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AUTOMATED
VOLUME I
REASONING

ALAN ROBINSON AND ANDREI VORONKOV
EDITORS

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HANDBOOK OF AUTOMATED REASONING

VOLUME I

Editors

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and

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HANDBOOK OF AUTOMATED REASONING

VOLUME I

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Preface

Automated reasoning has matured into one of the most advanced areas of computer science. During the half-century since the pioneering publications of the 1950s, significant progress has been achieved both in its theory and practice, culminating in a completely automatic solution of the Robbins Problem by the theorem prover EQP implemented by Bill McCune. This problem in algebra had remained open for over 50 years despite repeated attempts of mathematicians to solve it.

Several theoretical results, ideas, and techniques contributed to the Robbins Problem solution. We mention only a few: equational unification, and in particular AC-unification (Chapter 8 of this Handbook), completion procedures and notions of redundancy (Chapters 2 and 7), the basic strategy (Chapter 7), and term indexing (Chapter 26).

This Handbook presents overviews of the fundamental notions, techniques, ideas, and methods developed and used in automated reasoning and its applications, which are used in many areas of computer science, including software and hardware verification, logic and functional programming, formal methods, knowledge representation, deductive databases, and artificial intelligence..

The idea of making this Handbook originated during a visit by the first editor to the Computing Science Department of Uppsala University in 1996, where the second editor was working at the time. The idea was then presented at the Dagstuhl workshop on Deduction in 1997, after which the work began. It has taken four years to put together all the papers in their current form. Over 2000 email messages were exchanged between the editors and the authors.

The material included in the Handbook is intended to cover most of the areas in automated deduction, from theory to implementation. Nearly every chapter can be used as a basis for an undergraduate or a postgraduate course. In fact, some of them have already been so used. The chapters contain both basic and advanced material. It was deliberately decided also to include material that bridges the gap between the traditional automated reasoning (as presented at the CADE conferences) and related areas. Examples are model checking (Chapter 24), nonmonotonic reasoning (Chapter 19), numerical constraints (Chapter 12), description logics (Chapter 23), and implementation of declarative programming languages (Chapter 26).

To help the reader navigate through a large amount of material in the Handbook, the global concept index is provided at the end. It contains references to the pages containing the main notions and concepts introduced in different chapters.

The structure of the book is as follows.

Part I consists of a single chapter: an overview of the *early history* of automated deduction by Martin Davis.

Part II presents reasoning methods in *first-order logic*. Two most popular methods: *resolution* and *semantic tableaux* are discussed in Chapters 2 by Leo Bachmair and Harald Ganzinger and 3 by Reiner Hähnle. Nearly all existing implementations of first-order theorem provers are based on variants of one of these methods. The *inverse method*, both for classical and nonclassical logics, is introduced in Chapter 4 by Anatoli Degtyarev and Andrei Voronkov. Systems implementing first-order logic usually transform the goal formula into a clausal normal form. Chapters 5 by Matthias Baaz, Uwe Egly, and Alexander Leitsch, and 6 by Andreas Nonnengart and Christoph Weidenbach, discuss *short normal forms*, both from the theoretical and practical viewpoints.

Part III is dedicated to *equality and other built-in theories*. The first four chapters of this part discuss reasoning with equality and related subjects: Chapter 7 by Robert Nieuwenhuis and Albert Rubio deals with *paramodulation-based reasoning*, Chapter 8 by Franz Baader and Wayne Snyder presents *unification theory*, Chapter 9 by Nachum Dershowitz and David A. Plaisted overviews *rewriting*, and Chapter 10 by Anatoli Degtyarev and Andrei Voronkov discusses *equality reasoning in tableau-based and sequent-based calculi*. The next two chapters treat other important theories: Chapter 11 by Shang-Ching Chou and Xiao-Shan Gao presents *theorem proving in geometry*, while Chapter 12 by Alexander Bockmayr and Volker Weispfenning overviews methods of *solving numerical constraints*.

Part IV discusses methods of automated reasoning using *induction*. Chapter 13 by Alan Bundy gives a general introduction to induction, then Chapter 14 by Hubert Comon presents the so-called “*inductionless induction*” where induction is implemented using equational reasoning.

Part V discusses higher-order logic, which is used in a number of automatic and interactive proof-development systems. This part begins with two fundamental Chapters 15 by Peter Andrews and 16 by Gilles Dowek introducing, respectively, *classical type theory* and *higher-order unification*. The next two Chapters 17 by Frank Pfenning, and 18 by Henk Barendregt and Herman Geuvers discuss variants of higher-order logic used in two kinds of interactive systems: *logical frameworks* and *proof-assistants* using dependent type systems.

Part VI presents automated reasoning in *nonclassical logics*. Chapter 19 by Jürgen Dix, Ulrich Furbach, and Ilkka Niemelä is devoted to *nonmonotonic reasoning*, while Chapter 20 by Matthias Baaz, Christian G. Fermüller, and Gernot Salzer to reasoning in *many-valued logics*. The next two Chapters 21 by Hans Jürgen Ohlbach, Andreas Nonnengart, Maarten de Rijke, and Dov M. Gabbay, and 22 by Arild Waaler discuss reasoning methods for a wide range of logics whose semantics is characterized by the possible worlds semantics, for example, intuitionistic and modal logics. They discuss *translation into first-order classical logic* and the *connection method*,

respectively. Also highly relevant to reasoning in nonclassical logics are Chapters 4 and 23 put in other Parts of this Handbook.

Part VII deals with *decidable classes and model building*. The first two chapters concern two areas that have only recently emerged and which are now used in a number of applications: Chapter 24 by Edmund M. Clarke and Bernd-Holger Schlingloff gives an overview of *model checking*, Chapter 23 by Diego Calvanese, Giuseppe De Giacomo, Maurizio Lenzerini, and Daniele Nardi discusses reasoning in expressive *description logics*. In Chapter 25 Christian G. Fermüller, Alexander Leitsch, Ullrich Hustadt, and Tanel Tammet present *resolution-based decision procedures* for various classes of first-order formulas.

Part VIII deals with *implementation-related* questions. In Chapter 26 R. Sekar, I.V. Ramakrishnan, and Andrei Voronkov give an overview of *term indexing* used in implementing not only first-order theorem provers but also logic and functional programming languages. The next two Chapters 27 by Christoph Weidenbach and 28 by Reinhold Letz and Gernot Stenz discuss implementation of, respectively, *resolution-based* and *model elimination-based theorem provers*.

The Web page

<http://www.cs.man.ac.uk/~voronkov/handbook-ar/index.html>

contains further material related to this book. All comments and corrections should be sent to the second editor by email voronkov@cs.man.ac.uk or v@ronkov.com.

Alan Robinson, Andrei Voronkov
Greenfield and Manchester, 14 February 2001.

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