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MARK DUGOPOLSKI



Third Edition

Algebra for College Students

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ALGEBRA FOR COLLEGE STUDENTS, THIRD EDITION

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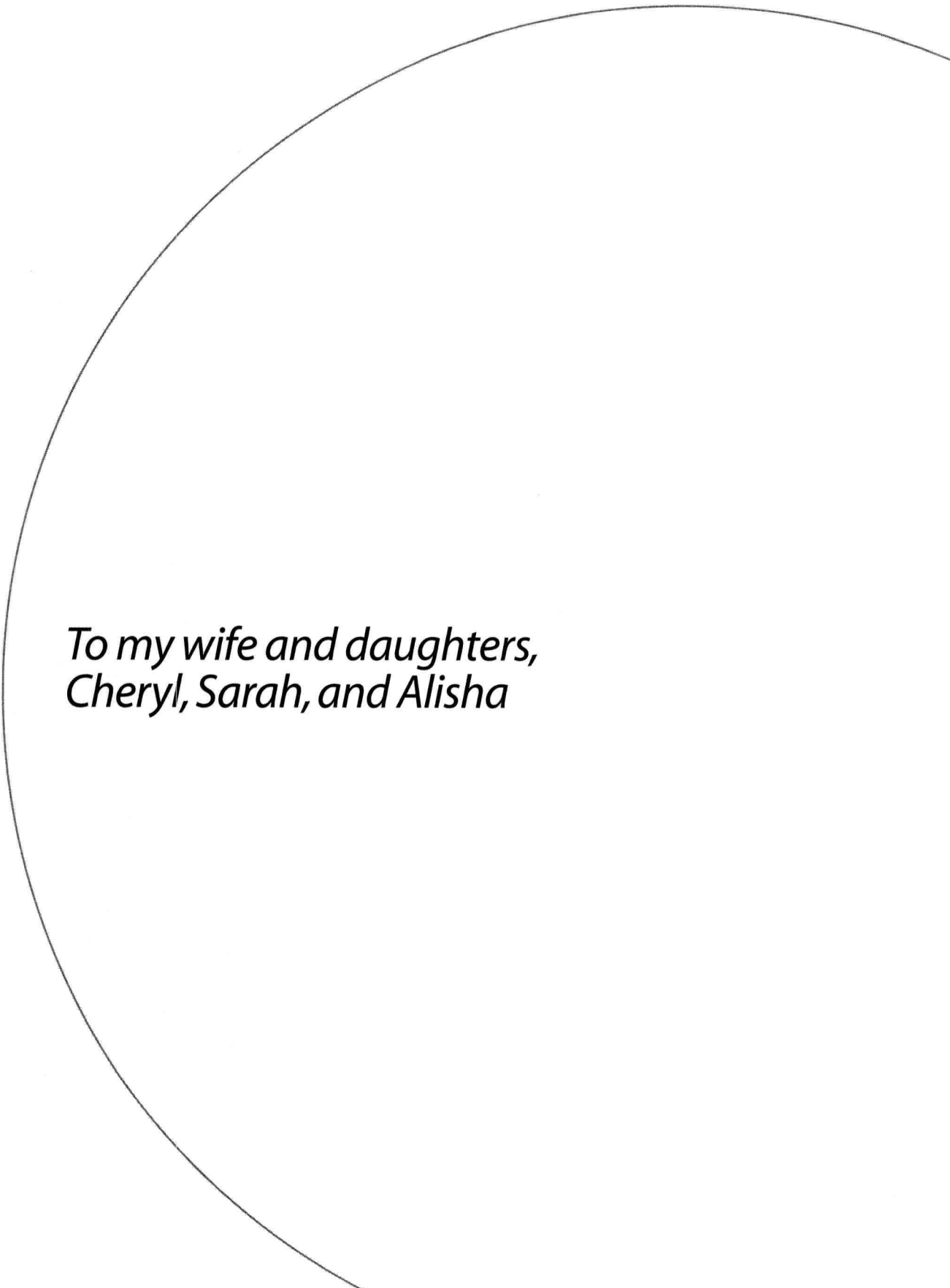
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*To my wife and daughters,
Cheryl, Sarah, and Alisha*

A *lgebra for College Students*, Third Edition, is designed to provide students with the algebra background needed for further college-level mathematics courses. The unifying theme of this text is the development of the skills necessary for solving equations and inequalities, followed by the application of those skills to solving applied problems. My primary goal in writing the third edition of *Algebra for College Students*, has been to retain the features that made the second edition so successful, while incorporating the comments and suggestions of second-edition users. As always, I endeavor to write texts that students can read, understand, and enjoy, while gaining confidence in their ability to use mathematics. Although a complete development of each topic is provided in *Algebra for College Students*, Third Edition, the text *Elementary Algebra*, Fourth Edition, in this series would be more appropriate for students with no prior experience in algebra.

Content Changes

While the essence of previous editions remains, the topics have been rearranged to reflect the current needs of instructors.

- Functions are introduced in Section 3.5. After that, function topics are revisited where appropriate. For example, polynomial functions appear in Section 5.3, the domain of rational expressions and functions is covered in Section 6.1, and the domain of radical functions is covered in Section 7.1. Functions also appear in Chapter 8, with quadratic equations, and again in Chapters 9, 10, and 11.
- Section 3.5 on functions and relations has been revised with expanded coverage of the concepts of domain and range and the different forms of a function. *Graphs of Functions*, previously Section 3.6, which included graphs of constant functions, quadratic functions, absolute value functions, and square-root functions has been moved to Chapter 9, Section 1.
- The distance and midpoint formulas now appear in Section 3.1, where graphing lines is introduced. The distance formula also appears in Section 12.2, where it is used to develop the equation of a parabola from the geometric definition.
- In Chapter 4, the two sections on determinants and Cramer's rule have been condensed into one, Section 4.5.
- To streamline Chapter 5, the section covering division of polynomials has been moved to Chapter 6 on rational expressions.
- Sections 7.1 and 7.2 have been swapped from the second edition so that radicals now appears as the first topic in Chapter 7, Section 1, and rational exponents now appears in Section 7.2.
- **NEW!** Transformations of graphs has been added to Chapter 9 and appears in Section 9.2.
- **NEW!** The factor theorem has been added to Chapter 10 and appears in Section 10.1.

- **NEW!** In Section 12.2, the discussion of parabolas was expanded to include parabolas that open right or left. In Section 12.4 the discussion of hyperbolas was expanded to include hyperbolas that are not centered at the origin.
- **NEW!** Appendix A Geometry Review has been updated to include review exercises to help students recall geometry concepts and formulas.

