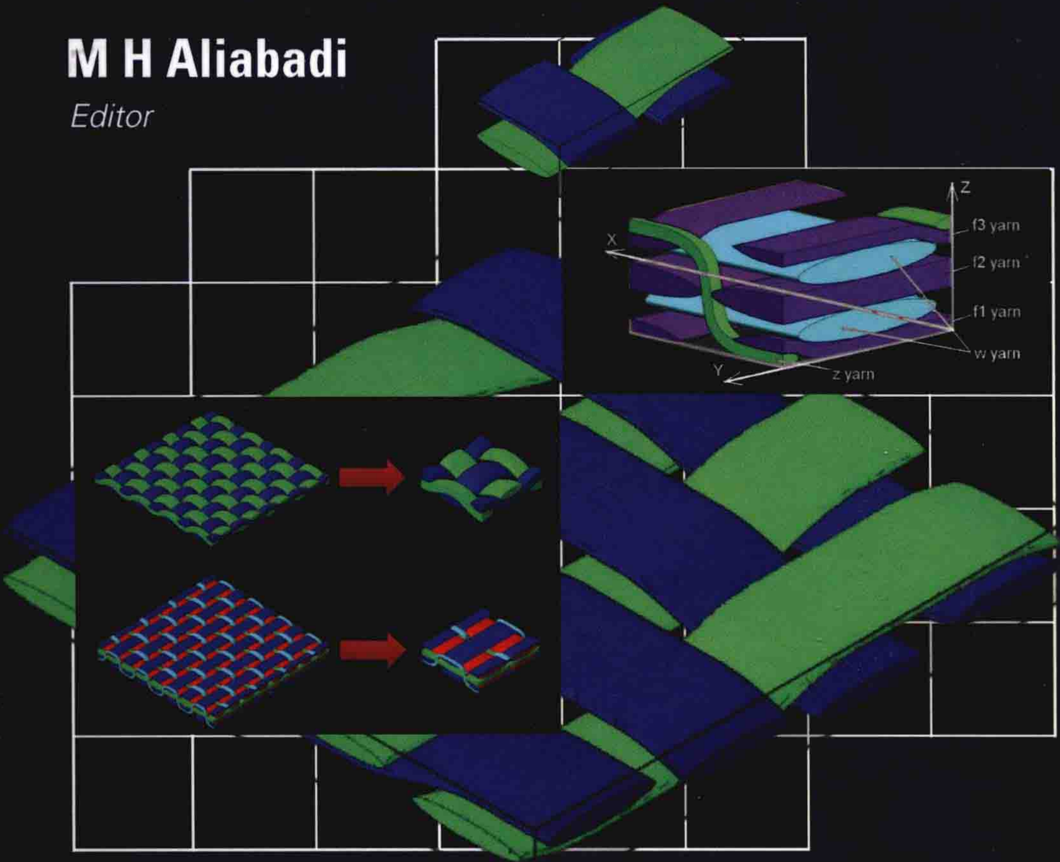
Computational and Experimental Methods in Structures – Vol. 6

Woven Composites

M H Aliabadi

Editor



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Woven Composites

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Chapter 1

MICROMECHANICAL MODELLING OF TEXTILE COMPOSITES

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1.1 Introduction

Composite materials make use of the different properties of their constituents, microstructure and interaction between constituents in order to improve their mechanical behaviour at higher scales.

Fibre-reinforced composites, and in particular, unidirectional (UD) composites, are widely utilised in different aerospace and automotive applications to reduce the overall weight of the components due to the high strength-to-weight ratio that they exhibit.

In recent years, textile composites such as woven, braided, knitted and stitched fabrics have increasingly been used as structural materials in industrial applications because they are efficient at reinforcing more directions within a single layer and their ability to conform to surfaces with complex curvatures. Furthermore, textile composites provide improved impact resistance, exceptional thermal, fatigue and corrosion resistance and better-balanced properties. Moreover, textile composites are easier and cheaper to handle and fabricate when compared with UD composites. The textile weave processes used to form 3D woven fabric composites (WFCs) are able to produce large volumes at an ever faster rate [1], allowing relatively easy and cost-efficient manufacturing.

