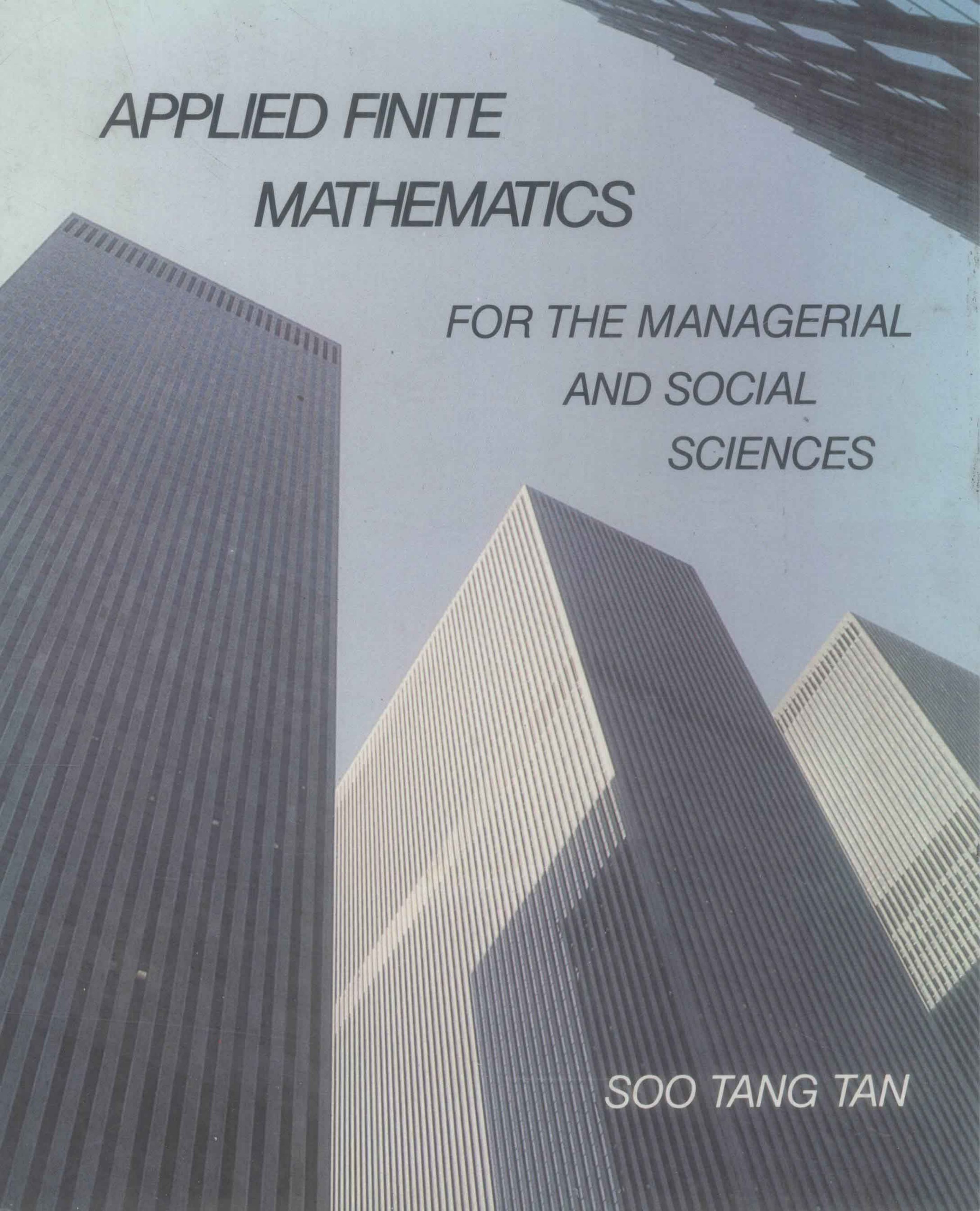
APPLIED FINITE MATHEMATICS

FOR THE MANAGERIAL AND SOCIAL SCIENCES



SOO TANG TAN

APPLIED FINITE MATHEMATICS

FOR THE MANAGERIAL AND SOCIAL SCIENCES

SOO TANG TAN

Stonehill College

PRINDLE WEBER & SCHMIDT
BOSTON, MASSACHUSETTS

PWS PUBLISHERS
Prindle, Weber & Schmidt • Willard Grant Press • Duxbury Press •
Statler Office Building • 20 Providence Street • Boston, Massachusetts 02116

PWS Publishers is a division of Wadsworth, Inc.

