McGraw Hill Education

数学图书 影印版系列

Miklós Bóna 著

计数组合学导引

Introduction to Enumerative Combinatorics

清华大学出版社

McGraw Hill

McGraw Hill Education

数学图书 影 印 版 系 列

Miklós Bóna 著

计 数 组 合 学 导 引

Introduction to Enumerative Combinatorics

清华大学出版社
北京

Miklós Bóna
Introduction to Enumerative Combinatorics
EISBN: 007-125415-3

北京市版权局著作权合同登记号　图字：01-2006-4342

图书在版编目（CIP）数据

计数组合学导引：英文/（匈）博瑙（Bona, M.）著. —影印本. —北京：清华大学出版社，2009.11
ISBN 978-7-302-21340-6

Ⅰ. 计…　Ⅱ. 博…　Ⅲ. 组合数学—高等学校—教材—英文　Ⅳ. O157

中国版本图书馆 CIP 数据核字(2009)第 187059 号

责任编辑：刘　颖
责任印制：王秀菊

出版发行：清华大学出版社　　**地　址**：北京清华大学学研大厦 A 座
http://www.tup.com.cn　　**邮　编**：100084
社　总　机：010-62770175　　**邮　购**：010-62786544
投稿与读者服务：010-62776969，c-service@tup.tsinghua.edu.cn
质　量　反　馈：010-62772015，zhiliang@tup.tsinghua.edu.cn
印 刷 者：北京密云胶印厂
装 订 者：三河市李旗庄少明装订厂
经　销：全国新华书店
开　本：185×230　**印　张**：34.25
版　次：2009 年 11 月第 1 版　　**印　次**：2009 年 11 月第 1 次印刷
印　数：1~3000
定　价：55.00 元

本书如存在文字不清、漏印、缺页、倒页、脱页等印装质量问题，请与清华大学出版社出版部联系调换。联系电话：010-62770177 转 3103　　产品编号：022574-01

Foreword

What could be a more basic mathematical activity than counting the number of elements of a finite set? The misleading simplicity that defines the subject of enumerative combinatorics is in fact one of its principal charms. Who would suspect the wealth of ingenuity and of sophisticated techniques that can be brought to bear on a such an apparently superficial endeavor? Miklós Bóna has done a masterful job of bringing an overview of all of enumerative combinatorics within reach of undergraduates. The two fundamental themes of bijective proofs and generating functions, together with their intimate connections, recur constantly. A wide selection of topics, including several never appearing before in a textbook, are included that give an idea of the vast range of enumerative combinatorics. In particular, for those with sufficient background in undergraduate linear algebra and abstract algebra there are many tantalizing hints of the fruitful connection between enumerative combinatorics and algebra that plays a central role in the subject of algebraic combinatorics. In a foreword to another book by Miklós Bóna I wrote, "This book can be utilized at a variety of levels, from random samplings of the treasures therein to a comprehensive attempt to master all the material and solve all the exercises. In whatever direction the reader's tastes lead, a thorough enjoyment and appreciation of a beautiful area of combinatorics is certain to ensue." Exactly the same sentiment applies to the present book, as the reader will soon discover.

Richard Stanley
Cambridge, Massachusetts
June 2005

Preface

Students interested in Combinatorics in general, and in Enumerative Combinatorics in particular, already have a few choices as to which books to read. However, the overwhelming majority of these books are either on General Combinatorics on the undergraduate level, or on Enumerative Combinatorics on the graduate level. The present book strives to be of a third kind. It focuses on *Enumerative Combinatorics*, attempts to be reasonably comprehensive, and is meant to be read primarily by *undergraduates*. We do understand that undergraduates need to learn various aspects of Combinatorics. Therefore, while in this book we will always count something, we will count objects from many areas of Combinatorics—trees, permutations, graphs, hypergraphs, sets, partitions, compositions, matrices, and so on—hopefully broadening the scope of the student's interest. In the process of counting these objects, we formally define them, and discuss the most important features of their structures. Our strong focus on enumeration allows us to reach the level of open problems in several chapters. New students of the field often find it fascinating that after only a year of learning, they can understand the questions attacked by experts. We want to encourage this process.