However, the complex architecture of these composites leads to difficulties in predicting the mechanical response necessary for product design. In particular, their complex structure, undulation and interfaces, and their hard to understand behaviour under different loading conditions, ensures that evaluating their constitutive properties for structural design remains challenging.

Woven fabric composites are a network of fibre tows which are later impregnated to create plies, before being stacked in predetermined orientations and finally cured to form the composite material. This process operates within the tow interlacing level of composite design [2]. The fibre yarns are identified as warp and weft tows; warp tows can be considered as lengthwise yarns (0°), while the weft tows (or fill tows) (90°) are inserted over and under warp yarns to produce the weave pattern. It is worth noting that these are not necessarily perpendicular to each other; the braid angle can be varied to alter the properties.

A unique property of WFCs is their ability to be moulded to fit complex curvatures while maintaining their desirable properties. The variation in weave type controls the mechanical interlocking of the fibres; this is the feature that defines the "drape" of a composite (the ability of the fabric to conform to the shape of the mould [3]). Similarly, both surface smoothness and stability are also characteristics affected predominantly by weave type. In Table 1.1 some of the most commonly used weaves with a comparison of a number of characteristics are listed.

Design remains difficult for WFCs due to the complex undulation in the geometry involved. The ability to accurately evaluate these properties numerically through computational methods such as finite element analysis

Table 1.1. Comparison of weave type and properties.

Weave type	Stability	Drape	Porosity	Smoothness	Symmetry	Low crimp
Plain	++++	++	+++	++	+++++	++
Twill	+++	++++	++++	+++	+++	+++
Satin	++	+++++	+++++	+++++	+	+++++
Basket	++	+++	++	++	+++	++
Leno	+++++	+	+	+	+	++

+++++ = Excellent, ++++ = Good, +++ = Acceptable, ++ = Poor, + = Very Poor

(FEA) will greatly aid the design process. It will help shape the future of composite design, as well as provide a non-destructive means of testing.

The predictions of mechanical properties for textile composites have been heavily researched with most studies focusing on plain woven composites. Early models for the analysis of woven lamina can be traced back to the 1970s, when Halpin *et al.* [5] investigated the stiffness of 2D and 3D composites. Later, the theory was improved by Chou and Ishikawa [6, 7], starting from modified classical laminate theories, developing the mosaic model, fibre undulation model and bridging model. These models were further improved by Naik and Ganesh [8], Shembekar and Naik [9], and Naik and Shembekar [10] by considering different parameters such as yarn thickness, undulation and the gap between adjacent layers. The unit cell was divided into slices and the slices were arranged in different combinations — series-parallel and parallel-series, to attain lower bound and upper bound properties. Later, Jiang *et al.* [11] applied the method of cells to determine the effective properties of plain woven composites, and Tanov and Tabiei [12] presented an efficient model with a simpler geometrical description. Tabiei and Ivanov [13] developed a model that allowed modelling of progressive failure. Other methods based on classical energy principles were applied by Kregers and Malbardis [14] for a random 3D reinforced composite, and Pastore and Gowayed [15] used a stiffness averaging technique.

Finite elements analysis has also been employed to evaluate the mechanical properties on the micro-level or constituent level, Zhang and Harding [16] presented the first simplified 2D numerical model for plain woven composites, later Paumelle *et al.* [17] investigated the 3D behaviour. Woo and Whitcomb [18] also proposed a 2D model, and Chapman and Whitcomb [19] extended an improved model to a 3D model. Zeman and Šejnoha [20] included imperfection in their analysis. A meshless implementation for the prediction of the mechanical behaviour of woven fabric composites was proposed by Wen and Aliabadi [21].

