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Convective HEAT and MASS transfer

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Preface

PRIOR TO World War II, convective heat and mass transfer were largely empirical sciences, and engineering design was accomplished almost exclusively by the use of experimental data, generalized to some degree by dimensional analysis. During the past two decades great strides have been made in developing analytical methods of convection analysis, to the point where today experiment is assuming more its classical role of testing the validity of theoretical models. This is not to say that direct experimental data are not still of vital importance in engineering design, but there is no question that the area of complete dependence on direct experimental data has been greatly diminished. With this change our understanding of convection phenomena has been greatly enhanced, and we find ourselves in a position to handle, with confidence, problems for which experiment would be time consuming and expensive. This book has been prepared as a response to this trend.

It is axiomatic that the engineering student must learn to reason from first principles so that he is not at a loss when faced by new problems. But time spent solving a complex problem from first principles is time wasted if the solution already exists. By their very nature analytic convection solutions often tend to be lengthy and difficult. Thus familiarity with, and an understanding of, some of the more important of the available analytic convection solutions should be an important part of the background of the heat transfer engineer. One of the objectives of this book is to bring together in an easily usable form some of the many solutions to the boundary layer equations. Although these are available in the heat transfer literature, they are not always readily accessible to the practicing engineer, for whom time is an important consideration.

The author feels that a study of these solutions, in a logical sequence, also provides the best way for a student to develop an understanding of convective heat and mass transfer. Thus it is hoped that this book will serve both as a classroom text and as a useful reference book for the engineer.

This book is the outgrowth of a set of notes which the author has developed over the past ten years to supplement lectures in the "convection" portion of a one-year course in heat transfer for first-year graduate students. The students in the course have been largely mechanical, nuclear, and aeronautical engineers, interested in problems associated with thermal power systems and thermal environmental control.

It is assumed that the student has a typical undergraduate background in applied thermodynamics, fluid mechanics, and heat transfer. Heat transfer, although not mandatory, is usually of considerable help in orienting the student's thinking and establishing a sense of need for a deeper study of the subject. In particular, some familiarity with the commonly employed empirical methods of calculating convection heat transfer rates is assumed, but only so that the student has an appreciation for the usefulness of a heat transfer coefficient and some grasp of the basic physics of the convection process.

The choice of subject matter reflects quite frankly the author's own interests, and the depth to which each topic is pursued represents a compromise made necessary by what can be practicably accomplished in approximately one semester (or perhaps two quarters). It will be found that the momentum boundary layer is heavily compressed with only sufficient material presented to support the heat and mass transfer sections. The student desiring to concentrate heavily in boundary layer theory will undoubtedly want to take a separate course on viscous fluid mechanics, for which adequate texts exist. And, for that matter, there is certainly a great deal more to convective heat and mass transfer than is presented here, not only in the topics considered but also in those not even mentioned. In the latter category the reader may miss such topics as natural convection, heat exchanger theory, rotating surfaces, nonsteady flows, two-phase flows, boiling and condensation, non-Newtonian fluids, internally radiating gases, rarefied gases, magnetohydrodynamic flows, and coupling between heat and mass transfer. But this only suggests why second editions are usually bigger than first editions.

Finally, I would like to acknowledge my indebtedness to some of my colleagues, without whose assistance, conscious or otherwise, this book could never have been written. First, Professor A. L. London taught me all that I profess to know about teaching, introduced me to heat transfer, and has been a constant source of help and inspiration. Professor W. C. Reynolds has worked with me on some of the research that is summarized in the book, and substantial parts of it are the result of his work alone. Several months spent with Professor D. B. Spalding at Imperial College in London were a rare privilege, and his influence will be found throughout the book. But most specifically, Spalding's generalization of the convective mass transfer problem forms the entire basis for the last three chapters. Although available in Spalding's many papers, it is hoped that its inclusion here will encourage its more extensive use. Lastly I would like to express appreciation to Mr. R. J. Moffat who read the manuscript and made many helpful suggestions.

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A note on problems

THE AUTHOR has rather strong feelings about the kinds of home problems that a student in a senior- or graduate-level course should be asked to attack and about the manner in which problems should be presented. That practice on a variety of problems is probably the effective route to proficiency in engineering analysis is not disputed. But the real world of engineering does not present itself as a series of neatly packaged and well-defined exercises. At some point, and the earlier the better, the student must face up to the analysis of problems that may require considerable time, possibly extensive outside study, and that are incompletely stated or specified so that individual judgment must be exercised. Above all, he must be weaned away from the notion that the end product of an engineering analysis is merely a number that is either correct or incorrect.

