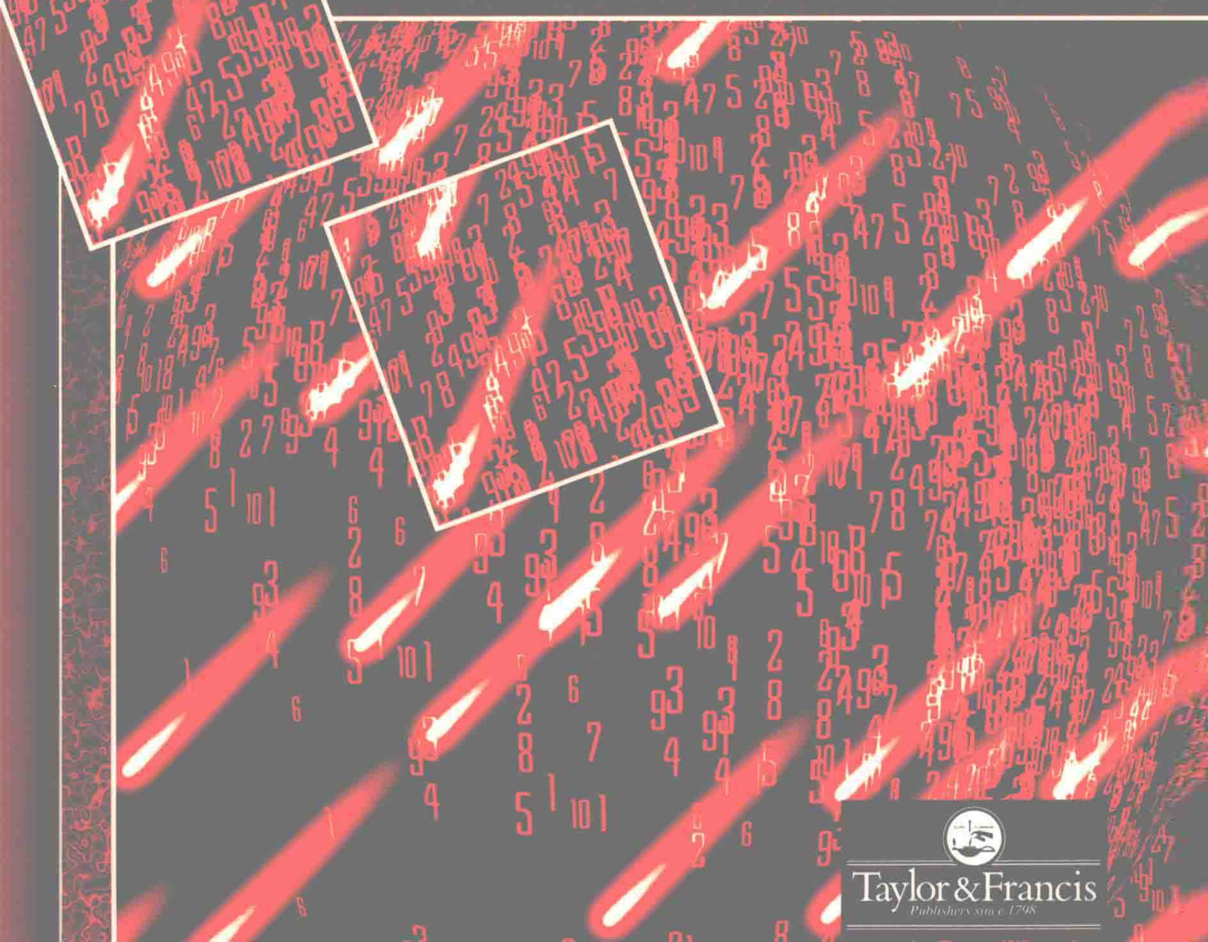
Modules in Life Sciences

INTRODUCTORY MATHEMATICS FOR THE LIFE SCIENCES

David Phoenix



Taylor & Francis
Publishers since 1798

Introductory Mathematics for the Life Sciences

David Phoenix

Department of Applied Biology
University of Central Lancashire
Preston, UK

Taylor & Francis
Publishers since 1798

UK Taylor & Francis Ltd., 1 Gunpowder Square, London EC4A 3DE.
USA Taylor & Francis Inc., 1900 Frost Road, Suite 101, Bristol, PA 19007.

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library
ISBN 0-7484-0428-7

Library of Congress Cataloging Publication Data are available

Cover design by Jim Wilkie

Typeset in Melior by Keyword Publishing Services Ltd

Printed in Great Britain by T.J. International, Padstow, UK

Introductory Mathematics for the Life Sciences

General Preface to the Series

The curriculum for higher education now presents most degree programmes as a collection of discrete packages or modules. The modules stand alone but, as a set, comprise a general programme of study. Usually around half of the modules taken by the undergraduate are compulsory and count as a core curriculum for the final degree. The arrangement has the advantage of flexibility. The range of options over and above the core curriculum allows the student to choose the best programme for his or her future.