Three-dimensional FEA was used by Whitcomb and Srirengan to predict the failure of plain weave composites [22]. Barbero *et al.* developed a numerical model capable of capturing damage using continuum damage mechanics (CDM) [23], Bacarreza *et al.* [24] used a semi-analytical method to predict the failure behaviour of plain woven composites at different scales, and Wen and Aliabadi [24] studied progressive failure of plain woven composites using meshfree simulations.

Twill woven composites have also been studied widely. Chaphalkar and Kelkar [25] proposed an analytical model based on the classical laminate theory to predict the mechanical behaviour of twill composites. Numerical models were proposed by Kwon and Roach [26], and Goyal *et al.* [2]. Nicoletto and Riva [27] analysed failure in twill weave laminates as well as Ng *et al.* [28].

For 3D woven composites in which tows are interlaced in multiple directions, the methods for predicting mechanical properties are more complicated than for 2D composites as 3D woven composites have a more complex architecture. In addition, it is reported that the lack of mechanical characterisation data and difficulties in testing methods have been significant issues for 3D woven composites. The level of complexity and lack of mechanical characterisation have led to slow progress of research in the field of predicting mechanical behaviour for 3D woven composites. However, some prediction approaches for elastic properties using the finite element method (FEM) have been introduced in recent years. Bogdanovich [29] proposed an FE approach using a 3D mosaic model, Stig and Hallström [30] performed FE analysis for fully interlaced 3D fabrics. Non-crimp fabric laminates were studied by Heß *et al.* [31] and knitted fabric composites by Huysmans *et al.* [32]. Meshless methods are proving to be very efficient in modelling these complex architectures and behaviours [33, 34]. Researchers are also working to combine extended finite element methods (XFEM) with binary modelling to achieve results at a faster rate [35].

Much research has been carried out to determine the properties of composites, and various methods and models have been developed. Broadly, these models can be classified into: analytical, semi-analytical and numerical.

Analytical methods include the rules of mixtures based on the Voigt [36] and Reuss [37] upper and lower bounds respectively, and their improvements such as those developed by Hashin and Shtrikman [38] which applies variational principles (variational bounding method); the improvements also contain asymptotic homogenisation [39, 40] and mean field approaches [41–43], also called effective medium approximation.

Semi-analytical methods are those where the global constitutive equations are evaluated from the local scale using explicit relations that link microscopic and macroscopic behaviour. Semi-analytical methods comprise the method of cells [44–46], transformation of fields analysis (TFA) [47–49] and its extension, non-uniform TFA [50].

Numerical approaches capable of dealing with the increasingly complex architectures of composites include unit cell methods [51–54] and methods based on computational homogenisation [55–66].

In this chapter, semi-analytical and numerical methods for the micromechanical analysis of textile composites are presented in Section 1.2. Computational methods presented include the FEM and newly developed meshfree method. The analysis of a unit cell for 2D plain woven with two different geometries is discussed in Section 1.3.1. The FEA of twill, satin and 3D woven composites are detailed in Sections 1.3.2, 1.3.3 and 1.3.4 respectively. A newly more realistic mathematical representation of 2D and 3D woven composites is also reported. Finally, damage at the unit cell scale of plain woven composites using meshfree methods is examined in Section 1.4.1 and multi-scale progressive failure analysis of plain woven composites using semi-analytical homogenisation with the FEM is explained in Section 1.4.2.

In this chapter repetitive unit cell (RUC) is defined as the smallest part of a woven fabric composite that includes all the features of the fabric which can be constructed by tessellating the RUC, i.e. the RUC repeats periodically. The representative volume element (RVE) is an approximation of the RUC using symmetries and antisymmetries.