The difficulty with using lengthy and comprehensive problems in an engineering course is that the student's time is limited, relatively fewer problems can be assigned, and thus it is usually not possible to cover many aspects of the subject with practice. The instructor must be extremely careful in his choice of assignments if the problems are to be a meaningful supplement to the other methods of learning.

A further difficulty is that such problems must be read and constructively criticized if full value is to be realized, which involves more work for the instructor than merely checking off answers.

The problems at the ends of the chapters have been selected to provide a choice of both relatively short exercises and much more lengthy tasks, many of which might better be described as projects rather than problems. Most of them have been used by the author in his classes, but only a fraction of them in any one class. Both the longer and shorter variety have been selected so that some represent engineering applications, whereas others are of a fundamental analysis variety designed to enhance understanding of the various developments described in the text. In either case the student should be encouraged to use his own initiative in varying the parameters of the problem, in using other methods than those suggested, and in supplementing his work by reference to journal papers and research reports.

Many of the problems require numerical or graphical integrations or iterative calculation procedures. The psychological barrier presented by such procedures should be battered down early. Although hand calculation is perfectly feasible in all cases, many engineering students will by this stage in their education have learned to program a computer; if they have access to a computer they ought to develop the habit of using it in their problem work.

An engineering analysis, if it is to be useful, must not only answer some specific questions, it must also communicate in a clear manner the line of thought leading to the answers, the decisions made enroute, and must give consideration to the validity and significance of the answers. A careful and neatly developed presentation of an analysis of a complex problem helps in itself to develop clear thinking. Fuzzy or sloppy thinking stands out in bold relief when it has to be completely explained to the reader. The author insists that his students present problem solutions in what might be described as a "technical paper" style, adhering to the following rules:

1. State the objectives of the problem in a concise form but include all the pertinent information.
2. Draw a diagram of the system, if it is indeed a physical system that is being analyzed.
3. Sketch a graphical representation (qualitative) of the anticipated system behavior on some suitable coordinate system (an expected temperature distribution, for example).
4. Develop the analysis from *stated* fundamental principles, definitions, postulates, or assumptions, or from specifically named basic equations.

5. Add sufficient verbal descriptive material to the analysis so that each step is easily understandable but omit all arithmetic and obvious algebra.
6. Terminate the analysis with a discussion and some specific conclusions.

To the engineering student accustomed only to numerical exercises, item 6 usually tends to be initially painful. But after a few attempts it is surprising how this part of the problems grows in both quality and quantity, especially if the student is amply rewarded for a meaningful discussion. The apparent lack of verbal ability in the engineering student is largely due to lack of practice and to the fact that he is seldom asked to write about things where his ideas can be backed up by facts and figures generated by an analysis in an area where he feels competent. The writing of the discussion, and the anticipation of having to write it, colors the entire development of the analysis and also provides the motivation for exercising initiative in varying the problem parameters, investigating other methods of analysis, and looking into the periodical literature for supplementary material. The net result may be a lot of work, but handled intelligently the rewards for this kind of problem-solving experience can be enormous, particularly if it is employed with comprehensive, open-ended problems where there is ample opportunity for choice and initiative. A depth of understanding is achieved which is seldom attainable by working numerical exercises and reading for examinations. And the effects carry over into laboratory reports, theses, and, hopefully, engineering practice.

Terminology

English letter symbols

A	area, surface area
A_c	flow cross-sectional area
A_m	see Eq. (8-39)
B	mass transfer driving force, defined by Eq. (14-34)
C_n	see Eq. (8-33)
c	specific heat at constant pressure
c_v	specific heat at constant volume
c_j	specific heat at constant pressure for component j of a mixture
D	inside diameter of a circular tube
D_h	hydraulic diameter, $4r_h = 4A_cL/A$