In addition to these changes, the text and exercise sets have been carefully revised where necessary. Many new, applied examples have been added to the text and many new, applied exercises have been added to the exercise sets. Particular care has been given to achieving an appropriate balance of problems that progressively increase in difficulty from routine exercises in the beginning of the set to more challenging exercises at the end of the set. As in earlier editions, fractions and decimals are used in the exercises and throughout the text discussions to help reinforce the basic arithmetic skills that are necessary for success in algebra.

Features

- Each chapter begins with a Chapter Opener that discusses a real application of algebra. The discussion is accompanied by a photograph and, in most cases by a real-data application graph that helps students visualize algebra and more fully understand the concepts discussed in the chapter. In addition, each chapter contains a Math at Work feature, which profiles a real person and the mathematics that he or she uses on the job. These two features have corresponding real data exercises.
- The third edition continues to emphasize real-data applications that involve graphs. Applications appear throughout the text to help demonstrate concepts, motivate students, and to give students practice using new skills. Many of the real-data exercises contain data obtained from the Internet. Internet addresses are provided as a resource for both students and teachers. An Index of Selected Applications listing applications by subject matter is included at the front of the text.
- Every section begins with In This Section, a list of topics that shows the student what will be covered. Because the topics correspond to the headings within each section, students will find it easy to locate and study specific concepts.
- Important ideas, such as definitions, rules, summaries, and strategies, are set apart in boxes for quick reference. Color is used to highlight these boxes as well as other important points in the text.
- The third edition contains margin features that appear throughout the text:

Calculator Close-ups give students an idea of how and when to use a graphing calculator. Some Calculator Close-ups simply introduce the features of a graphing calculator, where others enhance understanding of algebraic concepts. For this reason, many of the Calculator Close-ups will benefit even those students who do not use a graphing calculator. A graphing calculator is not required for studying from this text.

Study Tips are included in the margins throughout the text. These short tips are meant to continually reinforce good study habits and to remind students that it is never too late to make improvements in the manner in which they study.

Helpful Hints are short comments that enhance the material in the text, provide another way of approaching a problem, or clear up misconceptions.

- At the end of every section are Warm-up exercises, a set of ten simple statements that are to be answered true or false. These exercises are designed to provide a smooth transition between the ideas and the exercise sets. They help students understand that every statement in mathematics is either true or false. They are also good for discussion or group work.
- Most section-ending exercise sets in the third edition begin with six simple writing exercises. These exercises are designed to get students to review the definitions and rules of the section before doing more traditional exercises. For example, the student might simply be asked what properties of equality were discussed in this section.
- The end-of-section Exercises follow the same order as the textual material and contain exercises that are keyed to examples, as well as numerous exercises that are not keyed to examples. This organization enables the instructor to cover only part of a section if necessary and easily determine which exercises are appropriate to assign. The *keyed exercises* give the student a place to start practicing and building confidence, whereas the *nonkeyed exercises* are designed to wean the student from following examples in a step-by-step manner. *Getting More Involved exercises* are designed to encourage *writing, discussion, exploration, and cooperative learning*. *Graphing Calculator exercises* require a graphing calculator and are identified with a graphing calculator logo. Exercises for which a scientific calculator would be helpful are identified with a scientific calculator logo. Please refer to page xxiii for a visual guide of the icons.
- Every chapter ends with a four-part Wrap-up, which includes the following:

The chapter Summary lists important concepts along with brief illustrative examples.

Enriching Your Mathematical Word Power appears at the end of each chapter and consists of multiple-choice questions in which the important terms are to be matched with their meanings. This feature emphasizes the importance of proper terminology.

The Review Exercises contain problems that are keyed to the sections of the chapter as well as numerous miscellaneous exercises.

The Chapter Test is designed to help the student assess his or her readiness for a test. The Chapter Test has no keyed exercises, thus encouraging the student to work independently of the sections and examples.

- **UPDATED!** At the end of each chapter is a Collaborative Activities feature that is designed to encourage interaction and learning in groups. Many of the Collaborative Activities for the third edition have been updated. Instructions and suggestions for using these activities and answers to all problems can be found in the Instructor's Solutions Manual.
- Making Connections, at the end of Chapters 2–13, are cumulative exercises designed to help students review and synthesize new material with ideas from previous chapters, and in some cases, review material necessary for success in upcoming chapters. Every Making Connections exercise set includes at least one applied exercise that requires ideas from one or more of the previous chapters.

Topic Reinforcement

Several methods are employed in this text to help students retain what they learn and expand and build on previously learned concepts. Most notably, the Making Connections appear at the end of each chapter through Chapter 13 and beginning

with Chapter 2. Making Connections are cumulative sets of exercises that help students to continually practice what they learn. Also, functions are introduced in Section 3.5 and revisited where appropriate throughout the text. This constant reinforcement helps students retain and strengthen their understanding of this important concept. The distance formula is covered in Section 3.1 and reviewed in Section 12.2 where it is used to develop the equations of the conic sections. In Chapter 9, solving quadratics by factoring and the square root property are reviewed prior to the new ideas of completing the square and the quadratic formula. Through these, and similar methods, students retain what they learn and apply what they learn to new concepts.

Supplements for the Instructor

ANNOTATED INSTRUCTOR'S EDITION

This ancillary includes answers to all section ending exercises, review exercises, Making Connections exercises, and chapter tests. Each answer is printed next to each problem on the page where the problem appears. The answers are printed in a second color for ease of use by instructors.

INSTRUCTOR'S TESTING AND RESOURCE CD-ROM

This CD-ROM contains a computerized test bank that utilizes Brownstone Diploma® testing software. The computerized test bank enables instructors to create well-formatted quizzes or tests using a large bank of algorithmically generated and static questions. When creating a quiz or test, the user can manually choose individual questions or have the software randomly select questions based on section, question type, difficulty level, and other criteria. Instructors also have the ability to add or edit test bank questions to create their own customized test bank. In addition to printed tests, the test generator can deliver tests over a local area network or the World Wide Web, with automatic grading.

Also available on the CD-ROM are pre-formatted tests that appear in two forms: Adobe Acrobat (pdf) and Microsoft Word files. These files are provided for convenient access to “ready to use” tests. The tests can also be downloaded as a Word (.doc) file or can be viewed and printed as a (.pdf) file at www.mhhe.com/dugopolski.

INSTRUCTOR'S SOLUTIONS MANUAL

Prepared by Mark Dugopolski, this supplement contains detailed worked solutions to all of the exercises in the text. The solutions are based on by the techniques used in the text. Instructions and suggestions for using the Collaborative Activities feature in the text are also included in the Instructor's Solutions Manual.

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Supplements for the Student

STUDENT'S SOLUTIONS MANUAL

Prepared by Mark Dugopolski, the *Student's Solutions Manual* contains complete worked-out solutions to all of the odd-numbered exercises in the text. It also contains solutions for all exercises in the Chapter Tests. It may be purchased from McGraw-Hill.

