# Chemical Engineering in Medicine and Biology

Edited by
Daniel Hershey
*Department of Chemical Engineering*
*University of Cincinnati*
*Cincinnati, Ohio*

Proceedings of the
Thirty-Third Annual Chemical Engineering Symposium
of the Division of Industrial and Engineering Chemistry
of the American Chemical Society, Held at the
University of Cincinnati, on October 20-21, 1966

PLENUM PRESS · NEW YORK · 1967

Library of Congress Catalog Card Number 67-17376


*A Division of Plenum Publishing Corporation*
*227 West 17 Street, New York, N. Y. 10011*


*Printed in the United States of America*

# Chemical Engineering in Medicine and Biology

# INTRODUCTION

There is much discussion today concerning "Bioengineering" (or "Biomedical Engineering"). It is not exactly clear what these names signify, particularly in chemical engineering. Some have suggested retreading the old war horse "Biochemical Engineering" (or was it "Biomedical Chemical Engineering).

In an effort to demonstrate the on-going activities of chemical engineers in the life science area, we accepted the invitation of the Industrial and Engineering Division of the American Chemical Society to organize the 33rd Annual Chemical Engineering Symposium. We decided to call the symposium, Chemical Engineering in Medicine and Biology, and hence avoided the problem of having to decide which "bio" prefix to use.

Many chemical engineers in the academic and industrial world were contacted. From these contacts and a good deal of publicity arose the Symposium.

The two-day meeting was held at the University of Cincinnati in the Losantiville Room of the Student Union Building on October 20-21, 1966. Twenty-one papers were presented on topics relating chemical engineering to medicine and biology. The papers were representational of the scope of the activities across the country with presenters coming from Washington, California, Massachusetts, New York, South Carolina, Wisconsin, Iowa, Pennsylvania, Michigan, Indiana and Texas. Topics ranged over blood flow properties, diffusion in blood phenomena, mass transfer in the eye, artificial kidney analysis, separation of bacteria by ion exchange, mathematical modeling of drug distribution, carbon dioxide respiration, photosynthetic kinetics, water in frozen tissues, electrophoretic separation of proteins, and outerspace research on life support systems.

There were eighty-six registrants from outside of Cincinnati, with also about ten faculty and industrial persons from the local community. In addition about twenty students from the University of Cincinnati attended. The out-of-town registrants were from California, Michigan, Pennsylvania, Indiana, New York, Virginia, New Jersey, Texas, Massachusetts, Ohio, South Carolina, Canada, Maryland, Arizona, Missouri, Washington, Florida, Delaware, Washington, D.C., Illinois, Wisconsin, Iowa.

Chemical Engineers by virtue of their background, education, and training in non-Newtonian flow, chemistry, chemical reactor kinetics, mass transfer, mathematics and process dynamics are well qualified to examine problems in the engineering-life sciences interface. The papers presented during the Symposium have been expanded considerably by the authors for publication in these Proceedings volumes. The Proceedings represent an up-to-date progress report on each area reported upon. The references, background information, detailed derivations and experimental data have been included so that it should be possible for one relatively new in the field to read the material and acquire a "classroom" acquaintance.

Daniel Hershey

# CONTENTS

Chemical Engineering
in Medicine and Biology

# DIGITAL COMPUTER SIMULATION OF ARTERIAL BLOOD FLOW


## B. T. Fairchild

Central Research Department
Monsanto Company
St. Louis, Missouri

## L. J. Krovetz

Department of Pediatrics
University of Florida
Gainesville, Florida

## and
## C. E. Huckaba

Department of Chemical Engineering
Drexel Institute of Technology
Philadelphia, Pennsylvania


## INTRODUCTION

The advent of high speed electronic computers has relaxed the necessity for restrictive assumptions demanded by classical mathematical methods, and numerical solution techniques now provide a means for investigation of complex mathematical systems. Thus, it is desirable to re-evaluate physiological models in terms not limited by traditional techniques; to search for new areas of application; and to show how these models may be used in clinical and research studies.

The number of parameters of arterial blood flow that can be readily measured in the intact animal or human without surgical intercession is quite limited. One of the most useful and accurate of these is intravascular pressure. Simple and rugged pressure transducers provide powerful tools, and a great deal can be learned from a graph of pressure as a function of time. An equal advance will be made when simultaneous measurements of instantaneous flow can be obtained. Since current flow transducers are not accurate without surgical exposure of the vessel, computation of arterial blood flow from more readily measured parameters such as pressure is clearly desirable.

Study of the propagation of flow and pressure waves in a viscous fluid contained in an elastic tube has a long history. Witzig (1914) derived an approximate solution of the equations of viscous flow, neglecting the non-linear terms, and Karreman (1952) extended this analysis, as did Morgan and Kiely (1954).