D_j	mass diffusion coefficient for component j in a multicomponent mixture
d	outside diameter of a circular tube
$\mathcal{D}_{ij}$	mass diffusion coefficient for a binary (2-component) mixture. Note that $\mathcal{D}_{ij} = \mathcal{D}_{ji}$.
$\dot{E}$	rate of energy transfer by convection across a control surface
e	internal thermal and chemical energy, per unit of mass
F	a factor defined by Eq. (11-34)
$\vec{\mathrm{F}}$	resultant of all external forces acting on a control volume
f	friction coefficient in general, and mean friction coefficient with respect to length
f_x	local friction coefficient, defined by Eq. (5-10)
$\bar{f}_{\mathrm{app}}$	apparent mean friction coefficient, defined by Eq. (6-19)
G	mass flux, or mass velocity; see Eqs. (2-2) and (2-3)
G_∞	mass velocity in the free stream, $\rho_\infty u_\infty$
$\vec{\mathrm{G}}$	mass flux or mass velocity vector, $\rho\vec{\mathrm{V}}$, at any point in the stream
G_x, G_y, etc.	x, y, etc., components of the mass flux vector
$\vec{G}_{\mathrm{diff},j}$	mass flux of component j transported by diffusion; see Eq. (3-12)
G_n	see Eq. (8-34)
g	mass transfer conductance, defined by Eq. (14-33)
g	acceleration of gravity
g_c	Newton constant relating force and mass; see Appendix B
g^*	value of mass transfer conductance for very small mass transfer rate, Eq. (14-36)
H	boundary-layer shape factor, defined by Eq. (7-39)
H_0	heat of combustion, per unit of fuel mass, at a temperature t_0
h	heat transfer coefficient, or convection heat transfer conductance, Eq. (1-1)
h_x	local heat transfer coefficient, evaluated at some point x, Eq. (5-22)
i	enthalpy, and enthalpy of a mixture, $e + P/(\rho J)$
i_j	partial enthalpy of component j of a mixture

i_s	stagnation enthalpy, $i + u^2/(2g_cJ)$
J	mechanical-to-thermal energy conversion factor; see Appendix B
k	thermal conductivity, Eq. (3-8)
k_T	thermal diffusion ratio, Eq. (3-9)
L	flow length of a tube
Le_j	Lewis number, γ_j/Γ, $\mathrm{Pr}/\mathrm{Sc}_j$
l	mixing length
M	a blowing rate parameter; see Eq. (11-38)
M	Mach number, Eq. (13-27)
m	mass
m	an exponent; see Eqs. (7-22), (7-23)
m_j	mass concentration (mass fraction) of substance j in a mixture
$\dot{m}$	mass flow rate
$\dot{m}''$	total mass flux (mass flow rate per unit of area) at surface or phase interface
$\dot{m}_j''$	mass flux of substance j at surface or phase interface
$\dot{m}_j'''$	rate of creation of substance j, per unit of volume, by chemical reaction
$\mathcal{M}$	molecular weight
n_α	mass fraction of element α in a mixture of compounds
$n_{\alpha,j}$	mass fraction of element α in a compound substance j
Nu	Nusselt number, hD/k, $4r_hh/k$, hD_h/k, xh/k
P	pressure
P_a	partial pressure of substance a in a gas mixture
$\mathcal{P}$	conserved property of the second kind; see Eq. (14-21)
Pr	Prandtl number, $\mu c/k$, μ/Γ, ν/α
q	heat, energy in transit by virtue of a temperature gradient
$\dot{q}$	heat transfer rate
$\vec{\dot{q}}''$	heat flux vector, heat transfer rate per unit of area
$\dot{q}_0''$	heat flux (heat transfer rate per unit of area) at surface or phase interface

R	radius of a body of revolution, Fig. 5-4; radius of cylinder or sphere, Fig. 10-2
R	gas constant; see Appendix B
R_n	see Eq. (8-33)
r	radial distance in cylindrical or spherical coordinates
r	boundary-layer thickness ratio, Δ/δ
r	mass ratio of oxident-to-fuel in a simple chemical reaction
r_c	recovery factor, Eq. (13-16)
r_h	hydraulic radius, A_cL/A; see Eq. (6-17)
r_0	radius of a circular tube
r^+	nondimensional radial coordinate, r/r_0
Re	Reynolds number, $4r_hG/\mu$, DG/μ, $xu_\infty\rho/\mu$, etc.
S	source function, thermal energy created per unit of volume
Sc_j	Schmidt number, μ/γ_j, $\mu/\rho D_j$
St	Stanton number, $h/(Gc)$, $h/(u_\infty\rho c)$, etc.
T	absolute temperature
T	boundary-layer shape factor, defined by Eq. (7-39)
T^*	reference temperature for evaluation of fluid properties, Eqs. (12-16), (13-35)
t	temperature, on any arbitrary scale
t_{aw}	adiabatic wall temperature, Eq. (13-17)
t_e	fluid temperature at entrance to a tube
t_m	mixed mean temperature, defined by Eq. (8-5)
t_0	fluid temperature at surface or phase interface
t_∞	temperature in the free stream
t^*	stagnation temperature, defined by Eq. (13-5)
t_{db}	"dry bulb" temperature
t_{wb}	"wet bulb" temperature
u	velocity component in x direction
u^+	nondimensional velocity in a turbulent shear layer, defined by Eq. (6-22); also nondimensional velocity in a tube, u/V
u_∞	velocity in the free stream
V	volume