1.2 Calculation of Effective Elastic Properties

1.2.1 *Homogenisation using semi-analytical methods*

Paley and Aboudi [45] introduced the generalised method of cells for the micromechanical analysis of fibrous composites. It is capable of modelling multi-phase composites with different types of phase arrangements and architectures. Tabiei and Jiang [67] presented a model for plain woven composites which divides the unit cell into subcells and then each subcell is homogenised through the thickness via iso-stress and iso-strain compatibility conditions, the composite is then represented as an array of 2D subcells with homogenised properties. The homogenisation of the 2D subcell array is equivalent to the generalised method of cells [68].

Homogenisation of the representative volume element's (RVE) mechanical properties can be performed using semi-analytical homogenisation based on the four-cell model proposed by Tanov and Tabiei [12], which showed good agreement with experimental results for obtaining effective elastic moduli. This micromechanical model is based on the method of cells, which consists of dividing the RVE into different blocks or cells, each of which can be further divided into subcells. These subcells are an idealisation of actual subvolumnes of the RVE, i.e., part of a UD yarn bundle or a pure matrix region.

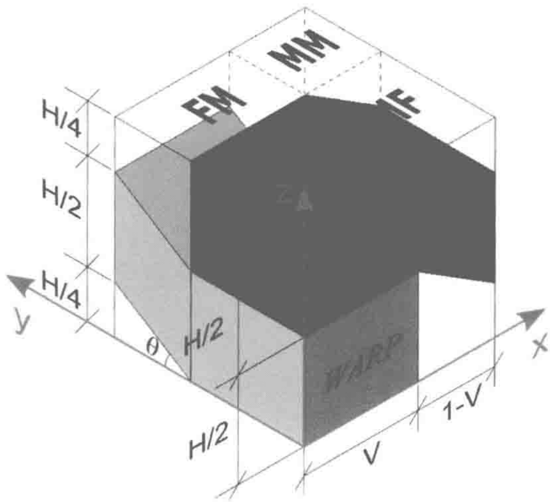

Fig. 1.1. Geometry of the RVE of the four-cell model for plain woven composites.

In Figure 1.1 the geometry and main features of the RVE or four-cell model, which can be used for homogenising elastic and damaged properties of plain woven composites, is presented. The four subcells or blocks can be distinguished: (1) Subcell "*FF*"; (2) Subcell "*FM*"; (3) Subcell "*MF*" and (4) Subcell "*MM*"; where "*M*" stands for matrix and "*F*" for fibre.

The following assumptions are used during homogenisation:

(i) The matrix is isotropic
(ii) The yarn bundles are transversely isotropic with the principal axis along the yarn axis
(iii) The contact between the constituents is perfect.

In order to homogenise the subcells, through the thickness, the material properties for yarn bundles, pure resin and the strains on the subcells should be known. The homogenised stiffness matrices for the subcells in their local co-ordinate systems are calculated based on the parallel-series compatibility conditions. The in-plane relations for these materials are in a parallel (Voigt) arrangement and the out-of-plane relations are in a series (Reuss) arrangement.

$$\Delta\varepsilon_x^{ab} = \Delta\varepsilon_x^a = \Delta\varepsilon_x^b \qquad \Delta\sigma_x^{ab} = \frac{1}{2}(\Delta\sigma_x^a + \Delta\sigma_x^b)$$

$$\Delta\varepsilon_y^{ab} = \Delta\varepsilon_y^a = \Delta\varepsilon_y^b \qquad \Delta\sigma_y^{ab} = \frac{1}{2}(\Delta\sigma_y^a + \Delta\sigma_y^b)$$