The book can be used in at least three ways. One can teach a one-semester course from it, choosing the most general topics. One can also use the book for a two-semester course, teaching most of the text and exploring the supplementary material that is given in form of exercises. If one has already taught a one-semester course using a general Combinatorics textbook and wants to follow up with a second semester that focuses on enumeration, one may use the last six chapters of this book. The book is also useful for teaching an introductory course for graduate students who do not have solid background in Combinatorics.

There are several topics here that are discussed in detail in an undergraduate textbook for a first time, such as acyclic and parking functions, unimodality, log-concavity, the real zeros property, and magic squares.

Therefore, we hope the book will provide a useful reference material for students interested in these topics.

Several topics, like pattern avoiding permutations, Ramsey numbers, or Hamiltonian cycles, are not discussed in the text, but they are the subjects of many of the exercises. This allows the instructor to cover these topics after all. About half of all exercises come with full solutions. We have decided to include so many full solutions due to very strong student feedback in this matter.

The book consists of three parts. The first part covers basic methods of enumeration, up to generating functions. This part should be covered in any undergraduate Combinatorics course. The second part applies the learned counting methods to central objects of Combinatorics, such as permutations, graphs, and hypergraphs. Chapters in this part begin with easy sections, but eventually reach more sophisticated theorems. It is up to the instructor to decide how far he or she wants to proceed within each chapter. The third part is a sampling of much more special topics, such as unimodality and log-concavity, and magic squares. This is meant to provide the students with a closer view of research problems.

Progress in any area of research or education always leads to new questions. We hope that the effect of this book will be no different, that is, students who read this book and grow to like Enumerative Combinatorics will be difficult to count.

Acknowledgments

I am indebted to the authors of the books from which I learned Combinatorics, such as Richard Stanley, for *Enumerative Combinatorics I and II*, László Lovász, for *Combinatorial Problems and Exercises,* Herb Wilf, for *Generatingfunctionology,* and countless others. I should also mention my gratitude to the authors of the books I used in teaching combinatorics, such as *Introductory Combinatorics* by Kenneth Bogart, and *A course in Combinatorics* by Richard Wilson and Jacobus Van Lint.

I am grateful to Richard Stanley, my thesis advisor, who taught me the foundations of Enumerative Combinatorics, Catherine Yan, who taught me many things about Parking Functions, and to my frequent co-author, Bruce Sagan, from whom I learnt a lot about log-concavity. My gratitude is extended to Miklós Simonovits, who gave me good advice on Extremal Graph Theory.

A significant part of the book was written during my stay in Hungary in Summer of 2004, when I enjoyed the hospitality of my parents, Miklós and Katalin Bóna.

My gratitude is extended to those colleagues who reviewed parts or all of the manuscript. At the University of Florida, this includes David Drake, Kevin Keating, Rebecca Smith, Andrew Vince, and Neil White.

I am grateful for the advice and comments of the following reviewers: K.T. Arasu, Wright State University; Joseph Bonin, George Washington University; Mihai Ciucu, Georgia Institute of Technology; Guoli Ding, Louisiana State University; Thomas Dowling, Ohio State University; Mark Ellingham, Vanderbilt University; Darren Glass, Columbia University; Henry Gould, West Virginia University; Frederick Hoffman, Florida Atlantic University; Cary Huffman, Loyola University of Chicago; Robert Hunter, Pennsylvania State University; Garth Isaak, Lehigh University; Norman Johnson, University of Iowa; Andre Kezdy, University of Louisville; John Konvalina, University of Nebraska–Omaha; Isabella Novik, University of Washington; James Propp, University of Wisconsin; Vladimir Tonchev, Michigan Technological University; Carl Wagner, University of Tennessee; Walter Wallis, Southern Illinois University; and Doron Zeilberger, Rutgers University.

Most of all, I must thank my wife Linda, who not only put up with my writing a third book, but also kept pace with me as explained by the introductory example of Chapter 9.

目　　录

Contents