87 86 85 84 83—10 9 8 7 6 5 4 3 2 1

ISBN 0-87150-336-0

Library of Congress Cataloging in Publication Data

Tan, Soo Tang.
Applied finite mathematics for the managerial and social sciences.

1. Mathematics—1961– . I. Title.
QA39.2.T34 510 82-5353
ISBN 0-87150-336-0

Printed in the United States of America

Cover photograph by Owen Franken/Stock Boston

Cover and text design by Sara Waller. Typeset by J. W. Arrowsmith Ltd. Artwork by Julie Gecha. Covers printed by New England Book Components, Inc. Text printed and bound by R. R. Donnelley & Sons Co.

INDEX OF APPLICATIONS

BUSINESS AND ECONOMICS

SOCIAL SCIENCES

LIFE SCIENCES

MISCELLANEOUS

To Pat, Bill and Michael

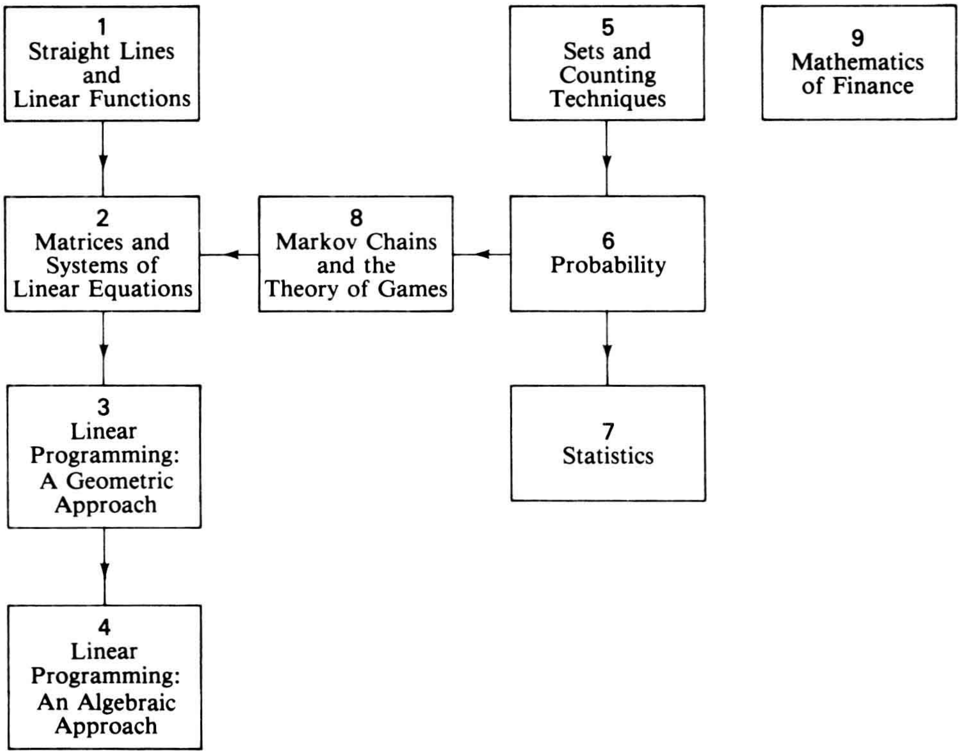
PREFACE

This book, which treats the standard topics in finite mathematics, is directed towards the student in the managerial, social and life sciences. The objective is two-fold. First, to provide the student with the background in the quantitative techniques that are necessary in order to better understand and appreciate the courses normally taken in one's undergraduate training. Second, to lay the foundation for more advanced courses, such as statistics and operations research. We have hoped to accomplish this by striking a careful balance between theory and applications.

Our approach is intuitive and we state the results informally. But we took special care to ensure that this does not compromise the mathematical content and accuracy. The applications are drawn from many fields and we made every effort to make them interesting, relevant and up-to-date. Numerous examples and solved problems are used to motivate each new concept or result in order to facilitate the student's comprehension of the new material. Each section is accompanied by an extensive set of exercises, which contains an ample set of problems of a routine computational nature to help the student master new techniques, followed by an extensive set of applications-oriented problems to test his or her mastery of the topics. The only pre-requisite for understanding this book is a year of high school algebra.

Since the book contains more than ample material for a one-semester or two-quarter course, the instructor may be flexible in choosing the topics most suitable for his or her course. The following chart on chapter-dependency is provided to help the instructor design a course that is most suitable for the intended audience.

