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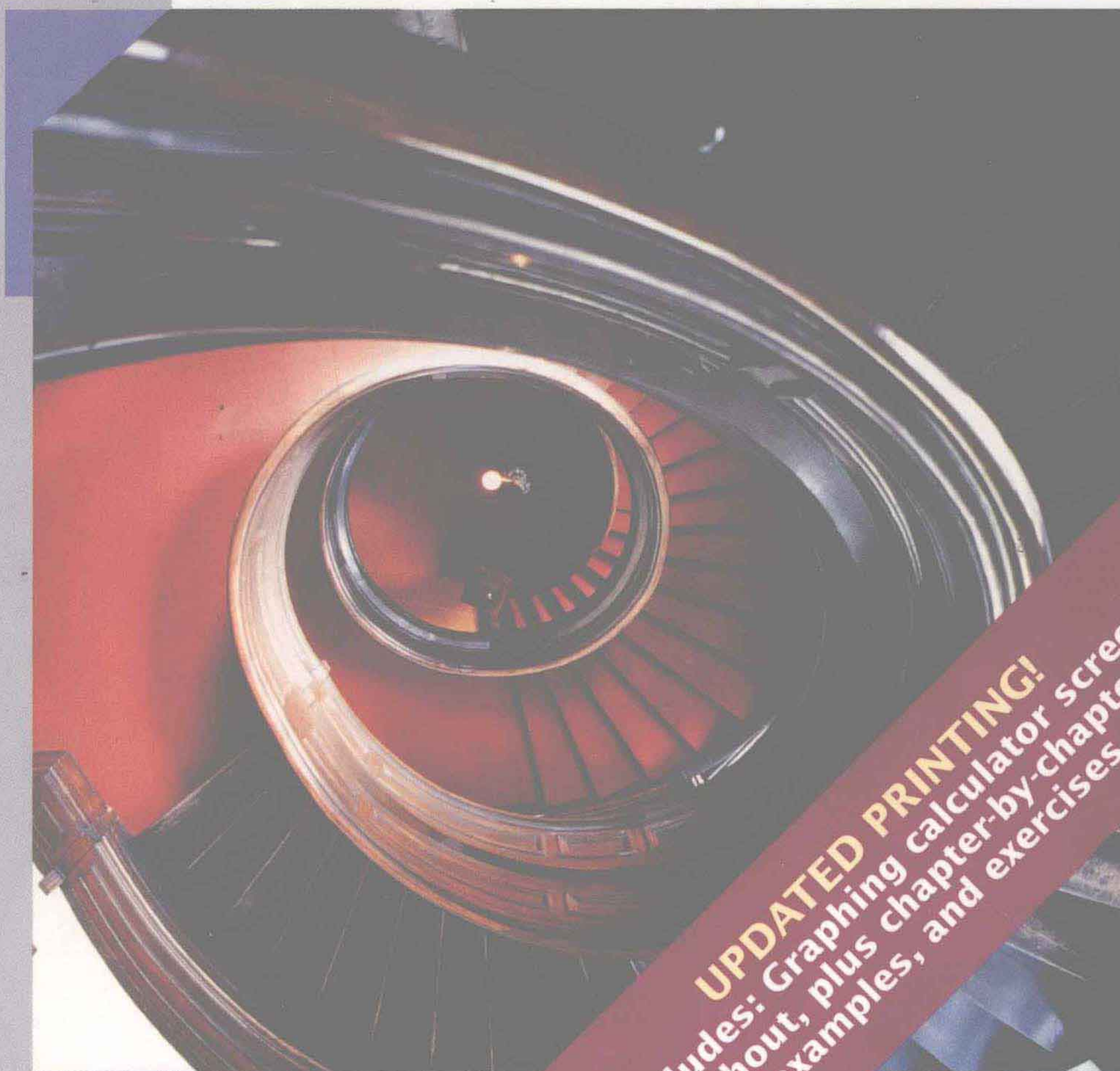
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# Intermediate Algebra

Concepts and Applications

FOURTH EDITION



**UPDATED PRINTING!**  
Includes: Graphing calculator screens  
throughout, plus chapter-by-chapter tips,  
examples, and exercises.



# Intermediate Algebra

Concepts and Applications

**FOURTH EDITION**

**MARVIN L. BITTINGER**

Indiana University–Purdue University at Indianapolis

**MERVIN L. KEEDY**

Purdue University

**DAVID ELLENBOGEN**

St. Michael's College

**BARBARA JOHNSON**

Indiana University–Purdue University at Indianapolis



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Production Supervisor	Jennifer Bagdigian
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# Intermediate Algebra

Concepts and Applications

**FOURTH EDITION**



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*For Kit*

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*An extraordinary mother, grandmother, and  
advocate for the educationally disadvantaged*

D.J.E.



