Mathematics Teacher Education 5

Roza Leikin
Rina Zazkis
Editors

Learning Through Teaching Mathematics

Development of Teachers' Knowledge and Expertise in Practice

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Editors
Roza Leikin
University of Haifa
Faculty of Education
31905 Haifa
Mount Carmel
Israel
rozal@construct.haifa.ac.il

Rina Zazkis
Simon Fraser University
Faculty of Education
8888 University Drive
Burnaby BC V5A 1S6
Canada
zazkis@sfu.ca

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MATHEMATICS TEACHER EDUCATION

VOLUME 5

SCOPE

The *Mathematics Teacher Education* book series presents relevant research and innovative international developments with respect to the preparation and professional development of mathematics teachers. A better understanding of teachers' cognitions as well as knowledge about effective models for preservice and inservice teacher education is fundamental for mathematics education at the primary, secondary and tertiary level in the various contexts and cultures across the world. Therefore, considerable research is needed to understand what facilitates and impedes mathematics teachers' professional learning. The series aims to provide a significant resource for teachers, teacher educators and graduate students by introducing and critically reflecting new ideas, concepts and findings of research in teacher education.

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Learning Through Teaching Mathematics

Contributors

Lara Alcock Loughborough University, Loughborough, Leicestershire, UK, L.J.Alcock@lboro.ac.uk

Marcelo C. Borba GPIMEM and UNESP, Rio Claro-SP, Brazil, mborba@ms.rc.unesp.br

Daniel Chazan University of Maryland, College Park, MD, USA, dchazan@umd.edu

Helen Doerr Syracuse University, Syracuse, NY, USA, hmdoerr@syr.edu

Shulamit Elikan University of Haifa, Haifa, Israel

José Guzmán Instituto Politécnico Nacional, Mexico City, México, jguzman@cinvestav.mx

Dave Hewitt University of Birmingham, Birmingham, UK, d.p.hewitt@bham.ac.uk

Nicholas Jackiw KCP Technologies, Emeryville, CA, USA, njackiw@keypress.com

Carolyn Kieran Université du Québec à Montréal, Québec, Canada, kieran.carolyn@uqam.ca

Roza Leikin University of Haifa, Haifa, Israel, rozal@construct.haifa.ac.il

Stephen Lerman South Bank University, UK, lermans@lsbu.ac.uk

Peter Liljedahl Simon Fraser University, Burnaby, BC, Canada, liljedahl@sfu.ca

Robin Marcus University of Maryland, College Park, MD, USA, marcus_robin@hotmail.com

John Mason The Open University (emeritus), Milton Keynes, Buckinghamshire; University of Oxford, Oxford, UK, j.h.mason@open.ac.uk

Nathalie Sinclair Simon Fraser University, Burnaby, BC, Canada, nathsinc@sfu.ca

Ron Tzur University of Colorado Denver, Denver, CO, USA, ron.tzur@ucdenver.edu

Michal Yerushalmy University of Haifa, Haifa, Israel, michalyr@construct.haifa.ac.il

Rina Zazkis Simon Fraser University, Burnaby, BC, Canada, zazkis@sfu.ca

Rúbia B.A. Zulatto GPIMEM, FIEL and FAAL, Limeira-SP, Brazil, rubiaba@uol.com.br

Introduction

The idea of teachers Learning through Teaching (LTT) – when presented to a naïve bystander – appears as an oxymoron. Are we not supposed to learn before we teach? After all, under the usual circumstances, learning is the task for those who are being taught, not of those who teach. However, this book is about the learning of teachers, not the learning of students.

It is an ancient wisdom that the best way to "truly learn" something is to teach it to others. Nevertheless, once a teacher has taught a particular topic or concept and, consequently, "truly learned" it, what is left for this teacher to learn? As evident in this book, the experience of teaching presents teachers with an exciting opportunity for learning throughout their entire career. This means acquiring a "better" understanding of what is being taught, and, moreover, learning a variety of new things. What these new things may be and how they are learned is addressed in the collection of chapters in this volume.

LTT is acknowledged by multiple researchers and mathematics educators. In the first chapter, Leikin and Zazkis review literature that recognizes this phenomenon and stress that only a small number of studies attend systematically to LTT processes. The authors in this volume purposefully analyze the teaching of mathematics as a source for teachers' own learning.

Research literature sometimes interprets LTT as learning by observing the teaching of others, for example, examining videotaped lessons of expert teachers (e.g., Brophy, 2003; Lampert & Ball, 1998) or having teachers complete retrospective analyses of their own teaching (e.g., Lampert, 2001). However, this book specifically addresses what teachers learn *while* they are teaching.