DUGOPOLSKI VIDEO SERIES (Videotapes or CD-ROMs)

The videos are text-specific and cover all chapters of the text. The videos feature an instructor who introduces topics and works through selected problems from the exercise sets. Students are encouraged to work the problems on their own and to check their results with those provided.

DUGOPOLSKI TUTORIAL CD-ROM

This interactive CD-ROM is a self-paced tutorial specifically linked to the text that reinforces topics through unlimited opportunities to review concepts and practice problem solving. The CD-ROM contains algorithmically generated chapter-, and section-specific questions. It requires virtually no computer training on the part of students and supports Windows and Macintosh computers.

ONLINE LEARNING CENTER

The Online Learning Center (OLC), located at www.mhhe.com/dugopolski, contains resources for students and instructors.

Through the Instructor Resource Site, instructors can access links to professional resources, a PowerPoint presentation (transparencies), printable tests, group projects, and a link to PageOut.

To access the Instructor Resource Site, instructors must have a passcode that can be obtained by contacting a McGraw-Hill Higher Education representative.

The Student Learning Site is also passcode-protected. Passcodes for students can be found at the front of their texts when newly purchased. *Passcodes are available free to students when they purchase a new text.* Students also have access to algorithmically generated “bookmarkable” practice exercises (including hints), section- and chapter-level testing, audiovisual tutorials, interactive applications, and links to NetTutor™ and other interesting websites.

The Information Center can be accessed by students and instructors without a passcode. Through the Information Center, users can access general information about the text and its supplements.

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Hammond, Louisiana

M.D.

CHAPTER

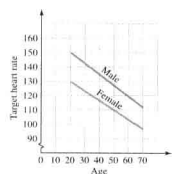
1

The Real Numbers

Everywhere you look people are running, riding, dancing, and exercising their way to fitness. In the past year more than \$25 billion has been spent on sports equipment alone, and this amount is growing steadily.

Proponents of exercise claim that it can increase longevity, improve body image, decrease appetite, and generally enhance a person's health. While many sports activities can help you to stay fit, experts have found that aerobic, or dynamic, workouts provide the most fitness benefit. Some of the best aerobic exercises include cycling, running, and even jumping rope. Whatever athletic activity you choose, trainers recommend that you set realistic goals and work your way toward them consistently and slowly. To achieve maximum health benefits, experts suggest that you exercise three to five times a week for 15 to 60 minutes at a time.

There are many different ways to measure exercise. One is to measure the energy used, or the rate of oxygen consumption. Since heart rate rises as a function of increased oxygen, another easier measure of intensity of exercise is your heart rate during exercise. The desired heart rate, or target heart rate, for beneficial exercise varies for each individual depending on conditioning, age, and gender. In Exercises 101 and 102 of Section 1.4 you will see how an algebraic expression can determine your target heart rate for beneficial exercise.



Chapter Opener

Each **chapter opener** features a real-world situation that can be modeled using mathematics. Each chapter contains exercises that relate back to the chapter opener.

1.4 Evaluating Expressions

(1-33) 33

Evaluate each expression for $a = -1$, $b = 3$, and $c = -4$. See Example 7.

57. $b^2 - 4ac$
58. $\sqrt{a^2 - 4bc}$
59. $\frac{a-b}{a-c}$
60. $\frac{b-c}{b-a}$
61. $(a-b)(a+b)$
62. $(a-c)(a+c)$
63. $\sqrt{c^2 - 2c} + 1$
64. $b^2 - 2b - 3$
65. $\frac{2}{a} + \frac{b}{c} - \frac{1}{c}$
66. $\frac{c}{a} + \frac{c}{b} - \frac{a}{b}$
67. $|a-b|$
68. $|b+c|$

Find the value of $\frac{y_2 - y_1}{x_2 - x_1}$ for each choice of y_1, y_2, x_1 , and x_2 . See Example 8.

69. $y_1 = 4, y_2 = -6, x_1 = 2, x_2 = -7$
70. $y_1 = -3, y_2 = -3, x_1 = 4, x_2 = -5$
71. $y_1 = -1, y_2 = 2, x_1 = -3, x_2 = 1$
72. $y_1 = -2, y_2 = 5, x_1 = 2, x_2 = 6$
73. $y_1 = 2, y_2 = 5, x_1 = 5, x_2 = 4$
74. $y_1 = -5, y_2 = 6, x_1 = 3, x_2 = 4$

Evaluate each expression without a calculator. Use a calculator to check.

75. $-2^2 + 5(3)^2$
76. $-3^2 + 3(6)^2$
77. $(-2 + 5)3^2$
78. $(-3 + 3)6^2$
79. $\sqrt{5^2 - 4(1)(6)}$
80. $\sqrt{6^2 - 4(2)(4)}$
81. $|13 + 2(-5)|^2$
82. $|6 + 2(-4)|^2$
83. $\frac{4 - (-1)}{-3 - 2}$
84. $\frac{2 - (-3)}{3 - 5}$
85. $3(-2)^2 - 5(-2) + 4$
86. $3(-1)^2 + 5(-1) - 6$
87. $-4\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2$
88. $3\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) + 1$
89. $-\frac{1}{2} \cdot 6 - 2$
90. $-\frac{1}{3} \cdot 9 - 6$
91. $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{2}$
92. $\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2}$
93. $|6 - 3 \cdot 7| + |7 - 5|$
94. $|12 - 4| - |3 - 4 \cdot 5|$
95. $3 - 7|4 - (2 - 5)|$
96. $9 - 2|3 - (4 + 6)|$
97. $3 - 4|2 - (4 - 6)|$
98. $3 - (-4) - (-5)$
99. $4|2 - (5 - (-3))|^2$
100. $[5 - (-3)]^2 + |4 - (-2)|^2$

Solve each problem. See Example 9.

101. **Female target heart rate.** The algebraic expression $0.65(220 - A)$ gives the target heart rate for beneficial exercise for women, where A is the age of the woman. How much larger is the target heart rate of a 25-year-old woman than that of a 65-year-old woman? Use the accompanying graph to estimate the age at which a woman's target heart rate is 115.

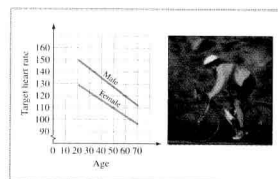


FIGURE FOR EXERCISES 101 AND 102

102. **Male target heart rate.** The algebraic expression $0.75(220 - A)$ gives the target heart rate for beneficial exercise for men, where A is the age of the man. Use the algebraic expression to find the target heart rate for a 20-year-old and a 50-year-old man. Use the accompanying graph to estimate the age at which a man's target heart rate is 115.

Solve each problem.

103. **Perimeter of a pool.** The algebraic expression $2L + 2W$ gives the perimeter of a rectangle with length L and width W . Find the perimeter of a rectangular swimming pool that has length 34 feet and width 18 feet.
104. **Area of a lot.** The algebraic expression for the area of a trapezoid, $0.5h(b_1 + b_2)$, gives the area of the property shown in the figure. Find the area if $h = 150$ feet, $b_1 = 260$ feet, and $b_2 = 220$ feet.

