

HAND BOOKS

IN ECONOMICS

Kenneth J. Arrow
Amartya Sen
Kotaro Suzumura

Social Choice and Welfare

VOLUME 2

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VOLUME TWO

HANDBOOK OF SOCIAL CHOICE AND WELFARE

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VOLUME TWO

HANDBOOK OF **SOCIAL CHOICE AND WELFARE**

PREFACE

The *Handbook of Social Choice and Welfare* presents, in two volumes, essays on past and ongoing work in social choice theory and welfare economics. The first volume, which consists of twelve chapters in four parts—Part I (*Arrovian Impossibility Theorems*), Part II (*Voting Schemes and Mechanisms*), Part III (*Structure of Social Choice Rules*), and Part IV (*Welfare, Justice, and Poverty*)—, was published in 2002. This is the long overdue second volume of the *Handbook*, which also consists of four parts.

In Part V (*Foundations*), some very basic issues in welfare economics and social choice theory are discussed. The topics covered include the initial insights behind and the basic functions of social choice theory, the informational basis of normative and positive social choice theory, the status of competitive market mechanisms as social choice procedures, and the main ingredients of nonwelfaristic approaches in welfare economics and social choice theory along the line of the theory of functionings and capabilities.

Part VI (*Developments of the Basic Arrovian Schemes*) consists of four chapters that are lineal descendants of Part I in the first volume. Recollect that three chapters in Part I discussed the basic Arrovian schemes in an abstract space of social alternatives. Here, Part VI addresses itself to the important variations of the basic schemes, such as impossibility theorems on economic domains, topological theories of social choice, nonbinary social choice theory, and social choice theory with fuzzy preferences.

In Part VII (*Fairness and Rights*), several new dimensions in welfare economics and social choice theory are introduced through the well-known conflicts of values such as equity–efficiency tradeoffs, compensation and responsibility, the tension between the requirement of social efficiency and the claim of individual liberties and rights, and the social choice theoretical analysis of freedom, opportunity, and well-being.

Finally, Part VIII (*Voting and Manipulation*) goes back to the issues discussed in Part II of the first volume. Here the basic issues of strategy-proof social choice rules are covered as well as probabilistic and spatial models of voting, and the geometry of voting.

The literature on social choice theory is now quite vast, and we hope this collection of critical surveys in two volumes will be of interest to readers who would like to be introduced to the writings on human well-being and collective decision-making that have been pursued within this literature.

The initial plan of this *Handbook* was conceived soon after the publication of the Proceedings of the International Economic Association Roundtable Meeting on Social Choice Theory by us in 1996/1997. We are most grateful to the authors of the two volumes of the *Handbook* for their contributions. The editors apologize for the fact that some of the authors had to wait long for publication and had to update their chapters as the

other essays were coming in. We should also express our sincere gratitude to those readers of the first volume who provided important suggestions for the second volume, aside from encouraging us to complete what proved to be a remarkably ambitious project. We very much hope that the completed *Handbook* will live up to the expectations of our colleagues and readers.

We conclude this “extra” preface by reiterating the last paragraph of the Preface to the *Handbook* as a whole, which was printed in the first volume: “The primary purpose of this Handbook is to provide an accessible introduction to the current state of the art in social choice theory and welfare economics. But we also believe that the expounded theory has a strong and constructive message for pursuing human well-being and facilitating collective decision-making.”

Kenneth Arrow
Amartya Sen
Kotaro Suzumura

A Statement from Kenneth Arrow and Amartya Sen

We would like to acknowledge our debt to Kotaro Suzumura for doing the lion’s share of the editorial work, and for doing this ungrudgingly in a way that can be done only by a person of the remarkable generosity that Kotaro has. We are immensely grateful to him.

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