The problem of pulsating flow, i.e., of a steady mean flow on which there is superimposed a periodic variation, was investigated by Schultz-Grunow (1940). The experimental arrangement consisted of a pipe which was fed with water at a constant head and whose end section was rhythmically increased and decreased in area. Such flows were handled theoretically by Sexl (1930) and by Uchida (1956). These authors solved a rigid tube model to obtain analytical solutions in the form of Bessel functions with complex arguments.

Womersley (1955) was the first, however, to apply these equations to arterial blood flow. Womersley, an applied mathematician, and McDonald, a gifted physiologist, collaborated to produce a series of excellent reports detailing the fit of experimental and calculated data (See esp. McDonald, 1960). Womersley improved his earlier model during a period of time spent at the Wright Air Development Center. Unfortunately he died in 1958, at the age of 50, shortly after completing a monograph (Womersley, 1957) extending his model to include elastic constraints, consideration of reflections occurring at junctions and discontinuities, and various ranges of viscosity. Solutions by his techniques are extremely laborious and few were attempted. Digital computation was employed apparently only for calculation of a set of tables for several of the recurring arguments in his equations.

Fry and Greenfield (1964) applied the digital computer to the Fourier analysis and solution of these equations. Their paper also furnishes an interesting critique of the method, making Womersley's assumptions more explicit than in the original reports.

Lambert (1956, 1958) developed a mathematical model for one dimensional flow of a non-viscous fluid in an elastic tube. This interesting model was selected by us for detailed analysis and we shall have more to say concerning this later. Lambert's approach was extended by Streeter, Keitzer, and Bohr (1963), who produced a more sophisticated model which considers viscous flow in an elastic tube. They employed the method of characteristics for solution and carried out the computations on an IBM 709 system. They extended their basic model to consider taper, and a continuously distributed outflow for simulation of branching. While the form of their frictional damping term can be determined from experimental flow data, their model presently requires a linear elastic wall relationship.

A simple, and probably currently the most widely employed, model was developed for an analog computer by Fry (1959). This model requires that the velocity in the aorta be considered zero during the latter part of the pulse cycle, a requirement that unfortunately cannot always be met. Consideration of measurement errors involved in this method and comparison with measured data is discussed by Greenfield and Fry (1962).

Although one of the most striking characteristics of the mammalian vascular bed is its large number of branches, all of these models consider only a straight unbranched arterial segment. The hemodynamic effects of branching have been considered earlier (Krovetz, 1965), and while branches produce significant disturbances to flow, there is as yet little quantitative data available.

Noordergraaf (1963) has developed an interesting electrical analog of the entire human systemic circulatory system including representative branches, and this model has been extended by Attinger and Anné (1966) with some interesting preliminary results.

Analog computer simulation of the entire cardiovascular system has also been reported (Warner, 1959; DeFares, et.al., 1963). These models are of interest at present primarily because they allow study of control mechanisms.

The present work is a study of basic mathematical models for description of the flow of blood in a length of unbranched artery, utilizing standard engineering concepts of unsteady momentum transport. Three models are considered: first, viscous flow in a rigid tube; second, nonviscous flow in an elastic tube; and third, a two-dimensional model that includes both viscous damping and elastic wall characteristics. Specific numerical solutions are compared with experimental measurements made in rigid and elastic tube models, and in the femoral artery of a dog.

## THEORETICAL ANALYSES

The development of a completely rigorous mathematical model describing the arterial system is, and may always be, an impossible task. In order that useful models may be obtained, certain simplifying assumptions are mandatory. It is necessary, however, that these assumptions be justified because of their negligible adverse effects. Some assumptions may prove to be restrictive but currently necessary for mathematical tractability, or due to lack of adequate quantitative data. The ultimate test of any model is a comparison of its predictions with the experimentally observed actions of the system for which it is an analogy.

In this section the assumptions which must be considered are discussed, and the development of basic equations for each of three models is outlined.

### Assumptions Considered

Prior to the development of the three models considered in this work, a comprehensive list of assumptions pertinent to the general problem of analysis of arterial blood flow is presented and assessed. Many of these assumptions have been discussed by previous investigators, including Lambert (1956, 1958), McDonald (1960), and Fox and Saibel (1963).

In the present work, some of the more restrictive traditional assumptions, such as steady flow, have been avoided. The first seven of the assumptions listed below are considered to be sufficiently valid for use throughout the present study. The remaining seven entries discuss effects which require critical examination, and are used in different ways in the model developments to follow.

1. Unsteady flow. In recent years it has become well recognized that there is much to be gained by considering oscillations in the vascular system, and, therefore, pulsatile effects must be included in any reasonable model.
2. Constant temperature. This assumption is usually well satisfied in mammalian systems under normal conditions. Recent applications of hypothermia may make it desirable to study the effect of temperature on blood flow, but for the present temperature is assumed to be constant.
3. Impervious wall. In larger vessels, transport of material across the wall is negligible in comparison with flow velocities.
4. No slip at the wall. Although evidence is not conclusive, this assumption is generally considered to be valid for all fluids in any solid conduit where fluid particles are small in comparison to the size of the conduit. In a stationary coordinate system it is not necessarily im-