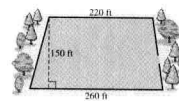


FIGURE FOR EXERCISE 104

105. **Saving for retirement.** The expression $P(1 + r)^t$ gives the amount of an investment of P dollars invested for t years at interest rate r compounded

Margin Notes

Margin notes include **helpful hints**, **study tips**, and **calculator close-ups**. The **helpful hints** point out common errors or reminders. The **study tips** provide practical suggestions for improving study habits. The optional **calculator close-ups** provide tips on using a graphing calculator to aid in your understanding of the material. They also include insightful suggestions for increasing calculator proficiency.

Study Tip

When you get a test back, don't simply file it in your notebook or the waste basket. While the material is fresh in your mind, rework all the problems that you missed. Ask questions about anything that you don't understand and save your test for future reference.

Linear Equation in One Variable

A **linear equation in one variable** x is an equation of the form $ax + b = 0$, where a and b are real numbers, with $a \neq 0$.

The equations in Examples 2 through 5 are called linear equations in one variable, or simply linear equations, because they could all be rewritten in the form $ax + b = 0$. At first glance the equations in Example 6 appear to be linear equations. However, they cannot be written in the form $ax + b = 0$, with $a \neq 0$, so they are not linear equations. A linear equation has exactly one solution. The strategy that we use for solving linear equations is summarized in the following box.

Strategy for Solving a Linear Equation

1. If fractions are present, multiply each side by the LCD to eliminate them.
2. Use the distributive property to remove parentheses.
3. Combine any like terms.
4. Use the addition property of equality to get all variables on one side and numbers on the other side.
5. Use the multiplication property of equality to get a single variable on one side.
6. Check by replacing the variable in the original equation with your solution.

Note that not all equations require all of the steps.

EXAMPLE 7

Using the equation-solving strategy

Solve the equation $\frac{y}{2} - \frac{y-4}{5} = \frac{23}{10}$.

Solution

We first multiply each side of the equation by 10, the LCD for 2, 5, and 10. However, we do not have to write down that step. We can simply use the distributive property to multiply each term of the equation by 10.

$$\begin{aligned} 10\left(\frac{y}{2}\right) - 10\left(\frac{y-4}{5}\right) &= 10\left(\frac{23}{10}\right) && \text{Multiply each side by 10.} \\ 5y - 2(y-4) &= 23 && \text{Divide each denominator into 10 to eliminate fractions.} \\ 5y - 2y + 8 &= 23 && \text{Be careful to change all signs: } -2(y-4) = -2y + 8. \\ 3y + 8 &= 23 && \text{Combine like terms.} \\ 3y + 8 - 8 &= 23 - 8 && \text{Subtract 8 from each side.} \\ 3y &= 15 && \text{Simplify.} \\ \frac{3y}{3} &= \frac{15}{3} && \text{Divide each side by 3.} \\ y &= 5 \end{aligned}$$

Check that 5 satisfies the original equation. The solution set is $\{5\}$.

The solution set is $(-\infty, 4]$, and its graph is shown in Fig. 2.13.



FIGURE 2.13

EXAMPLE 5

An inequality with fractions

Solve $\frac{1}{2}x - \frac{2}{3} \leq x + \frac{4}{3}$. State and graph the solution set.

Helpful Hint

Notice that we use the same strategy for solving inequalities as we do for solving equations. But we must remember to reverse the inequality symbol when we multiply or divide by a negative number. For inequalities it is usually best to isolate the variable on the left-hand side.

Solution

First multiply each side of the inequality by 6, the LCD:

$$\begin{aligned} \frac{1}{2}x - \frac{2}{3} &\leq x + \frac{4}{3} && \text{Original inequality} \\ 6\left(\frac{1}{2}x - \frac{2}{3}\right) &\leq 6\left(x + \frac{4}{3}\right) && \text{Multiplying by positive 6 does not reverse the inequality.} \\ 3x - 4 &\leq 6x + 8 && \text{Distributive property} \\ 3x &\leq 6x + 12 && \text{Add 4 to each side.} \\ -3x &\leq 12 && \text{Subtract 6x from each side.} \\ x &\geq -4 && \text{Divide each side by } -3 \text{ and reverse the inequality.} \end{aligned}$$

The solution set is the interval $[-4, \infty)$. Its graph is shown in Fig. 2.14.



FIGURE 2.14

In Example 6 we see an inequality that is satisfied by all real numbers and one that has no solution.

EXAMPLE 6

All or nothing

Solve each inequality and graph the solution set.

- a) $6 - 4x < -4x + 7$ b) $2(4x - 5) \geq 4(2x - 1)$

Solution

a) Adding $4x$ to each side will greatly simplify the inequality:

$$\begin{aligned} 6 - 4x &< -4x + 7 && \text{Original inequality} \\ 6 &< 7 && \text{Add } 4x \text{ to each side.} \end{aligned}$$

Since $6 < 7$ is correct no matter what real number is used in place of x , the solution set is the set of all real numbers $(-\infty, \infty)$. Its graph is shown in Fig. 2.15.

b) Start by simplifying each side of the inequality.

$$\begin{aligned} 2(4x - 5) &\geq 4(2x - 1) && \text{Original inequality} \\ 8x - 10 &\geq 8x - 4 && \text{Distributive property} \\ -10 &\geq -4 && \text{Subtract } 8x \text{ from each side.} \end{aligned}$$

Since $-10 \geq -4$ is false no matter what real number is used in place of x , the solution set is the empty set \emptyset and there is no graph to draw.

Helpful Hint

Making a guess can be a good way to become familiar with the problem. For example, let's guess that the answers to Example 2 are 50, 51, and 52. Since $50 + 51 + 52 = 153$, these are not the correct numbers. But now we realize that we should use $x + 1$ and $x + 2$ and that the equation should be

$$x + (x + 1) + (x + 2) = 228.$$

Since the sum of these three expressions for the consecutive integers is 228, we can write the following equation and solve it:

$$\begin{aligned} x + (x + 1) + (x + 2) &= 228 && \text{The sum of the integers is 228.} \\ 3x + 3 &= 228 \\ 3x &= 225 \\ x &= 75 \\ x + 1 &= 76 && \text{Identify the other unknown quantities.} \\ x + 2 &= 77 \end{aligned}$$

To verify that these values are the correct integers, we compute

$$75 + 76 + 77 = 228.$$

The three consecutive integers that have a sum of 228 are 75, 76, and 77. ■

General Strategy for Problem Solving

The steps to follow in providing a complete solution to a verbal problem can be stated as follows.