$$
\begin{aligned}
\Delta\varepsilon_z^{ab} &= \frac{1}{2}(\Delta\varepsilon_z^a + \Delta\varepsilon_z^b) & \Delta\sigma_z^{ab} &= \Delta\sigma_z^a = \Delta\sigma_z^b \\
\Delta\varepsilon_{xy}^{ab} &= \Delta\varepsilon_{xy}^a = \Delta\varepsilon_{xy}^b & \Delta\sigma_{xy}^{ab} &= \frac{1}{2}(\Delta\sigma_{xy}^a + \Delta\sigma_{xy}^b) \\
\Delta\varepsilon_{yz}^{ab} &= \frac{1}{2}(\Delta\varepsilon_{yz}^a + \Delta\varepsilon_{yz}^b) & \Delta\sigma_{yz}^{ab} &= \Delta\sigma_{yz}^a = \Delta\sigma_{yz}^b \\
\Delta\varepsilon_{zx}^{ab} &= \frac{1}{2}(\Delta\varepsilon_{zx}^a + \Delta\varepsilon_{zx}^b) & \Delta\sigma_{zx}^{ab} &= \Delta\sigma_{zx}^a = \Delta\sigma_{zx}^b
\end{aligned}
\tag{1.1}
$$

a, b can be a fibre or matrix, i.e., "*FF*", "*FM*" and "*MF*".

The above assumptions are independent of stress-strain relations and have been widely used in the estimation of mechanical properties of composite materials. They lead to the homogenised constitutive properties of each subcell in its local co-ordinate system.

Once the equivalent strains in the subcells are computed, the components of strains and stresses in the constituents, and the stresses on the subcells can be determined. The stresses and strains in each subcell and the average stress in the RVE can be calculated once the average incremental strains in the RVE are known.

The homogenisation of the RVE is then performed by means of the following micromechanical conditions based on the homogenisation of the 2D subcell array.

Continuity conditions of strains:

$$
\begin{aligned}
&\textstyle\sum_a V_a \Delta\varepsilon_x^{ab} = \Delta\bar{\varepsilon}_x && a = F, M; b = F, M \\
&\textstyle\sum_b V_b \Delta\varepsilon_y^{ab} = \Delta\bar{\varepsilon}_y && a = F, M; b = F, M \\
&\Delta\varepsilon_z^{ab} = \Delta\bar{\varepsilon}_z && a = F, M; b = F, M \\
&\Delta\varepsilon_{xy}^{ab} = \Delta\bar{\varepsilon}_{xy} && a = F, M; b = F, M \\
&\textstyle\sum_a \sum_b V_a V_b \Delta\varepsilon_{yz}^{ab} = \Delta\bar{\varepsilon}_{yz} && a = F, M; b = F, M \\
&\textstyle\sum_b V_b \Delta\varepsilon_{yz}^{Fb} = \sum_b V_b \Delta\varepsilon_{yz}^{Mb} && b = F, M \\
&\textstyle\sum_a \sum_b V_a V_b \Delta\varepsilon_{xz}^{ab} = \Delta\bar{\varepsilon}_{xz} && a = F, M; b = F, M \\
&\textstyle\sum_a V_a \Delta\varepsilon_{yz}^{aF} = \sum_a V_a \Delta\varepsilon_{yz}^{aM} && a = F, M
\end{aligned}
\tag{1.2}
$$

Continuity conditions of stresses:

$$
\Delta\sigma_x^{ab} = \Delta\sigma_x^{\widehat{a}b} \qquad a = F, M; b = F, M; \widehat{a} \neq a
$$

$$\begin{aligned}
&\textstyle\sum_b V_b \Delta\sigma_x^{ab} = \Delta\bar{\sigma}_x && a = F; b = F, M \\
&\Delta\sigma_y^{ab} = \Delta\sigma_y^{a\widehat{b}} && a = F, M; b = F, M; \widehat{b} \neq b \\
&\textstyle\sum_a V_a \Delta\sigma_y^{ab} = \Delta\bar{\sigma}_y && a = F, M; b = F \\
&\textstyle\sum_a \sum_b V_a V_b \Delta\sigma_z^{ab} = \Delta\bar{\sigma}_z && a = F, M; b = F, M \\
&\textstyle\sum_a \sum_b V_a V_b \Delta\sigma_{xy}^{ab} = \Delta\bar{\sigma}_{xy} && a = F, M; b = F, M \\
&\Delta\sigma_{yz}^{ab} = \Delta\sigma_{yz}^{a\widehat{b}} && a = F, M; b = F, M; \widehat{b} \neq b \\
&\textstyle\sum_a V_a \Delta\sigma_{yz}^{ab} = \Delta\bar{\sigma}_{yz} && a = F, M; b = F \\
&\Delta\sigma_{xz}^{ab} = \Delta\sigma_{xz}^{\widehat{a}b} && a = F, M; b = F, M; \widehat{a} \neq a \\
&\textstyle\sum_b V_b \Delta\sigma_{xz}^{ab} = \Delta\bar{\sigma}_{xz} && a = M; b = F, M
\end{aligned} \tag{1.3}$$