The chapters in this volume are written by authors from different countries: Brazil, Canada, Israel, Mexico, UK, and USA. They address teaching diverse contents: numerical literacy (Doerr & Lerman, Liljedahl), geometry (Borba & Zulatto, Leikin, and Jackiw & Sinclair), algebra (Yerushalmy & Elican, Marcus & Chazan, and Kieran & Guzmán), and Real Analysis (Alcock). The focus of analyses involves teaching which occurs at a variety of levels: elementary school (Liljedahl, Doerr, & Lerman), secondary school (Yerushalmy & Elican and Marcus & Chazan), university undergraduate mathematics courses (Alcock), and teacher education courses (Zazkis, Borba & Zulatto, and Hewitt). These authors employ different methodological tools and different theoretical perspectives as they consider teaching in different learning environments: lecturing (Alcock), small group work on problems and tasks (Hewitt, Liljeahl), mathematical explorations with the support of technological software (Jackiw & Sinclair, Kieran & Guzmán), or e-learning (Borba & Zulatto). However, despite these differences, each author discusses issues that support or impede teachers' learning and exemplify teachers' learning that occurred during their professional practice.

Research on teacher education, teacher knowledge, and teacher practice is an explicit focus of contemporary research in mathematics education. Numerous studies (too numerous to give justice by citing a few) describe the complexity of teachers' work, the fragility of teachers' knowledge, and the deficiencies and strengths within teacher education practices. The important and original contribution of this book is that it ties these notions together—presenting them through the lens of a relatively unexplored phenomenon: Learning through Teaching.

When conceiving this book, our initial idea was to focus on teachers' learning of mathematics. That is, we were interested in whether and how teachers' mathematical knowledge is enhanced as a result of their teaching practice. In Leikin and Zazkis' chapter we characterize the mathematics that teachers learn when teaching. However, whenever a case of LTT was considered, we asked ourselves: "Is it mathematics or is it pedagogy?" We quickly realized that answering this question was very complex. Teachers' learning of mathematics through teaching is usually either embedded in their pedagogical choices or results in pedagogical considerations. We found that even when the authors do not discuss pedagogical issues explicitly, the mathematics that is learned by the teachers is often mathematics *for* teaching. That is, chapters that focus on teachers' learning of mathematics are not devoid of pedagogical considerations.

In addressing this complexity we found the distinction between *mathematical pedagogy* and *pedagogical mathematics* as introduced by Mason (2007) to be very helpful. In his view, mathematical pedagogy involves strategies and useful constructs for teaching mathematics. In contrast, pedagogical mathematics involves mathematical explorations useful for, and arising from, pedagogical considerations. We use this distinction as a lens for organizing the chapters in this volume: Part II includes chapters that address mainly pedagogical mathematics, while the focus of chapters in Part III is mainly on mathematical pedagogy. The chapters in Part II and Part III are introduced by the editors in the beginning of each section (see Interlude 1 and Interlude 2). Here we focus on Part I.

Part I of this volume addresses issues related to the theory and the methodology of research on LTT. In particular, the first chapter, co-authored by Leikin and Zazkis, exemplifies and examines factors that contribute to or impede LTT – issues that are then echoed in subsequent chapters. However, in order to explore these factors, it is essential to understand what is meant by teachers' learning or by learning, in general. Mason, in his chapter, offers a concise definition: Learning is a transformation of attention. He elaborates further, stating that this transformation involves both "shifts in the form as well as in the focus of attention." Mason substantiates and instantiates this view using a series of phenomena in which learning occurred while teaching or doing mathematics. A common theme in examining these presented episodes is that of reflection, which is considered an essential condition for learning from experience.

Mason's theoretical discussion and the analysis of the phenomena are followed by some practical considerations which address the question of "What can teachers do?" Some suggestions are provided regarding what teachers can do for their students, for themselves, and for each other in order to enhance their learning. The theme of reflection is also at the heart of Tzur's chapter, who presents an explicit theoretical model entitled, "reflection on activity-effect relationship." This model is used to elaborate upon *what* and *how* teachers can learn from their teaching.

Mason suggests that "Learning about teaching from teaching is a lifetime process of refining sensitivities to students and to the conditions in which learning is fostered and sustained." Tzur elaborates further on this idea, claiming that the experience of student-teaching, or "practicum," is usually stressful and insufficient to develop efficient ways of engaging with students and to notice an impact on students' learning. As such, on-going LTT is the only possible solution for developing effective teaching practices. However, while Tzur believes that every teaching activity is a potential source for learning, he refers to LTT as "unrealized potential" and elaborates in great detail on the reasons for that. Based on the work of Simon (2006, 2007), both Tzur and Leikin and Zazkis identify "perception based perspective" – in contrast with "conception based perspective" – as one of the major factors that impede learning in general, and LTT, in particular. Tzur concludes with a detailed list of ideas and questions for further research on LTT. These ideas include explicating what "counts" as evidence of LTT and examining how it can be measured or occasioned.