Study Tip

Don't simply work exercises to get answers. Keep reminding yourself of what you are actually doing. Keep trying to obtain the big picture. How does this section relate to what we did in the previous section? Where are we going next? When is the picture complete?

Strategy for Solving Word Problems

1. Read the problem until you understand the problem. Making a guess and checking it will help you to understand the problem.
2. If possible, draw a diagram to illustrate the problem.
3. Choose a variable and write down what it represents.
4. Represent any other unknowns in terms of that variable.
5. Write an equation that models the situation.
6. Solve the equation.
7. Be sure that your solution answers the question posed in the original problem.
8. Check your answer by using it to solve the original problem (not the equation).

We will now see how this strategy can be applied to various types of problems.

Geometric Problems

Any problem that involves a geometric figure may be referred to as a **geometric problem**. For geometric problems the equation is often a geometric formula.

EXAMPLE 3 Finding the length and width of a rectangle

The length of a rectangular piece of property is 1 foot more than twice the width. If the perimeter is 302 feet, find the length and width.

Solution

First draw a diagram as in Fig. 2.4. Because the length is 1 foot more than twice the width, we let

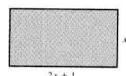


FIGURE 2.4

and $x = \text{the width}$
 $2x + 1 = \text{the length.}$

Math at Work

The **Math at Work** feature that appears in each chapter explores the careers of individuals who use the mathematics presented in the chapter in their work. Students are referred to exercises that directly relate to the occupation highlighted in **Math at Work**.

Strategy Boxes

The **strategy boxes** provide a numbered list of concepts from a section or a set of steps to follow in problem solving. They can be used by students who prefer a more structured approach to problem solving or they can be used as a study tool to review important points within sections.

Solution

If x represents the number of AM ads and y represents the number of FM ads, then x and y must satisfy the inequality $50x + 75y \leq 3000$. Because the number of ads cannot be negative, we also have $x \geq 0$ and $y \geq 0$. So we graph only points in the first quadrant that satisfy $50x + 75y \leq 3000$. The line $50x + 75y = 3000$ goes

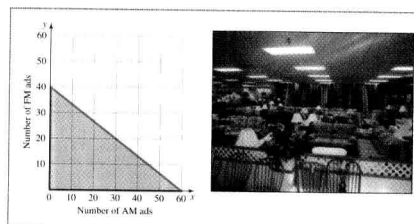


FIGURE 3.35

MATH AT WORK

"We will return after these messages." We often hear these words on television and radio just before several minutes of commercials. Carolanne Johnson, Account Executive and Media Salesperson for WBOQ, a classical radio station, is involved in every step of creating such advertisements.



MEDIA
SALESPERSON

The first step is finding clients that are consistent with the station's image. Ms. Johnson generates her own leads from a number of sources, such as print ads and billboards. The next steps are sitting down with the client, gathering information about the product or service, assessing the competition, and finally determining how much of the client's advertising budget should be spent on radio. Typically, this can be 2% to 4% of the total budget.

Radio ads usually run for 60 seconds, but reminder ads can be as short as 30 seconds. Some of the radio spots are time-sensitive and run 40 to 60 times a month for a specific month. Other clients are concerned with image building and may sponsor one particular broadcast every day for the whole year.

Ms. Johnson is concerned that the clients receive an adequate return on their investment. She is constantly reviewing the budget and making sure that the commercials present what the client wishes to project.

Example 7 and Exercise 77 of this section give problems that involve allocation of advertising dollars.

Warm-ups

Warm-ups appear before each set of exercises at the end of every section. They are true or false statements that can be used to check conceptual understanding of material within each section.



32. **Buying texts.** Melissa purchased an English text, a math text, and a chemistry text for a total of \$276. The English text was \$20 more than the math text and the chemistry text was twice the price of the math text. What was the price of each text?
33. **Three-day drive.** In three days, Carter drove 2196 miles in 36 hours behind the wheel. The first day he averaged 64 mph, the second day 62 mph, and the third day 58 mph. If he drove 4 more hours on the third day than on the first day, then how many hours did he drive each day?
34. **Three-day trip.** In three days, Katy traveled 146 miles down the Mississippi River in her kayak with 30 hours of paddling. The first day she averaged 6 mph, the second day 5 mph, and the third day 4 mph. If her distance on the third day was equal to her distance on the first day, then for how many hours did she paddle each day?
35. **Diversification.** Ann invested a total of \$12,000 in stocks, bonds, and a mutual fund. She received a 10% return on her stock investment, an 8% return on her bond investment, and a 12% return on her mutual fund. Her total return was \$1,230. If the total investment in stocks and bonds equaled her mutual fund investment, then how much did she invest in each?
36. **Paranoia.** Fearful of a bank failure, Norman split his life savings of \$60,000 among three banks. He received 5%, 6%, and 7% on the three deposits. In the account earning 7% interest, he deposited twice as much as in the account earning 5% interest. If his total earnings were \$3,760, then how much did he deposit in each account?
37. **Weighing in.** Anna, Bob, and Chris will not disclose their weights but agree to be weighed in pairs. Anna and Bob together weigh 226 pounds. Bob and Chris together weigh 210 pounds. Anna and Chris together weigh 200 pounds. How much does each student weigh?

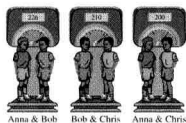


FIGURE FOR EXERCISE 37

38. **Big tipper.** On Monday Hesley paid \$1.70 for two cups of coffee and one doughnut, including the tip. On Tuesday he paid \$1.65 for two doughnuts and a cup of

coffee, including the tip. On Wednesday he paid \$1.30 for one coffee and one doughnut, including the tip. If he always tips the same amount, then what is the amount of each item?

39. **Three coins.** Nelson paid \$1.75 for his lunch with 13 coins, consisting of nickels, dimes, and quarters. If the number of dimes was twice the number of nickels, then how many of each type of coin did he use?

40. **Packet change.** Harry has \$2.25 in nickels, dimes, and quarters. If he had twice as many nickels, half as many dimes, and the same number of quarters, he would have \$2.50. If he has 27 coins altogether, then how many of each does he have?

41. **Working overtime.** To make ends meet, Ms. Farnsby works three jobs. Her total income last year was \$48,000. Her income from teaching was just \$6,000 more than her income from house painting. Royalties from her textbook sales were one-seventh of the total money she received from teaching and house painting. How much did she make from each source last year?

42. **Lunch-box special.** Salvador's Fruit Mart sells variety packs. The small pack contains three bananas, two apples, and one orange for \$1.80. The medium pack contains four bananas, three apples, and three oranges for \$3.05. The family size contains six bananas, five apples, and four oranges for \$4.65. What price should Salvador charge for his lunch-box special that consists of one banana, one apple, and one orange?