where V_a, V_b are the volumes of the constituents "F" or "M".

Using the original continuity conditions from [12] leads to premature failure of the RVE in out-of-plain shear failure, even when the specimen is loaded purely in the normal direction. Equations (1.2) and (1.3) already include the improvements in the continuity conditions of the out-of-plane shear stresses and strains.

1.2.2 *Homogenisation using FE Analysis*

The procedure of determining the effective elastic properties using FE analysis can be performed using computational homogenisation and following the four-step homogenisation scheme proposed by Suquet [55]:

(i) Identification of the RVE where the mechanical behaviour of each individual constituent is known
(ii) Application of correct microscopic boundary conditions on the RVE from the macroscopic variables (macro–micro transition)
(iii) Calculation of the macroscopic response from the deformed microstructural behaviour of the RVE (micro–macro transition)
(iv) Finding the implicit relationship between macroscopic input and output variables.

1.2.2.1 *Geometric modelling*

The accuracy of the predicted properties and damage characterisation of the numerical model is strongly dependent on the quality of the geometrical description. Often the models rely on accurate information of the tow geometric parameters such as cross-sectional shape, path and position within the textile structure, undulation function, etc. It has to be noted that in practical cases, the modelled tow or preform may not replicate the actual ones, due to various manufacturing parameters encountered, starting from the fibre manufacturing to moulding and curing. However, past studies have demonstrated that the basic assumptions are quite sufficient to characterise the properties and failure modes to acceptable levels [69].

The actual choice of the repetitive unit cell (RUC) is an important task. The RUC should be large enough to represent the microstructure without introducing non-existing properties, and at the same time, it should be small enough to allow efficient computational modelling, it can be defined as the smallest microstructural volume that represents the overall macroscopic properties of the material accurately enough. Figure 1.2 shows the RUC according to the previous definition for plain woven and 3D orthogonal composites.

Periodicity is not the only aspect in woven composites, symmetries and antisymmetries can also be exploited to reduce the size of the RUC [70, 71], this reduced RUC or representative volume element (RVE) is widely used

