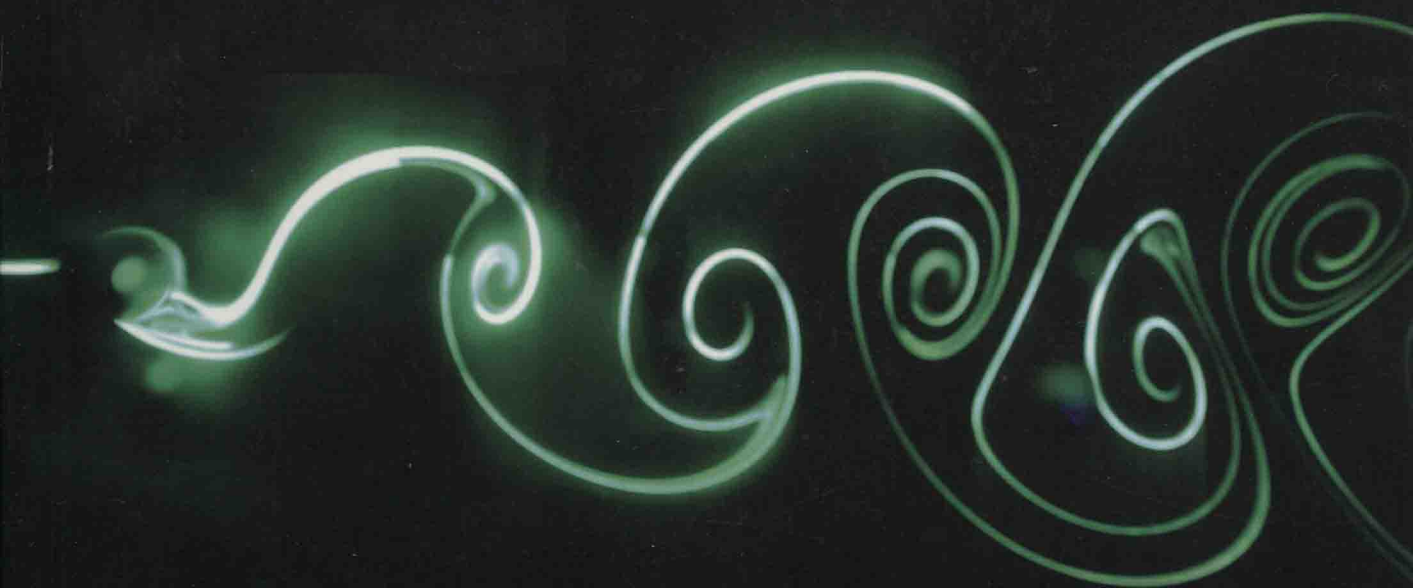


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# Physical Hydrodynamics

SECOND EDITION

ETIENNE GUYON

JEAN-PIERRE HULIN

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# Physical Hydrodynamics

*Second Edition*

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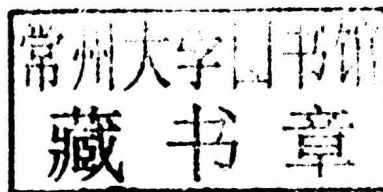
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## PHYSICAL HYDRODYNAMICS



# Foreword

Fluid mechanics is a subject with a long history, but yet is young with regard to recent discoveries, and has many applications which affect everyday life. Its history is a parade of the great names of science: from the eighteenth century the Bernoullis, Euler and Lagrange; from the nineteenth century Cauchy, Navier, Stokes, Helmholtz, Rayleigh, Reynolds and Lamb; and from the twentieth century Couette, Prandtl, G.I. Taylor and Kolmogorov.

In the natural environment, we can now rely on the 5-day weather forecasts and tornado warnings; early success in predicting tides is today applied to automated tsunami warnings; understanding the circulation in the oceans and atmosphere is applied to pollution, the ozone hole and climate change. In the interior of the earth, fluid mechanics is important in mantle convection, volcanoes and their dust clouds, oil reservoirs and possible CO<sub>2</sub> sequestration.

Fluid mechanics is central to many industries. The design of aircraft developed from simple ideas in the early twentieth century to low-drag forms of wings with winglets at their tips and improved body profiles by the end of the century. At the same time, jet-noise was dramatically reduced with the introduction of wide entrance bypass fans which shield the fast jet. Simple and complex fluids are processed in various manufacturing industries: for glass and other materials, chemical engineering and food processing.