Finally, I wish to express my personal appreciation to each of the following reviewers whose many suggestions have helped make a much improved end product: Professors Ronald D. Baker, University of Delaware; Jerry Davis, Johnson State College; Sharon S. Hewlett, University of New Orleans; James D. Nelson, Western Michigan University and Richard D. Porter, Northeastern University.



I also wish to thank my colleagues: Professors A. Atmar and W. Norko for reading portions of my manuscript and for their helpful suggestions; Professor L. Hegarty for class testing my manuscript and Dean R. Kruse for his enthusiastic support of this project.

My thanks also go to the editorial and production staff of Prindle Weber & Schmidt: David Pallai, Mary LeQuense, and Sara Waller, for their assistance and cooperation in the development and production of this book.

CONTENTS

1

STRAIGHT LINES AND LINEAR FUNCTIONS

1.1

The Cartesian Coordinate System

The system of real numbers will play a fundamental role throughout this book. This system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division. We shall assume that you are familiar with the rules governing these algebraic operations (see Appendix 1).

number line

origin

It is convenient and useful to have a geometrical representation of the set of real numbers. This is realized through the use of the *number line* which is constructed as follows. Arbitrarily select a point on a straight line to represent the number 0. This point is called the *origin*. If the line is placed horizontally, then a point at a convenient distance to the right of the origin is chosen to represent the number 1. Points to the right of 0 that are integral multiples of this unit length represent the positive integers while such points to the left of 0 represent the negative integers. Nonintegral numbers are represented by points whose distances from 0 are in the proper proportions. In this manner a one-to-one correspondence is set up between the set of real numbers and the set of points on the line with all the positive numbers lying to the right of the origin and all the negative numbers lying to the left of the origin (see Figure 1.1).

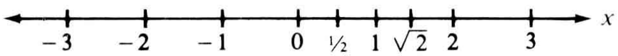


Figure 1.1

Cartesian coordinate system

A similar representation for points in the plane (a two-dimensional space) is realized through the *Cartesian coordinate system* which is constructed as follows.

Take two perpendicular lines, one of which is normally chosen to be horizontal. These lines intersect at a point O called the *origin* (see Figure 1.2).

origin

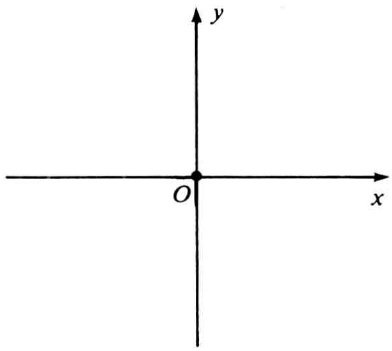


Figure 1.2

The horizontal line is called the *axis of abscissas* or, more simply, the x-*axis*. The vertical line is called the *axis of ordinates* or the y-*axis*. A number scale is set up along the x-axis with the positive numbers lying to the right of the origin and the negative numbers lying to the left of the origin. Similarly, a number scale is set up along the y-axis with the positive numbers lying above the origin and the negative numbers lying below the origin. Observe that the number scales need not be the same. Indeed, in many applications different quantities are represented by x and y; for example, x may represent the number of units of typewriters sold and y the total revenue resulting from the sales. In such cases it is often desirable to choose different number scales to represent the different quantities. Observe that, by construction, the zeros of both number scales coincide with the origin of the two-dimensional coordinate system.

axes

A point in the plane can now be represented uniquely in this coordinate system by an *ordered pair* of numbers; that is, a pair (x, y) where x is the first number and y the second number. To see this, let P be any point in the plane (see Figure 1.3). Draw perpendiculars from P to the x-axis and y-axis, respectively. Then the number x is precisely the number corresponding to the point on the x-axis at which the perpendicular through P cuts the x-axis. Similarly, y is the

ordered pair

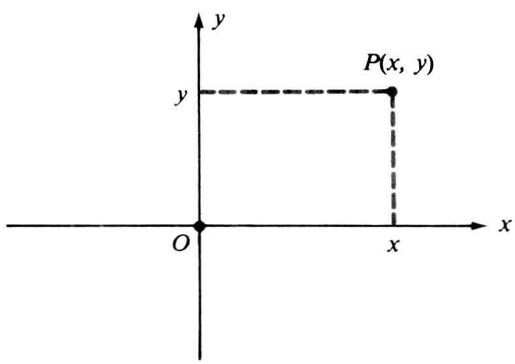


Figure 1.3

number corresponding to the point on the y-axis at which the perpendicular through P crosses the y-axis.