# P R E F A C E

Appropriate for a one-term course in intermediate algebra, this text is intended for students who have completed a first course in algebra. It is the second of two texts in an algebra series that also includes *Elementary Algebra: Concepts and Applications*, Fourth Edition, by Bittinger/Keedy/Ellenbogen. *Intermediate Algebra: Concepts and Applications*, Fourth Edition, is a significant revision of the Third Edition with respect to design, contents, pedagogy, and an expanded supplements package. This series is designed to prepare students for any mathematics course at the college algebra level.

## A P P R O A C H

Our approach, which has been developed over many years, is designed to help today's students both learn and retain mathematical concepts. Our goal in preparing this revision was to address the major challenges for teachers of developmental mathematics courses that we have seen emerging during the early 1990s. The first challenge is to prepare students of developmental mathematics to make the transition from "skills-oriented" elementary and intermediate algebra courses to the more "concept-oriented" presentation of college algebra or other college-level mathematics courses. The second is to teach these same students critical-thinking skills: to reason mathematically, to communicate mathematically, and to solve mathematical problems. The third challenge is to reduce the amount of content overlap between elementary algebra and intermediate algebra texts.

Following are some aspects of the approach that we have used in this revision to help meet the challenges we all face teaching developmental mathematics.

### PROBLEM SOLVING

One distinguishing feature of our approach is our treatment of and emphasis on problem solving. We use problem solving and applications to motivate the material wherever possible, and we include real-life applications and problem-solving techniques throughout the text. We feel that problem solving encourages students to think about how mathematics can be used. It also challenges students and helps to prepare them for more difficult material in later courses.



- In Chapter 1, we introduce the five-step process for solving problems: (1) Familiarize, (2) Translate, (3) Carry out, (4) Check, and (5) State the answer. These steps are used throughout the text whenever we encounter a problem-solving situation. Repeated use of this algorithm gives students a sense that they have a starting point for any type of problem they encounter, and frees them to focus on the mathematics necessary to successfully translate the problem situation. (See pages 26–33, 265, and 449.)

## FUNCTIONS AND GRAPHING

To retain skills and to apply them at a more conceptual level in later courses, students must have an intuitive understanding of the material. A visual interpretation of mathematical concepts can provide this type of understanding to those students with a visual, rather than symbolic, orientation. In addition, familiarity and practice with functions and graphing techniques make students more comfortable with this essential tool when they move on to later courses.

- We introduce functions and graphing in Chapter 2, which is substantially earlier than in many intermediate algebra texts. We then present functions and graphs throughout the text to help students develop an intuitive understanding of different types of equations and their solutions. For instance, examples of polynomial and rational functions are introduced along with polynomial and rational expressions and equations in Chapters 5 and 6. (See pages 64–71 and 72–80.)
- The inclusion of *Technology Connections*, an optional feature that allows students to use a graphing calculator or computer to help visualize concepts, provides additional opportunities for students to see the usefulness of functions and graphing. (See pages 69, 262, and 556.)
- Many examples in the text are also illustrated on a grapher. The graphs appear in the margin by the example and aid the student in using a grapher to visualize the concepts. (See pages 68, 200, 266, and 507.)

## CONTENT

Many intermediate algebra texts contain a substantial review of elementary algebra topics. This lulls students into complacency and does not allow instructors sufficient time to cover the intermediate algebra topics necessary to prepare students for later courses.

- By introducing graphing and functions in Chapter 2, we present students with “intermediate algebra” topics almost immediately. These topics are then used throughout the text to give students familiarity and practice with concepts that will be critical to them at the college level.
- Systems of equations are introduced in Chapter 3 to provide students with a valuable problem-solving tool. Students can then translate problem situations into systems of equations throughout the remainder of the text. This approach provides a useful alternative to always translating problems into equations in which only one variable is used.



## PEDAGOGY

*Skill Maintenance Exercises and Cumulative Reviews.* Retention of skills is critical to the future success of our students. In nearly all exercise sets, we include carefully chosen exercises that review skills and concepts from preceding chapters of the text. Each chapter test includes Skill Maintenance Exercises selected from the three or four text sections that are identified at the beginning of each chapter. After every three chapters, and at the end of the text, we have also included a Cumulative Review, which reviews skills and concepts from all preceding chapters of the text. (See pages 164, 170, 171, and 205.)

*Synthesis Exercises.* Each exercise set ends with a set of synthesis exercises. These problems can offer opportunities for students to synthesize skills and concepts from earlier sections with the present material, or can provide students with deeper insights into the current topic. Synthesis exercises are generally more challenging than those in the main body of the exercise set. (See pages 95, 121, 122, and 315.)