43. **Three generations.** Edwin, his father, and his grandfather have an average age of 53. One-half of his grandfather's age, plus one-third of his father's age, plus one-fourth of Edwin's age is 65. If 4 years ago, Edwin's grandfather was four times as old as Edwin, then how old are they all now?

44. **Three-digit number.** The sum of the digits of a three-digit number is 11. If the digits are reversed, the new number is 46 more than five times the old number. If the hundreds digit plus twice the tens digit is equal to the units digit, then what is the number?

GETTING MORE INVOLVED

45. **Exploration.** Draw diagrams showing the possible ways to position three planes in three-dimensional space.
46. **Discussion.** Make up a system of three linear equations in three variables for which the solution set is $\{(0, 0, 0)\}$. A system with this solution set is called a *homogeneous* system. Why do you think it is given that name?

Helpful Hint

A problem involving two unknowns can often be solved with one variable as in Chapter 2. Likewise, you can often solve a problem with three unknowns using only two variables. Solve Example 5 by letting a , b , and $2a$ be the rent for a one-bedroom, two-bedroom, and a three-bedroom condo.

equation for the total repairs, and a third equation expressing the fact that the rent for the three-bedroom condo is twice that for the one-bedroom condo:

$$\begin{aligned}x + y + z &= 1240 \\0.1x + 0.2y + 0.3z &= 276 \\z &= 2x\end{aligned}$$

Substitute $z = 2x$ into both of the other equations to eliminate z :

$$\begin{aligned}x + y + 2x &= 1240 \\0.1x + 0.2y + 0.3(2x) &= 276 \\3x + y &= 1240 \\0.7x + 0.2y &= 276 \\-2(3x + y) &= -2(1240) \quad \text{Multiply each side by } -2 \\10(0.7x + 0.2y) &= 10(276) \quad \text{Multiply each side by } 10 \\-6x - 2y &= -2480 \\7x + 2y &= 2760 \quad \text{Add} \\x &= 280 \\z = 2(280) &= 560 \quad \text{Because } z = 2x \\280 + y + 560 &= 1240 \quad \text{Because } x + y + z = 1240 \\y &= 400\end{aligned}$$

Check that $(280, 400, 560)$ satisfies all three of the original equations. The condos rent for \$280, \$400, and \$560 per week. ■

WARM-UPS

True or false? Explain your answer.

- The point $(1, -2, 3)$ is in the solution set to the equation $x + y - z = 4$.
- The point $(4, 1, 1)$ is the only solution to the equation $x + y - z = 4$.
- The ordered triple $(1, -1, 2)$ satisfies $x + y + z = 2$, $x - y - z = 0$, and $2x + y - z = -1$.
- Substitution cannot be used on three equations in three variables.
- Two distinct planes are either parallel or intersect in a single point.
- The equations $x - y + 2z = 6$ and $x - y + 2z = 4$ are inconsistent.
- The equations $3x + 2y - 6z = 4$ and $-6x - 4y + 12z = -8$ are dependent.
- The graph of $y = 2x - 3z + 4$ is a straight line.
- The value of x nickels, y dimes, and z quarters is $0.05x + 0.10y + 0.25z$ cents.
- If $x = -2$, $z = 3$, and $x + y + z = 6$, then $y = 7$.

Exercises

The theme of mathematics in everyday situations is carried over to the exercise sets. Applications based on real-world data are included in each set. The **Index of Selected Applications** can help students to quickly identify exercises that associate the mathematics that may be used in their areas of interest.

MISCELLANEOUS

55. The points $(3, \quad)$ and $(\quad, -7)$ are on the line that passes through $(2, 1)$ and has slope 4. Find the missing coordinates of the points.
56. If a line passes through $(5, 2)$ and has slope $\frac{1}{2}$, then what is the value of y on this line when $x = 8$, $x = 11$, and $x = 12$?
57. Find k so that the line through $(2, k)$ and $(-3, -5)$ has slope $\frac{1}{2}$.
58. Find k so that the line through $(k, 3)$ and $(-2, 0)$ has slope 3.
59. What is the slope of a line that is perpendicular to a line with slope 0.247?
60. What is the slope of a line that is perpendicular to the line through $(3.27, -1.46)$ and $(-5.48, 3.61)$?

GETTING MORE INVOLVED

61. **Writing.** What is the difference between zero slope and undefined slope?
62. **Writing.** Is it possible for a line to be in only one quadrant? Two quadrants? Write a rule for determining whether a line has positive, negative, zero, or undefined slope from knowing in which quadrants the line is found.

63. **Exploration.** A rhombus is a quadrilateral with four equal sides. Draw a rhombus with vertices $(-3, -1)$, $(0, 3)$, $(2, -1)$, and $(5, 3)$. Find the slopes of the diagonals of the rhombus. What can you conclude about the diagonals of this rhombus?

64. **Exploration.** Draw a square with vertices $(-5, 3)$, $(-3, -3)$, $(1, 5)$, and $(3, -1)$. Find the slopes of the diagonals of this square. What can you conclude about the diagonals of this square?

GRAPHING CALCULATOR EXERCISES

65. Graph $y = 1x$, $y = 2x$, $y = 3x$, and $y = 4x$ together in the standard viewing window. These equations are all of the form $y = mx$. What effect does increasing m have on the graph of the equation? What are the slopes of these four lines?
66. Graph $y = -1x$, $y = -2x$, $y = -3x$, and $y = -4x$ together in the standard viewing window. These equations are all of the form $y = mx$. What effect does decreasing m have on the graph of the equation? What are the slopes of these four lines?

In This Section

- Point-Slope Form
- Slope-Intercept Form
- Standard Form
- Using Slope-Intercept Form for Graphing
- Applications

3.3 THREE FORMS FOR THE EQUATION OF A LINE

In Section 3.1 you learned how to graph a straight line corresponding to a linear equation. The line contains all of the points that satisfy the equation. In this section we start with a line or a description of a line and write an equation corresponding to the line.

Point-Slope Form

Figure 3.20 shows the line that has slope $\frac{2}{3}$ and contains the point $(3, 5)$. In Section 3.2 you learned that the slope is the same no matter which two points of the line are used to calculate it. So if we find the slope m for this line using an arbitrary point of the line, say (x, y) , and the specific point $(3, 5)$, we get

$$m = \frac{y - 5}{x - 3}$$

Getting More Involved appears within selected exercise sets. This feature may contain



Writing,



Cooperative Learning,



Exploration, and/or



Discussion exercises. Each of these components is designed to give students an opportunity to improve and develop the ways in which they express mathematical ideas.

The exercise sets contain exercises that are keyed to examples, as well as exercises that are not keyed to examples.

- c) Write the equation of the line through $(0, 3)$ and $(-5, 0)$ in intercept form.
- d) Which lines cannot be written in intercept form?