Conversely, given an ordered pair (x, y) with x as the first number and y as the second number, a point P in the plane is uniquely determined as follows: locate the point on the x-axis represented by the number x, then draw a line through that point parallel to the y-axis. Next, locate the point on the y-axis represented by the number y and draw a line through that point parallel to the x-axis. The point of intersection of these two lines is the point P. In the ordered pair (x, y), x is called the *abscissa* or *x-coordinate*, y is called the *ordinate* or *y-coordinate* and together x and y are referred to as the coordinates of the point P.

abscissa

ordinate

The points $A = (2, 3)$, $B = (-2, 3)$, $C = (-2, -3)$, $D = (2, -3)$, $E = (3, 2)$, $F = (4, 0)$, and $G = (0, -5)$ are plotted in Figure 1.4. The fact that in general $(x, y) \neq (y, x)$ is clearly illustrated by points A and E in Figure 1.4.

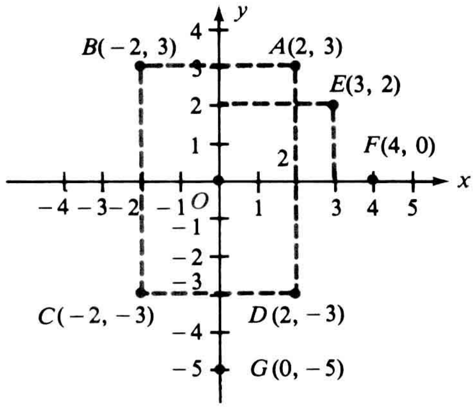


Figure 1.4

The axes divide the plane into four quadrants. Quadrant I consists of points P with coordinates x and y denoted by $P(x, y)$ satisfying $x > 0$ and $y > 0$; Quadrant II, the points $P(x, y)$ where $x < 0$ and $y > 0$; Quadrant III, the points $P(x, y)$ where $x < 0$ and $y < 0$; and Quadrant IV, the points $P(x, y)$ where $x > 0$ and $y < 0$ (see Figure 1.5).

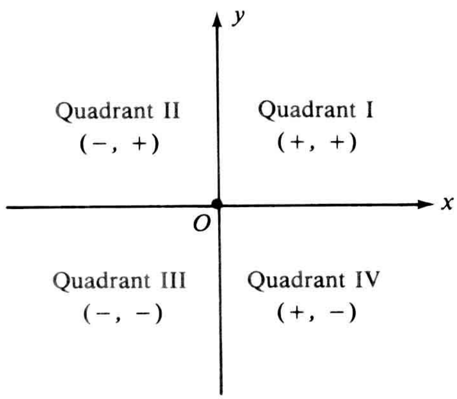
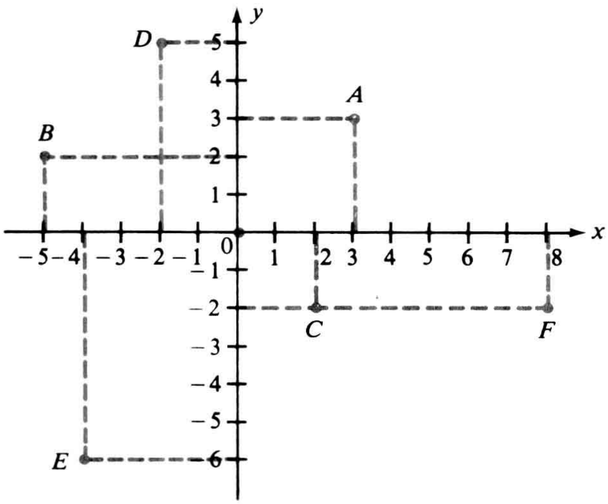


Figure 1.5

1.1 EXERCISES

1. Refer to the figure below and determine the coordinates of the following points:

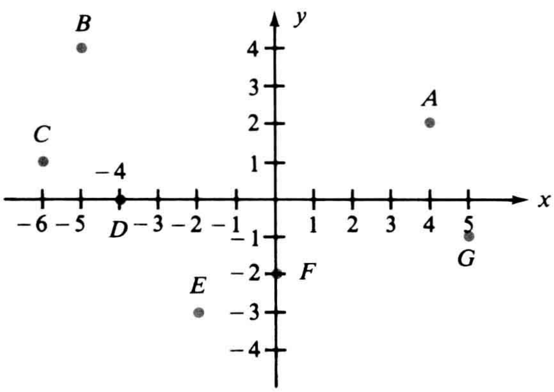
(*a*) *A* (*b*) *B* (*c*) *C* (*d*) *D* (*e*) *E* (*f*) *F*