Usually, the subject of the core curriculum, for example biochemistry, has a general textbook that covers the material at length. Smaller specialist volumes deal in depth with particular topics, for example photosynthesis or muscle contraction. The optional subjects in a modular system, however, are too many for the student to buy the general textbook for each and the small in-depth titles generally do not cover sufficient material. The new series *Modules in Life Sciences* provides a selection of texts which can be used at the undergraduate level for subjects optional to the main programme of study. Each volume aims to cover the material at a depth suitable to the year of undergraduate study with an amount appropriate to a module, usually around one-quarter of the undergraduate year. The life sciences was chosen as the general subject area since it is here, more than most, that individual topics proliferate. For example, a student of biochemistry may take optional modules in physiology, microbiology, medical pathology and even mathematics.

Suggestions for new modules and comments on the present volume will always be welcomed and should be addressed to the series editor.

John Wrigglesworth, Series Editor
King's College, London

Preface

Students are entering A-level and undergraduate life science courses with only GCSE mathematics. Many students do not possess a thorough understanding of the basic mathematical principles which are required in these courses and those that do understand the mathematics often have difficulty applying the principles to biological problems. These deficiencies are difficult to correct and can involve the need for intensive tutorial-based courses, but with increasing student numbers and decreasing staff time the support for material which lies 'outside' the standard life science curriculum is limited. This leads to many students struggling with basic concepts, such as concentration, and if courses include areas with a strong mathematical orientation such as kinetics, energetics or even pH calculations students tend to gain little, since their time is spent struggling with the mathematics; thus they often miss the biological importance of the material.

This book has been written after discussion with undergraduates to find out the areas with which they want help. It is intended to introduce essential mathematical ideas from first principles but without the use of mathematical proofs. In the body of each chapter are worked examples so that readers can apply the mathematics and develop their confidence. At the end of each chapter are a number of questions taken from biology and these allow students to try to apply the mathematics they have learnt. The emphasis is on essential mathematics, i.e. that which students will need at some time in most courses and some of which will be applied on a daily basis. Once the mathematics has been learnt, students need to apply it. It is useful to perform the following steps when facing a numerical problem:

(a) look at the problem and write down all the information that you have;
(b) write down what it is you want to know;
(c) work out what information is actually required and what is superfluous;

(d) establish the link between what is wanted and what is known;

(c) apply the mathematics and find the answer!

David Phoenix
Department of Applied Biology
University of Central Lancashire

Contents

1 Numbers

1.1 Introduction

Scientists must be able to take quantitative measurements and look for correlations within their experimental data. A scientist should therefore be able to manipulate numbers and have an appreciation of their relevance. The objectives of this chapter are:

(a) to introduce real numbers;

(b) to develop rules for the manipulation of numbers.

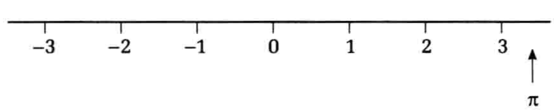
1.2 Real numbers

Real numbers may be represented by their position on a **number line** (Figure 1.1). All the numbers which lie on this line are termed **real numbers** and the set is represented by the symbol $\mathbb{R}$. Whole numbers (**integers**) are represented by the symbol $\mathbb{Z}$ and can be sub-grouped into positive ($\mathbb{Z}+$) or negative ($\mathbb{Z}-$) integers.

Negative numbers are written to the left of zero. The further a number is to the right, the bigger it is, so for exam-

On the number line, the further the number is to the right the bigger it is

Figure 1.1



$\mathbb{R}$ represents the group of all numerical values which can be represented on the number line (i.e. the real numbers)
$\mathbb{Z}$ represents the set of intergers $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$\mathbb{Z}+$ represents the set of positive integers, sometimes called natural numbers ($\mathbb{N}$) $\{1, 2, 3, 4, \ldots\}$
$\mathbb{Z}-$ represents the negative integers $\{-1, -2, -3, -4, \ldots\}$

ple -2 is greater than -5. Addition therefore indicates that you move to the right, since the number is getting bigger; subtraction indicates that you move to the left.

It is obviously important that you are able to manipulate both positive *and* negative numbers. It is useful to remember that if you are adding a negative number to a positive number you can treat this as a subtraction, as shown in Example 1.1.

Example 1.1

$$(-2) + 3$$
$$= 3 - 2 = 1$$

It may help to remember the number line. In Example 1.1 you start at position minus two (-2) and plus three $(+3)$ tells you to move to the right three places, which takes you to position one on the number line. In Example 1.2 you start at position minus four and move one place to the left, thus giving the answer minus five.

Example 1.2

If you subtract a negative number it becomes positive

$$-4 - 1 = -5$$

When dealing with negative numbers, the only rule that must be remembered is that if you subtract a negative number it becomes positive. This can be seen in Example 1.3.

Example 1.3

$1 - (-3)$
$= 1 + 3 = 4$ *Subtraction of a negative gives a positive*

Multiplying or dividing numbers of the same sign gives a positive answer

A similar rule applies when multiplying or dividing; if both numbers have the same sign the answer is positive, if their signs are different the answer is negative. This is illustrated in Box 1.1 and Example 1.4(a)–(c).