99. Graph $y = x - 3000$, using a viewing window that shows both the x -intercept and the y -intercept.

100. Graph $y = 2x - 400$ and $y = -0.5x + 1$ on the same screen, using the viewing window $-500 \leq x \leq 500$ and $-1000 \leq y \leq 1000$. Should these lines be perpendicular? Explain.



GRAPHING CALCULATOR EXERCISES

98. Graph the equation $y = 0.5x - 1$ using the standard viewing window. Adjust the range of y -values so that the line goes from the lower left corner of your viewing window to the upper right corner.

101. The lines $y = 2x - 3$ and $y = 1.9x + 2$ are not parallel. Find a viewing window in which the lines intersect. Estimate the point of intersection.

In This Section

- Definition
- Graphing Linear Inequalities
- The Test Point Method
- Graphing Compound Inequalities
- Applications

3.4 LINEAR INEQUALITIES AND THEIR GRAPHS

In the first three sections of this chapter you studied linear equations. We now turn our attention to linear inequalities.

Definition

A linear inequality is a linear equation with the equal sign replaced by an inequality symbol.

Linear Inequality

If A , B , and C are real numbers with A and B not both zero, then

$$Ax + By \leq C$$

is called a **linear inequality**. In place of \leq , we can also use \geq , $<$, or $>$.

Graphing Linear Inequalities

Consider the inequality $-x + y > 1$. If we solve the inequality for y , we get

$$y > x + 1.$$

Which points in the xy -plane satisfy this inequality? We want the points where the y -coordinate is larger than the x -coordinate plus 1. If we locate a point on the line $y = x + 1$, say $(2, 3)$, then the y -coordinate is equal to the x -coordinate plus 1. If we move upward from that point, to say $(2, 4)$, the y -coordinate is larger than the x -coordinate plus 1. Because this argument can be made at every point on the line, all points above the line satisfy $y > x + 1$. Likewise, points below the line satisfy $y < x + 1$. The solution sets, or graphs, for the inequality $y > x + 1$ and the inequality $y < x + 1$ are the shaded regions shown in Figs. 3.24(a) and 3.24(b) on the next page. In each case the line $y = x + 1$ is dashed to indicate that points on the line do not satisfy the inequality and so are not in the solution set. If the inequality symbol is \leq or \geq , then points on the boundary line also satisfy the inequality, and the line is drawn solid.

Study Tip

Working problems one hour per day every day of the week is better than working problems for 7 hours on one day of the week. It is usually better to spread out your study time than to try and learn everything in one big session.

Calculator Exercises

Calculator Exercises are optional. They provide an opportunity for students to learn how a scientific or graphing calculator might be useful in solving various problems.

Collaborative Activities

Collaborative Activities appear at the end of each chapter. The activities are designed to encourage interaction and learning in a group setting.

96. **Cooperative learning.** Work with a group to examine the following solution to $x^2 - 2x = -1$:

$$\begin{aligned} x(x-2) &= -1 \\ x-1 &= 0 \quad \text{or} \quad x-2 = -1 \\ x &= -1 \quad \text{or} \quad x = 1 \end{aligned}$$

Is this method correct? Explain.

97. **Cooperative learning.** Work with a group to examine the following steps in the solution to $5x^2 - 5 = 0$:

$$\begin{aligned} 5(x^2 - 1) &= 0 \\ 5(x-1)(x+1) &= 0 \\ x-1 &= 0 \quad \text{or} \quad x+1 = 0 \\ x &= 1 \quad \text{or} \quad x = -1 \end{aligned}$$

What happened to the 5? Explain.

COLLABORATIVE ACTIVITIES

Magic Tricks

Jim and Sadar are talking one day after class.

Sadar: Jim, I have a trick for you. Think of a number between 1 and 10. I will ask you to do some things to this number. Then at the end tell me your result, and I will tell you your number.

Jim: Oh, yeah you probably rig it so the result is my number.

Sadar: Come on Jim, give it a try and see.

Jim: Okay, okay, I thought of a number.

Sadar: Good, now write it down, and don't let me see your paper. Now add 3. Got that? Now multiply everything by 2.

Jim: Hey, I didn't know you were going to make me think! This is algebra!

Sadar: I know, now just do it. Okay, now square the polynomial. Got that? Now subtract $4x^2$.

Jim: How did you know I had a $4x^2$? I told you this was rigged!

Sadar: Of course it's rigged, or it wouldn't work. Do you want to finish or not?

Jim: Yeah, I guess so. Go ahead, what do I do next?

Sadar: Divide by 4. Okay, now subtract the x -term.

Grouping: Two students per group

Topic: Practice with exponent rules, multiplying polynomials

Jim: Just any old x -term? Got any particular coefficient in mind?

Sadar: Now stop teasing me. I know you only have one x -term left, so subtract it.

Jim: Ha, ha, I could give you a hint about the coefficient, but that wouldn't be fair, would it?

Sadar: Well you could, and then I could tell you your number, or you could just tell me the number you have left after subtracting.

Jim: Okay, the number I had left at the end was 25. Let's see if you can tell me what the coefficient of the x -term I subtracted is.

Sadar: Ah, then the number you chose at the beginning was 5, and the coefficient was 10!

Jim: Hey, you're right! How did you do that?

In your group, follow Sadar's instructions and determine why she knew Jim's number. Make up another set of instructions to use as a magic trick. Be sure to use variables and some of the exponent rules or rules for multiplying polynomials that you learned in this chapter. Exchange instructions with another group and see whether you can figure out how their trick works.

WRAP-UP

SUMMARY

Definitions

Definition of negative integral exponents

If a is a nonzero real number and n is a positive integer, then

$$a^{-n} = \frac{1}{a^n}$$

Examples

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Wrap-up

Every chapter ends with a four-part **Wrap-up**:

The **Summary** lists important concepts along with brief illustrative examples.

Enriching Your Mathematical Word Power enables students to review terms introduced in each chapter. It is intended to help reinforce students' command of mathematical terminology.

Review Exercises contain problems that are keyed to each section of the chapter as well as **miscellaneous exercises**, which are not keyed to the sections. The *miscellaneous exercises* are designed to test the student's ability to synthesize various concepts.

WRAP-UP

SUMMARY

Quadratic Equations
Quadratic equation

An equation of the form $ax^2 + bx + c = 0$, where a , b , and c are real numbers, with $a \neq 0$

Examples

$$\begin{aligned} x^2 &= 11 \\ (x-5)^2 &= 99 \\ x^2 + 3x - 20 &= 0 \end{aligned}$$

$$\begin{aligned} x^2 + x - 6 &= 0 \\ (x+3)(x-2) &= 0 \\ 6x^2 - 5x - 2 &= 0 \end{aligned}$$

ENRICHING YOUR MATHEMATICAL WORD POWER

For each mathematical term, choose the correct meaning.