V	mean velocity in a tube, defined by Eq. (6-3); also uniform velocity upstream of a blunt body
$\vec{\mathbf{V}}$	velocity vector
v	velocity component in y direction
v	fluid specific volume
v_r	velocity component in r direction
W	mechanical or electrical work
$\dot{W}$	rate of doing mechanical work
w	velocity component in z direction
X	body force acting on a fluid, in the x direction, per unit of volume
X_a	mole fraction of component a in a mixture, Eq. (16-1)
x	a spatial coordinate in cartesian and cylindrical systems, see Appendix D; also flow length in a tube, or distance measured along the surface of a body
x^+	nondimensional axial distance inside a tube, $(x/r_0)/(\mathrm{Re}\ \mathrm{Pr})$
y	a spatial coordinate in cartesian system
y^+	nondimensional distance from wall in a turbulent shear layer, defined by Eq. (6-22)
z	a spatial coordinate in cartesian system; also elevation with respect to a given datum

Greek letter symbols

α	molecular thermal diffusivity, $k/(\rho c)$
β	wedge angle; see Fig. (7-2)
β	beta function, Eq. (10-34) and Appendix C
Γ	thermal diffusion coefficient, k/c, Eq. (3-10)
γ	ratio of specific heats, c/c_v
γ_j	mass diffusion coefficient for substance j in a mixture, ρD_j, Eq. (3-14)
γ_m	see Eq. (8-39)
Δ	designates a difference when used as a prefix
Δ	thickness of a thermal boundary layer (for example, see Eq. 10-24)

Δ_2	enthalpy thickness of a boundary layer, defined by Eq. (5-14)
Δ_4	conduction thickness, k/h, Eq. (5-16)
δ	thickness of a momentum boundary layer (for example, see Eq. (7-34))
δ_1	displacement thickness of momentum boundary layer, defined by Eq. (5-5)
δ_2	momentum thickness of boundary layer, defined by Eq. (5-6)
δ_4	shear thickness of boundary layer, defined by Eq. (7-38)
ε	roughness size, Fig. 6-13
ϵ	turbulent eddy diffusivity, Eqs. (9-1), (9-2)
ϵ_H	eddy diffusivity for heat transfer, Eq. (9-8*a*)
ϵ_M	eddy diffusivity for momentum transfer, Eqs. (6-27), (9-7*a*)
ζ	dependent variable in Blasius equation, $u/u_\infty = \zeta'(\eta)$, Eq. (7-8)
η	similarity parameter, Eq. (7-11), independent variable in Blasius and other similarity solution equations
η	film cooling effectiveness; see Eq. (11-38)
θ	time
θ	nondimensional fluid temperature in a tube, $(t_0 - t)/(t_0 - t_e)$, a solution to Eq. (8-29)
θ	nondimensional fluid temperature in an external boundary layer, $(t_0 - t)/(t_0 - t_\infty)$, for the case of a step change in surface temperature
θ	nondimensional fluid temperature in a high-velocity boundary layer, defined by Eq. (13-13)
θ	a temperature difference; see Eq. (10-24)
θ	angular coordinate in a spherical coordinate system; see Appendix D
λ	a nondimensional boundary layer parameter defined by Eq. (7-41)
λ	a diffusion coefficient, Γ or γ_j
λ_n	see Eq. (8-34)
μ'	absolute viscosity coefficient; see Eqs. (3-1) to (3-3)
μ	dynamic viscosity coefficient, $\mu' g_c$