Example 1.4

(a) $3 \times 2 = 6$ *Both signs are the same, therefore the answer is positive*

(b) $3 \times (-2) = -6$ *The signs are different, therefore the answer is negative*

(c) $(-9) \div (-3) = 3$ *Both signs are the same, therefore the answer is positive*

Box 1.1 **Sign rules for multiplication and division.**

(positive) × (positive)	= positive	(positive)	÷ (positive)	= positive
(positive) × (negative)	= negative	(positive)	÷ (negative)	= negative
(negative) × (positive)	= negative	(negative)	÷ (positive)	= negative
(negative) × (negative)	= positive	(negative)	÷ (negative)	= positive

If you have more than two terms in the calculation, then to apply the sign rules in Box 1.1 you need to break the calculation down into parts as shown in Example 1.5.

Example 1.5

$2 \times (-3) \times (-1)$ $2 \times -3 = -6$: *The different signs imply that the answer is negative*

$(-6) \times (-1)$ $-6 \times -1 = 6$: *The same signs imply that the the answer is positive*

$= 6$

Worked examples 1.1

Evaluate:
(i) 2×-5 (ii) -6×-3 (iii) $3 - 5$ (iv) $-2 - 6$
(v) $-3 - (-4)$ (vi) $-6 \div -6$ (vii) $6 \div -12$.

1.3 Modulus

On some occasions it may be the size of the value that is important, rather than its sign. For example, suppose you are measuring the height of a seedling in centimetres. The exact height is 4.7 cm and you take two measurements which are recorded in Table 1.1 along with the error.

Table 1.1

Reading (cm)	Error (cm)
4.5	−0.2
4.7	0.2

With the first reading you have under-estimated the height by 0.2 cm but the second reading is too large by 0.2 cm. The error in both cases is of the same size or **magnitude**; it is only the direction that is different, i.e. one is an under-estimate and the other an over-estimate. In this case it may be worthwhile considering the **absolute values**. The absolute value takes into account the magnitude or size of the change but

Modulus measures the absolute value without the sign

it does not take into account the direction of the change. It is denoted by two straight lines (i.e. $|-2| = 2$) and is usually called the **modulus**. In the example given above you can say that the **absolute error** in both measurements is 0.2 cm.

Worked examples 1.2

Evaluate:
(i) $-2 - |-2|$ (ii) $|3 - 5|$ (iii) $1 - 4 - |3|$
(iv) $3 + |2 - 3|$

1.4 Functions with multiple operations

You often have to deal with functions which contain more than one mathematical operation and it is important to know in what order to perform these operations. In general, if an expression contains brackets you always evaluate whatever is in the brackets first, then you perform multiplication and division and finally addition and subtraction (Box 1.2).

Box 1.2 **Priority of operations.**

1 Brackets
2 Multiplication and division
3 Addition and subtraction

If there is more than one set of brackets you start on the inside and work outwards.

Example 1.6

$((3 - 2) \times 4 + 4) \div 2$ *Innermost brackets first, so* $3 - 2 = 1$
$= (1 \times 4 + 4) \div 2$ *Brackets; multiplication, so* $1 \times 4 = 4$
$= (4 + 4) \div 2$ *Brackets; addition* $4 + 4 = 8$
$= 8 \div 2$
$= 4$

It is essential that these rules are applied since failure to do so will greatly influence the outcome of the calculation, as can be seen in the following examples.

Example 1.7

$3 + 4 \times 5 = 3 + 20$ *Compared with* $(3 + 4) \times 5 = 7 \times 5$
$= 23$ $= 35$

Example 1.8

$$6 - 4 \div 2 = 6 - 2 \qquad \textit{Compared with} \qquad (6-4) \div 2 = 2 \div 2$$
$$= 4 \qquad\qquad\qquad\qquad\qquad = 1$$

Note that in Example 1.8 the expressions can be rewritten to emphasise their difference:

$$6 - 4 \div 2 = 6 - \frac{4}{2} \quad \textit{and} \quad (6-4) \div 2 = \frac{6-4}{2}$$

In general, although the list of priorities tells you which operation to perform first, it is always best to use brackets to clarify what is required.

Example 1.9

$$6 - 4 \div 2 = 6 - (4 \div 2) = 6 - \frac{4}{2}$$

In Example 1.9 the brackets are not needed but their presence can help prevent confusion and this decreases the chance of error.

Worked examples 1.3

Evaluate:
(i) $3 - 9 \div 3$ (ii) $4 \times (2 - 3)$ (iii) $((4 + 6) \div 5 + 3) \times 3$
(iv) $10 \times 5 + 4 \times 5$ (v) $((15 - 5) + 2 \times 2) \div 7$.

1.5 Commutative and associative laws of addition and multiplication

The **commutative law** (Box 1.3) states that:

> *The order in which two numbers are added or multiplied may be interchanged.*

Box 1.3 **Commutative laws.**

$a + b = b + a$
$ab = ba$

If this law holds then the order in which we add or multiply two numbers does not matter since the order can be interchanged. Examples 1.10 and 1.11 show this to be true.