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Lucien Le Cam, Grace Lo Yang

Asymptotics in Statistics

Some Basic Concepts

Second Edition

统计学中的渐近性

基本概念

第2版

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Asymptotics in Statistics

Some Basic Concepts

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Springer

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Dedicatory Note

On April 25, 2000, as the last editorial touches were added to our book, Lucien Le Cam passed away. At Berkeley, he worked without stopping on the final revision of our book up until the time when he was hospitalized, just four days before his death.

His passing is a great loss to the statistical profession. He was a fundamental thinker in mathematical statistics and a principal architect of the modern asymptotic theory of statistics, following and extending the path of Neyman and Wald. Among the numerous seminal concepts that he introduced, the Le Cam distance between experiments provides a coherent statistical theory that links asymptotics and decision theory.

I was privileged and extremely fortunate to be his student and later his collaborator on several projects, including this book. Statisticians all know that Professor Le Cam was a brilliant mathematician. His students, colleagues, and friends also know of his kindness, generosity, integrity and patience.

This book is dedicated to the memory of Professor Le Cam. We feel his loss keenly. He will be greatly missed.

Grace Lo Yang
College Park, Maryland

May 2000

Preface to the Second Edition

This is a second edition, "reviewed and enlarged" as the French used to say. The 1990 edition was generally well received, but complaints about the conciseness of the presentation were heard. We have tried to accommodate such concerns, aiming at being understood by a "good" second-year graduate student.

The first edition contained some misleading typos. They have been removed, but we cannot guarantee that the present version is totally free of misprints.

Among substantial changes, we have introduced a Chapter 4 on gaussian and Poisson experiments. The gaussian ones are used everywhere because of mathematical tractability. Poisson experiments are less tractable and less studied, but they are probably more important. They will loom large in the new century.

The proof of asymptotic sufficiency of the "centerings" of Chapter 6 has been reworked and simplified. So has Chapter 7, Section 4. Section 5 of the same chapter has been augmented by a derivation of the Cramér-Rao lower bound imitated from Barankin [1949].

We thought of adding material on the von Mises differentiable functions and similar entities; we did not. The reader can refer to the recent book by van der Vaart [1998]. Similarly, we did not cover the use of "empirical processes" even though, following Dudley's paper of 1978, they have been found most valuable. A treatment of that material is expected to appear in a book

by Sara van de Geer. Also, we did not include material due to David Donoho, Iain Johnstone, and their school. We found ourselves unprepared to write a distillate of the material. We did touch briefly on "nonparametrics," but not on "semiparametrics." This is because we feel that the semiparametric situation has not yet been properly structured.

We hope that the reader will find this book interesting and challenging, in spite of its shortcomings.

The material was typed in LaTeX form by the authors themselves, borrowing liberally from the 1990 script by Chris Bush. It was reviewed anonymously by distinguished colleagues. We thank them for their kind encouragement. Very special thanks are due to Professor David Pollard who took time out of a busy schedule to give us a long list of suggestions. We did not follow them all, but we at least made attempts.

We wish also to thank the staff of Springer-Verlag for their help, in particular editor John Kimmel, who tried to make us work with all deliberate speed. Thanks are due to Paul Smith, Te-Ching Chen and Ju-Yi-Yen, who helped with the last-minute editorial corrections.

Lucien Le Cam
Berkeley, California
Grace Lo Yang
College Park, Maryland

February 2000

Preface to the First Edition

In the summer of 1968 one of the present authors (LLC) had the pleasure of giving a sequence of lectures at the University of Montreal. Lecture notes were collected and written out by Drs. Catherine Doléans, Jean Haezendonck and Roch Roy. They were published in French by the Presses of the University of Montreal as part of their series of *Séminaires de Mathématiques Supérieures*. Twenty years later it was decided that a Chinese translation could be useful, but upon prodding by Professor Shanti Gupta at Purdue we concluded that the notes should be updated and rewritten in English and in Chinese. The present volume is the result of that effort.

We have preserved the general outline of the lecture notes, but we have deleted obsolete material and sketched some of the results acquired during the past twenty years. This means that while the original notes concentrated on the LAN situation we have included here some results of Jeganathan and others on the LAMN case. Also included are versions of the Hájek-Le Cam asymptotic minimax and convolution theorems with some of their implications. We have not attempted to give complete coverage of the subject and have often stated theorems without indicating their proofs.

What we have attempted to do is to present a few concepts and tools in an elementary manner refering the reader to the general literature for further information. We hope that this will provide the reader with a way of thinking about asymptotic

problems in statistics that is somewhat more coherent than the traditional reliance upon maximum likelihood.

We wish to extend our thanks to the Presses of the University of Montreal for the permission to reuse some copyrighted material and to Springer-Verlag for the production of the present volume.

We also extend all our thanks to Professor Kai-Tai Fang whose efforts with Science Press are very much appreciated.

The English version of the manuscript was typed at Berkeley by Ms. Chris Bush whose patience and skill never cease to amaze us.

As Chris can attest, producing the typescript was no simple task. We were fortunate to have the help of Ruediger Gebauer and Susan Gordon at Springer-Verlag. We are very grateful for the assistance of Mr. Jian-Lun Xu in the preparation of the Chinese version.

Lucien Le Cam
Berkeley, California
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October 1989

Contents

1

Introduction

In this volume we describe a few concepts and tools that we have found useful in thinking about asymptotic problems in statistics. They revolve largely around the idea of approximating a family of measures, say, $\mathcal{E} = \{P_\theta; \theta \in \Theta\}$ by other families, say, $\mathcal{F} = \{Q_\theta; \theta \in \Theta\}$ that may be better known or more tractable.

For example, contemplate a situation where the statistician observes a large number of independent, identically distributed variables $X_1, X_2, \ldots, X_n$ that have a common Cauchy distribution with density

$$f(x, \theta) = \frac{1}{\pi} \frac{1}{1 + (x - \theta)^2}$$

on the line.

Let $P_{\theta,n}$ be the joint distribution of $X_1, \ldots, X_n$. Let Z_n be another variable that has a gaussian distribution $G_{\theta,n}$ with expectation θ and variance $2/n$ on the real line.

The theory expounded in later chapters says that, for n large, the two families $\mathcal{E}_n = \{P_{\theta,n}; \theta \in \Re\}$ and $\mathcal{F}_n = \{G_{\theta,n}; \theta \in \Re\}$ are, for most statistical purposes, very close to each other.

For another example, suppose that $Y_1, Y_2, \ldots Y_n$ are independent with a common density $[1 - |x - \theta|]^+$ on the line. Let $Q_{\theta,n}$ be their joint distribution and let $H_{\theta,n}$ be gaussian with mean θ and variance $1/(n \log n)$. Then, for n large, $\{Q_{\theta,n}; \theta \in \Re\}$ and $\{H_{\theta,n}; \theta \in \Re\}$ are very close to each other.

Chapter 2 introduces distances that are intended to make precise the above "close to each other." The ideas behind the possible introduction of such distances go back to Wald [1943]. They are also related to the "comparison of experiments" described by Blackwell [1951] and others. Here, following Blackwell, we shall use the name "experiment" for a family $\mathcal{E} = \{P_\theta; \theta \in \Theta\}$ of probability measures P_θ carried by a σ-field $\mathcal{A}$ of subsets of a