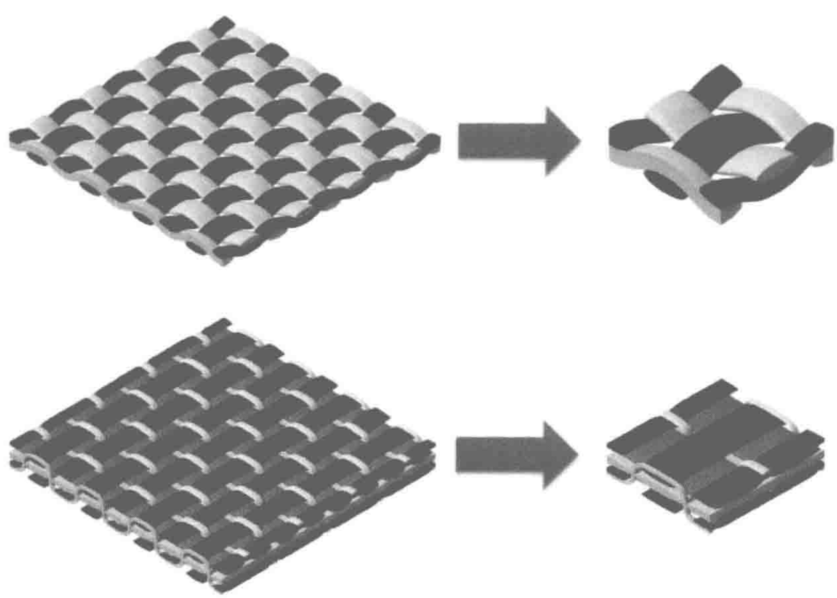

Fig. 1.2. Repetitive unit cell.

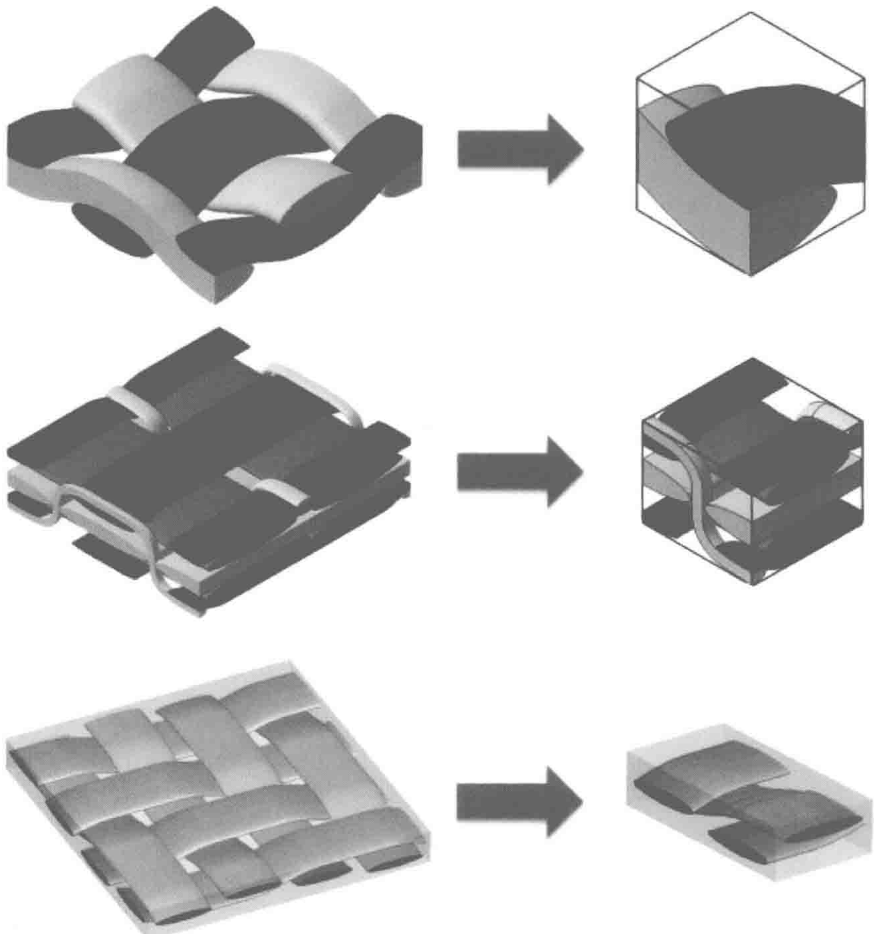

Fig. 1.3. Representative volume element.

in the analysis of advanced composites like the ones shown in Figure 1.3 for different types of composites.

Complex geometries are difficult to model using standard pre-processing software for FEM. In order to create these architectures CAD systems are recommended. Several CAD codes provide associative interfaces with commercial FEM codes. This feature is very useful while carrying out design iterations.

Unfortunately, degradation of the geometry quality can be observed during translation. This shortcoming is illustrated in Figure 1.4 where edges of the tow and matrix overlap after the CAD assembly is imported to the