Recent research in fluid mechanics includes: microfluidics at the micron scale with the possibility of multiple simultaneous tests of small biological samples; similar scales and effects of wettability in ink-jet-printing; the ideas of convection allowing the design of energy-efficient buildings by using natural convection; and the control of instabilities and turbulence.

With such a wealth of ideas and applications, there is a major challenge of how to teach the subject. Some material is best left to specialized Masters courses. But the basic core has to be taught in a way to help students' progress to the advanced topics, current and future. The authors of this book have adopted in my opinion an approach and style which should interest and educate students, preparing them for the future. I fear that some alternative approaches fail on this: some engineering courses have an over-reliance on Computational Fluid Dynamics, which can be unsafe in novel applications; some mathematical courses are lost in the enormous difficulty of proving the governing equations have, or do not have, solutions in the simplest of situations, an open Clay prize problem. The approach in this book is grounded in experiment and reality. The chosen structure of the presentation helps students come to deep insights into the subject.

In my opinion, the subject of fluid mechanics has benefited in the last 30 years from the contributions of French physicists such as the authors of this book, bringing a fresh approach to the subject along with novel experimental techniques and an appreciation of practicalities.

*John Hinch*  
*University of Cambridge*



# Introduction

The place of fluid mechanics has been poorly defined in the scientific world. On the one hand, it has a strong connection with applied mathematics, in particular in France where the impact of such research has been extremely important from the nineteenth century onward. In recent times, this tendency has been reinforced by significant developments of computational science in such domains as turbulence and instabilities. On the other hand, engineering communities deal with sophisticated technical problems regarding flows and transfer of heat and matter, which require approximate solutions, but where some basic understanding is often missing. A consequence of this state of affairs is that both physicists and chemists, who are also involved in these issues, have remained on the sidelines. This was the case in the basic training of the authors of the present book. Yet another possible reason has been the strong polarization of physicists towards problems on the quantum scale, and relativity, although the pioneering giants of the field (Einstein, Bohr, Heisenberg . . .) had a good background in continuum mechanics. For a long time, physicists did not follow closely the important developments in the field, or followed them only indirectly: for example, the information on perfect flows and vorticity, of three of the authors of the present volume was triggered by their experimental work on superfluid Helium. The study of hydrodynamic instabilities was approached by analogies with phase transitions. Physical chemists dealt with complex materials such as liquid crystals, colloids, or polymers: they had to apply mechanics but also other approaches like scaling laws.

This schematic description of a mutual ignorance applies less and less frequently nowadays. Over the last decades, the interaction between scientists trained in physics and in mechanics has continued to increase through a great deal of joint research. The first edition of *Physical Hydrodynamics*, which was published in English 13 years ago, is presented in a completely renewed and expanded form and content while keeping the same physical, pedagogical approach. Moreover, 36 exercises have been added at the end of selected chapters and their correction is provided at the end of the book.

Indeed, in the course of the last 30 years, physicists and chemists have approached the subject in a manner complementary to classical mechanics. This implied the use of experimental and theoretical tools which had been developed for domains of physics such as condensed matter, statistical physics and material science. International conferences such as the APS-DFD or Euromech meetings led to interactions between participants with very diverse backgrounds. A broader field of applications has also been considered, such as in the life sciences, in addition to more classical engineering ones. But this also implies less formal ways of reasoning, such as for example, scaling approaches making extensive use of the classical tools of experimental physics.

The present book is based on our experience as experimental physicists. Actually, the word “hydrodynamics” is somewhat misleading and we use it in the sense that was given in *Hydrodynamica* by Daniel Bernoulli. We deal here not only with liquids but also with gas flows such that compressibility effects cannot be ignored. We have therefore excluded such problems as high-velocity gas flows, which involve a coupling between the equation of motion and thermodynamics. One specificity of our approach is to attempt to tie in, as often as possible, the macroscopic behavior of fluids to their microscopic properties. We also rely, as often as is reasonable, on order of magnitude arguments rather than just



on formal derivations. Thus, in introducing dimensionless numbers, as is customary in the field, we stress the fact they are ratios (in Greek, αναλογία, *analogy*, which means ratio) of similar quantities such as characteristic lengths, times, energies, and so on, rather than relying on algebraic manipulations based on dimensional analysis. This was the approach of Osborne Reynolds in introducing the dimensionless quantity which bears his name. Rather than “cookbook recipes”, we look for a deeper physical understanding of the mechanisms at play in a field extremely rich in experimental observations.