*Verbalization Skills.* Wherever appropriate throughout the text, we have discussed how mathematical terms are used in language. The Summary and Review sections emphasize key terms and important properties and formulas. In addition, thinking and writing exercises are included in the Synthesis Exercises. These encourage students to verbalize mathematical concepts, leading to better understanding. (See pages 390 and 406.)

## WHAT'S NEW IN THE FOURTH EDITION?

We have rewritten many key topics in response to user and reviewer feedback and have made significant improvements in design and pedagogy. Detailed information about the content changes is available in the form of a Conversion Guide. Please ask your local Addison-Wesley sales representative for more information. Following is a list of the major changes in this revision.

### *New Design*

- The new design is more open and readable. Pedagogical use of color makes it easier to see where exercises, explanations, and examples begin and end.
- The entire art program is new for this edition. We have ensured the accuracy of the graphical art through the use of computer-generated graphs. Color in the graphical art is used pedagogically and precisely to help the student visualize the mathematics. (See pages 475 and 546.)

### *Technology Connections*

- These features integrate technology, increase the understanding of concepts through visualization, encourage exploration, and motivate discovery learning. Optional Technology Connection exercises occur in many exercise sets. (See pages 277 and 546.)



*Graphing Calculator Appendix*

- The Graphing Calculator Appendix is an introduction to a graphing calculator, as well as a supplement to the text. The appendix corresponds to the sections in the text, so that students can use a grapher to visualize the text concepts as they are developed. Each section contains illustrated examples, describes the procedures generally, and gives specific keystrokes for the TI-82 calculator. Short exercise sets are provided for practice; answers are at the end of the appendix.

*Writing Exercises*

- Nearly every set of Synthesis Exercises begins with two writing exercises. These exercises are usually not as difficult as other synthesis exercises, but require written answers that aid in student comprehension, critical thinking, and conceptualization. Because some instructors may collect answers to writing exercises, and because more than one answer may be correct, answers to writing exercises are not listed at the back of the text. (See pages 19, 406, and 550.)

*Content Changes.* A variety of content changes have been made. Some of the more significant changes are listed below.

- Rational exponents are now presented early in Chapter 7. Doing so has enabled us to use rational exponents in our subsequent work with radical notation. (See pages 343–346.)
- Although our fear of students performing “illegal” cancellations is as acute as ever, we now use canceling as a way to simplify rational expressions. We do so in recognition of the fact that we use canceling when working on our own. Whenever canceling is used, we point out that we are effectively “removing” a factor of 1. (See pages 280–285.)
- In response to numerous requests, we have included a new section on “Geometric Applications,” in which the important properties of  $30^\circ$ – $60^\circ$ – $90^\circ$  and  $45^\circ$ – $45^\circ$ – $90^\circ$  triangles are developed and used. (See pages 343–346.)
- Because so many students remain convinced that they “cannot do word problems,” we have made increased use of guessing as a means of familiarizing oneself with a problem-solving situation. By checking to see if a guess is correct, students can more easily discover an algebraic translation of the problem. (See pages 114 and 115.)
- Throughout the text, we have included a variety of new applications that appeal to a large cross section of the student population. By emphasizing applications that students and faculty find interesting, we hope that we have made the text enjoyable to use. (See pages 164, 327, and 479.)

**S U P P L E M E N T S   F O R   T H E   I N S T R U C T O R**

**INSTRUCTOR'S SOLUTIONS MANUAL**  
by Judith A. Penna

This supplement contains worked-out solutions to all exercises in the text.



## **INSTRUCTOR'S RESOURCE GUIDE** by Donna DeSpain

This supplement contains the following:

- Extra practice problems for challenging topics in the text
- Black-line masters of grids and number lines for transparency masters or test preparation
- Videotape index and section cross references to the tutorial software packages available with this text
- Conversion guide from the Third Edition to the Fourth Edition

## **PRINTED TEST BANK** by Donna DeSpain

This supplement contains the following:

- Six alternative test forms for each chapter and six final examinations
- Two multiple-choice versions of each chapter test

All test forms have been completely rewritten.

## **COMPUTERIZED TESTING**

**Omnitest II (for IBM and Macintosh).** This computerized test bank allows you to create up to 99 versions of a customized test with just a few keystrokes, and allows the option of choosing items by chapter, section, or objective. It contains over 400 multiple-choice and open-ended algorithms. You may enter your own test items, edit existing items, and define the level of difficulty of problems.