The study by Leikin – that concludes Part I – provides a partial answer to the question of occasioning LTT. She describes a methodology of systematic exploration of LTT through employing multiple solution tasks in teaching experiments and longitudinal teacher-development experiments. Multiple solution tasks are used both as a didactical tool to engage teachers' learning, as well as a research tool used to intensify unforeseen (for teachers) situations and to analyze teachers' LTT. Leikin's conclusions point to an interrelationship between teachers' mathematical and pedagogical knowledge – the issue that is explored in further detail in all the chapters of this volume.

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Contents

Part I
Theoretical and Methodological Perspectives on Teachers' Learning Through Teaching

Teachers' Opportunities to Learn Mathematics Through Teaching

Roza Leikin and Rina Zazkis

Every teacher's greatest opportunity for further learning in mathematics education is her classroom teaching. (Simon, 2006, p. 137).

Introduction

Numerous studies on mathematics teacher development have demonstrated that mathematics teachers learn through their teaching experiences (e.g., Cobb & McClain, 2001; Kennedy, 2002; Lampert & Ball, 1999; Lesh & Kelly, 1994; Mason, 1998, 2002; Ma, 1999; Shulman, 1986; Wilson, Shulman, & Richert, 1987). Several articles explored teachers' learning in the course of their teaching careers, among other elements of their professional growth (e.g., Franke, Carpenter, Levi, & Fennema, 2001; Chazan, 2000; Kennedy, 2002; Lampert, 2001; Ma, 1999; Mason, 1998; Schifter, 1998). Other studies investigated different approaches to professional development and stumbled upon learning through teaching (LTT) while focusing on other issues (e.g., Cobb & McClain, 2001; Lesh & Kelly, 1994). Only a few studies have been devoted to the systematic investigation of teachers' learning in their own classrooms (Leikin, 2006; Leikin, 2005; Leikin & Rota, 2006; Nathan & Knuth, 2003).

The main body of research that addresses LTT focuses mainly on teachers' continual inquiry into the students' thinking and learning, but little is known about the teachers' learning of mathematics in their own classrooms. Perrin-Glorian, DeBlois, and Robert (2008) stressed that research concerning LTT (they used the term "indirect learning") is scant and that the educational community has a relatively limited understanding of what it is that changes in teachers' knowledge, or how these changes come about in an authentic classroom situation, especially in the domain of mathematical knowledge.

R. Leikin (✉)
University of Haifa, Haifa, Israel

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Evidence for Learning Through Teaching

Theories of Teacher Knowledge and Teaching

In their seminal work that analyzes the epistemological structure of teachers' knowledge, Wilson et al. (1987) described the complexity of teachers' knowledge and of its sources. The researchers argued that teachers' reasoning begins with comprehension of the set of core ideas related to the topic to be taught, and that teachers' knowledge related to the topic is transformed while teaching. This process of transformation is associated with the planning and design of instructional activities, evaluation, and reflection. As a result, after teaching, teachers attain new comprehension, enriched by fresh understanding and increased awareness of the purposes of instruction, its subject matter, and the participants.

The teaching/learning process can be modeled as a sequence of situations that result in new knowledge construction by students (Brousseau, 1997). So-called *a-didactic situations,* in which the teacher passes some of the responsibilities for the learning process onto the students, are considered to be most effective for students' learning. We consider such situations to be most effective for teachers' learning as well. The teacher is responsible for the *devolution* of a meaningful task that supports the design of an a-didactic situation. When an a-didactic situation is created, students are responsible for realizing the learning purposes by approaching the task, and the teacher's role is to facilitate this realization. In a situation of this type, the teachers adjust their plans to the students' ideas and learn together with the students.

Based on the theory of didactic situations, Simon (1997) suggested a Mathematics Teaching Cycle model for the teaching process. According to the model, teachers design a hypothetical learning trajectory based on the various types of knowledge they possess. The trajectory includes three interrelated elements: learning goals, the teacher's plan for learning activities (tasks), and the teacher's hypothesis of the learning process. When implementing this hypothetical trajectory in the classroom, teachers need to adjust it based on their interactions with students. These adjustments lead teachers to new understandings that precede the next hypothetical learning trajectory they design.

Following the analysis of elements of epistemological knowledge of mathematics teachers, Steinbring (1998) devised a two-ring model of teaching and learning mathematics as autonomous systems. According to this model, teachers use their knowledge of content and of students to design and devolve learning tasks for the students, and the students cope with these tasks by activating their own knowledge and devising their own interpretations of the tasks. Students approach the tasks, reflect on the process, and, as a result, construct their knowledge. Simultaneously, the teacher observes and supports the learning process, reflects on the learning situation, adjusts the task to the situation, and transforms the teacher's own knowledge. New learning opportunities are based on knowledge enriched by the teaching experiences. The cyclic view of teaching (e.g., Artzt & Armour-Thomas, 2002; Steinbring, 1998; Simon, 1997; Wilson et al., 1987) does not claim that teachers learn through teaching, but demonstrates that teaching has a great *potential* for teachers' learning.