The book can be roughly divided into three parts:

The first five chapters (1 to 5) are devoted to the basic and classical elements of fluid mechanics. In the first chapter, we give a schematic description typical of elementary out-of-equilibrium statistical physics on non-equilibrium transport processes and of the spectroscopic tools used for these studies. Diffusive versus convective processes are discussed in the following chapter. Kinematics and dynamics of flows as well as the use of conservation laws provide the classical foundation of the book.

Different regimes of flows are analyzed in the following four chapters (6 to 9): potential flows, flows governed by vorticity (with an extension to rotating flows as encountered in geostrophic conditions); quasi-parallel flows and low Reynolds numbers flows.

The last chapters (10 to 12) are devoted to more detailed and complex phenomena which simultaneously involve different mechanisms: boundary-layer flows lead to many applications as, in particular, those found in chemical engineering. Unstable flows are presented in a simple way emphasizing the coupling between the different mechanisms involved and, finally, the chapter on turbulent flows in which mathematical developments have been kept to a minimum.

As stated above, our physicists’ approach was initiated by our research work on various themes on fluid mechanics not often dealt with by specialists of classical mechanics (such as, sound in superfluid  $\text{He}^4$ , instabilities in liquid crystals, percolation in porous media). Our basic training made use of references such as Landau’s *Fluid Mechanics*. However, over the course of time, we benefited from other communities and, in particular, the British school of G.I. Taylor (the exquisite four volumes of his complete works) and direct contacts with G.K. Batchelor who provided us an access to a pragmatic approach to fluids where ingenious experimental discoveries are accompanied with rigorous reasoning and more mathematical treatment well connected with experimental reality. The teaching of John Hinch, who has accepted to write a “Foreword” to this book, and Keith Moffatt, who introduced fluid mechanics to a number of us in a famous summer institute in Les Houches, in 1973, have provided us with a number of fine tools suitable both for our research as well as for undergraduate and graduate teaching at the origin of this book. In the USA, connections with A. Acrivos, J. Brady, G. Homsy, J. Koplik, L. Mahadevan, H. Stone and many other colleagues have broadened our vision of the field. Our initial curiosity was stirred up by several films of the NCFMF (National Committee for Fluid Mechanics Films), and by the Album of Fluid Motion of M. Van Dyke (which inspired our “Ce que disent les fluides” in French). More recently, the Multimedia Fluid Mechanics project directed by G. Homsy has broadened the range of documents available for teaching purposes. Our English speaking colleagues which have used the recent French edition of the book in their classes have encouraged us to produce this new, second edition of *Physical Hydrodynamics*.

Above all, it is in the everyday life of class and laboratory activities that the book has been constructed. The research and teaching of P. Bergé, B. Castaing, C. Clanet, Y. Couder, M. Farge, M. Fermigier, P. Gondret, E. Guazzelli, J.F. Joanny, F. Moisy, B. Perrin, Y. Pomeau, M. Rabaud, D. Salin, B. Semin, J.E. Wesfreid and many others are at the origin of elements of this book. We wish to thank particularly several colleagues who have directly contributed to specific parts of this book: C. Allain (non-Newtonian flows),

A. Ambari (polarography), A.M. Cazabat (dynamics of wetting), M. Champion (flames), C. Clanet and D. Quéré (capillarity), P. Gondret (exercises), F. Moisy (text and exercises on turbulence and rotating flows), C. Nore (MHD), N. Ribe (free liquid jets) and J. Teixeira (spectroscopy of fluids).

The translation, as well as the necessary adjustment to the English scientific form, has been made essentially by C.D. Mitescu. We express our gratitude to Dr. Natalie Reinert, DVM, who has edited the book during the last year. She has also contributed greatly to the style of the translation. Our gratitude goes to Mrs Nicole Mitescu, wife of C.D. Mitescu and mother of Nathalie for proof reading the book. We thank her as well as Marie Yvonne, Danièle and Christine for their patience during the preparation of this new edition.

E. Guyon, J.P. Hulin, L. Petit, C.D. Mitescu



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