## S U P P L E M E N T S   F O R   T H E   S T U D E N T

## **STUDENT'S SOLUTIONS MANUAL** by Judith A. Penna

This manual contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the text, and answers for all even-numbered exercises in the text.

## **VIDEOTAPES**

Developed especially for the Bittinger/Keedy/Ellenbogen texts, these videotapes feature an engaging team of lecturers presenting material from each section of the text in an interactive format that includes a group of students. The lecturers' presentation also incorporates slides, sophisticated computer-generated graphics, and a white board to support an approach that emphasizes visualization and problem solving.

## **TUTORIAL SOFTWARE**

**THE MATHLAB<sup>+</sup>** (IBM and Macintosh). This software combines a unique combination of drill and practice modules with an interactive and easy-to-use graphing



tool. The drill and practice segments feature feedback for wrong answers and detailed record keeping. The graphing tool allows students to graph and explore a wide variety of two-dimensional functions.

**Algebra Problem Solver (IBM).** After selecting a topic and an exercise type, students can enter their own exercises or request an exercise from the computer. In each case, the student is given detailed, annotated, step-by-step solutions.

## ACKNOWLEDGMENTS

We wish to express our appreciation to the many people who helped with the development of this book. Barbara Johnson and Laurie A. Hurley deserve special thanks for their many fine suggestions. Their proofreadings of the text, in spite of almost endless time pressure, contributed immeasurably to the accuracy and readability of the text. Judy Penna also merits special thanks for her preparation of the *Student's Solution Manual*, the *Instructor's Solution Manual*, and the indexes. Judy's work is always performed with a thoroughness that amounts to another proofreading of the book and for that we are grateful. We are also indebted to Stuart Ball for his expert guidance in preparing the Technology Connections and the associated artwork.

This book's sponsoring editor, Melissa Acuña, performed admirably in coordinating the many intricacies of this project; George and Brian Morris of Scientific Illustrators generated a remarkable set of graphs and illustrations that are both precise and easily understood; and Leo Harrington drew the many fine sketches that enhance our exercises and examples. Geri Davis and Martha Morong of Quadrata, Inc., provided design, editorial, and production services second to none, ensuring that every last detail has been taken care of. To all of these people, we offer our deepest thanks.

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M.L.B., M.L.K., and D.J.E.



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# Algebra and Problem Solving

## AN APPLICATION

A commercial jet, flying from Chicago to Los Angeles, has been instructed to climb from its present altitude of 8000 ft to a cruising altitude of 29,000 ft. If the plane ascends at a rate of 3500 ft per minute, how long will it take to reach the cruising altitude?

This problem appears as Exercise 9 in Section 1.4.



### Carole Perry

COMMERCIAL COPILOT

*"A good working knowledge of math is essential to the success of each flight. I must be able to calculate quickly and accurately without a calculator."*



The principal theme of this text is problem solving in algebra. An overall strategy for solving problems is presented in Section 1.4. Additional and increasing emphasis on problem solving appears throughout the book.

This chapter begins with a short review of algebraic symbolism and properties of numbers. As you will see, the manipulations of algebra, such as simplifying expressions and solving equations, are based on the properties of numbers.

## 1.1

### The Beginnings of Algebra

Algebraic Expressions and Their Use • Translating to Algebraic Expressions • Evaluating Algebraic Expressions • Solutions to Equations • Sets of Numbers • Set Notation • Notation for Rational Numbers

This section is intended to introduce some of the basic concepts of algebra. We will study the use of algebraic expressions in problem solving and some of the types of numbers needed for problem solving.

#### Algebraic Expressions and Their Use

In arithmetic, you worked with expressions like

$$42 + 58, \quad 9 \times 12, \quad 17 - 5, \quad \text{and} \quad \frac{5}{7}.$$

In algebra, we will work with expressions like

$$42 + x, \quad l \cdot w, \quad 17 - t, \quad \text{and} \quad \frac{d}{y}.$$

Sometimes a letter can stand for various numbers. In that case, we call the letter a **variable**. Sometimes a letter can stand for just one number. In that case, we call the letter a **constant**. Let  $b$  = your date of birth. Then  $b$  is a constant. Let  $a$  = your age. Then  $a$  is a variable since  $a$  changes from year to year.

An **algebraic expression** consists of variables, numbers, and operation signs. Thus all of the expressions above are examples of algebraic expressions.

Algebraic expressions frequently arise in problem-solving situations. For example, consider the chart at the top of page 3. Suppose we want to determine how many Americans were uninsured in 1992. We might use algebra to translate the problem into an equation, with  $x$  representing the number of uninsured Americans.

Number of Americans with health insurance	plus	Number of uninsured Americans	is	Total number of Americans
↓		↓		
212,000,000	+	$x$	=	249,000,000