1. quadratic

a. a parabola

b. a function

c. a curve

d. a line

$$87. \frac{x-4}{x+2} \approx 0$$

$$88. \frac{x-3}{x+5} < 0$$

$$89. \frac{x-2}{x+3} < 1$$

$$90. \frac{x-3}{x+4} > 2$$

$$91. \frac{3}{x+2} > \frac{1}{x+1}$$

$$92. \frac{1}{x+1} < \frac{1}{x-1}$$

MISCELLANEOUS

In Exercises 93–104, find all real or imaginary solutions to each equation.

$$93. 144x^2 - 120x + 25 = 0$$

quadratic function

a. $y = ax + b$ with $a \neq 0$

b. a parabola

c. $y = ax^2 + bx + c$ with $a \neq 0$

d. the quadratic formula

quadratic in form

$ax^2 + bx + c = 0$

Chapter 8 Review Exercises

(8-55) 507

$$103. x^{1/2} - 15x^{1/4} + 50 = 0$$

$$104. x^{-2} - 9x^{-1} + 18 = 0$$

Find exact and approximate solutions to each problem.

105. **Mixing numbers.** Find two positive real numbers that differ by 4 and have a product of 4.

106. **One on one.** Find two positive real numbers that differ by 1 and have a product of 1.

107. **Big screen TV.** On a 19-inch diagonal measure television picture screen, the height is 4 inches less than the width. Find the height and width.

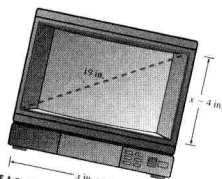


FIGURE FOR EXERCISE 107

108. **Boxing match.** A boxing ring is in the shape of a square. 20 ft on each side. How far apart are the fighters when they are in opposite corners of the ring?

CHAPTER 8 TEST

Calculate the value of $b^2 - 4ac$, and state how many real solutions each equation has.

1. $2x^2 - 3x + 2 = 0$
2. $-3x^2 + 5x - 1 = 0$
3. $4x^2 - 4x + 1 = 0$

Solve by using the quadratic formula.

4. $2x^2 + 5x - 3 = 0$
5. $x^2 + 6x + 6 = 0$

Solve by completing the square.

6. $x^2 + 10x + 25 = 0$
7. $2x^2 + x - 6 = 0$

Solve by any method.

8. $xtx + 1 = 12$
9. $a^4 - 5a^2 + 4 = 0$
10. $x - 2 - 8\sqrt{x-2} + 15 = 0$

Find the complex solutions to the quadratic equations.

11. $x^2 + 36 = 0$
12. $x^2 + 6x + 10 = 0$
13. $3x^2 - x + 1 = 0$

Graph each quadratic function. State the domain and range.

14. $f(x) = 16 - x^2$

15. $g(x) = x^2 - 3x$

Write a quadratic equation that has each given pair of solutions.

16. $-4, 6$
17. $-5i, 5i$

Solve each inequality. State and graph the solution set.

18. $w^2 + 3w < 18$

19. $\frac{2}{x-2} < \frac{3}{x+1}$

Find the exact solution to each problem.

20. The length of a rectangle is 2 ft longer than the width. If the area is 16 ft^2 , then what are the length and width?
21. A new computer can process a company's monthly payroll in 1 hour less time than the old computer. To really save time, the manager used both computers and finished the payroll in 3 hours. How long would it take the new computer to do the payroll by itself?

Solve each problem.

22. Find the x -intercepts for the parabola $y = x^2 - 6x + 5$.
23. The height in feet for a ball thrown upward at 48 feet per second is given by $h(t) = -16t^2 + 48t$, where t is the time in seconds after the ball is tossed. What is the maximum height that the ball will reach?

Chapter Test

This is designed to help the student assess his or her readiness for a test. The **Chapter Test** has no keyed exercises, which affords students an opportunity to synthesize concepts found within the chapter.

Making Connections

These nonkeyed exercises are designed to help students synthesize new material with ideas from previous chapters and, in some cases, review material necessary for success in the upcoming chapter. They may serve as a cumulative review.

MAKING CONNECTIONS CHAPTERS 1-8

Solve each equation.

1. $2x - 15 = 0$
2. $2x^2 - 15 = 0$

3. $2x^2 + x - 15 = 0$

4. $2x^2 + 4x - 15 = 0$

5. $|4x + 11| = 3$

6. $|4x^2 + 11x| = 3$

7. $\sqrt{x} = x - 6$

8. $(2x - 5)^{2/3} = 4$

Solve each inequality. State the solution set using interval notation.

9. $1 - 2x < 5 - x$

10. $(1 - 2x)(5 - x) \leq 0$

11. $\frac{1-2x}{5-x} \leq 0$

12. $|5 - x| < 3$

13. $3x - 1 < 5$ and $-3 \leq x$

14. $x - 3 < 1$ or $2x \geq 8$

Solve each equation for y .

15. $2x - 3y = 9$

16. $\frac{y-3}{x+2} = \frac{1}{2}$

17. $3y^2 + cy + d = 0$

18. $my^2 - ny = w$

19. $\frac{1}{3}x - \frac{2}{5}y = \frac{5}{6}$

20. $y - 3 = -\frac{2}{3}(x - 4)$

Let $m = \frac{y_2 - y_1}{x_2 - x_1}$. Find the value of m for each of the following choices of x_1, x_2, y_1 , and y_2 .

21. $x_1 = 2, x_2 = 5, y_1 = 3, y_2 = 7$

22. $x_1 = -3, x_2 = 4, y_1 = 5, y_2 = -6$

23. $x_1 = 0.3, x_2 = 0.5, y_1 = 0.8, y_2 = 0.4$

24. $x_1 = \frac{1}{2}, x_2 = \frac{1}{3}, y_1 = \frac{3}{5}, y_2 = -\frac{4}{3}$

Solve each problem.

25. **Ticket prices.** If the price of a concert ticket goes up, then the number sold will go down, as shown in the figure. If you use the formula $n = 48,000 - 400p$ to predict the number sold depending on the price p , then how many will be sold at \$20 per ticket? How many will be sold at \$25 per ticket? Use the bar graph to estimate the price if 35,000 tickets were sold.

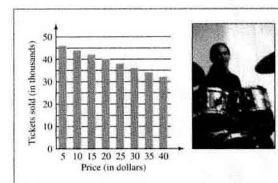


FIGURE FOR EXERCISE 25

26. **Increasing revenue.** Even though the number of tickets sold for a concert decreases with increasing price, the revenue generated does not necessarily decrease. Use the formula $R = p(48,000 - 400p)$ to determine the revenue when the price is \$20 and when the price is \$25. What price would produce a revenue of \$1.28 million? Use the graph to find the price that determines the maximum revenue.

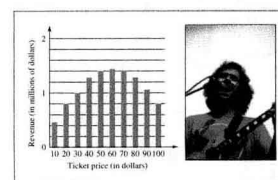


FIGURE FOR EXERCISE 26