2. Refer to the figure in problem 1 to determine the quadrant in which each of the following points is located.

(*a*) *A* (*b*) *B* (*c*) *C* (*d*) *D* (*e*) *E* (*f*) *F*

3. Refer to the figure below and answer (*a*)–(*e*).



(*a*) Which point has coordinates (4, 2)?

(*b*) What are the coordinates of the point B?

(*c*) Which points have negative y-coordinates?

(*d*) Which point has a negative x-coordinate and a negative y-coordinate?

(*e*) Which point has an x-coordinate that is equal to zero?

4. Sketch a set of coordinate axes and plot the following points:

(*a*) (1, 3) (*b*) (−2, 5) (*c*) (3, −4) (*d*) (3, −1)

5. Sketch a set of coordinate axes and plot the following points:

(*a*) (−5/2, 3/2) (*b*) (8, −7/2) (*c*) (1.2, −3.4) (*d*) (4.5, −4.5)

The distance between any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the plane is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use this formula to complete problems 6 through 14.

6. Find the distance between the points (1, 0) and (4, 4).

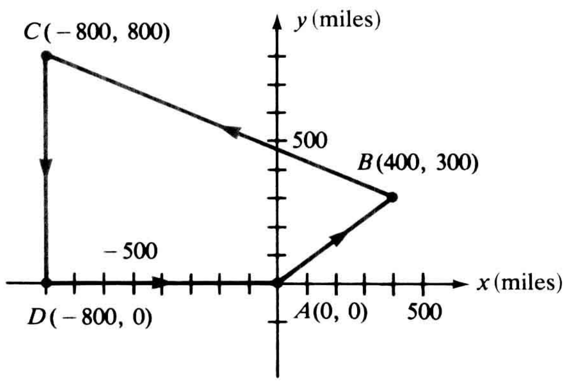
7. Find the distance between the points (1, 3) and (4, 7).

8. Find the distance between the points (−2, 1) and (10, 6).

9. Find the distance between the points (−1, 3) and (4, 9).

10. A furniture store offers free set-up and delivery services to all points within a 25 mile radius of its warehouse distribution center. If you live 20 miles east and 14 miles south of the warehouse, will you incur a delivery charge for furniture purchased from this store? Justify your answer.

11. A grand tour of four cities begins at city A with successive stops at cities B, C, and D before returning to city A. If the cities are located as shown in the figure below, find the total distance covered on the tour.

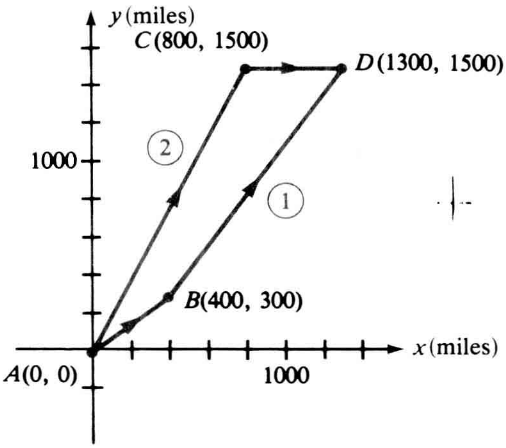


12. Mr. Barclay wishes to determine which antenna he should purchase for his home. The T.V. store has supplied him with the following information:

Range in Miles			
VHF	UHF	Model	Price
30	20	A	\$40.00
45	35	B	\$50.00
60	40	C	\$60.00
75	55	D	\$70.00

He wishes to receive programs from channel 17 (VHF), which is located 25 miles east and 35 miles north of his home; and programs from channel 38 (UHF), which is located 20 miles south and 32 miles west of his home. Which model will allow him to receive both channels at the least cost? (Assume there is flat terrain between his home and both broadcasting stations.)

13. Towns A, B, C, and D are located as shown in the following figure. There are two highways linking town A to town D. Route 1 runs from town A to town D via town B and Route 2 runs from town A to town D via town C. A salesman wishes to drive from town A to town D. If traffic conditions are such that he could expect to average the same speed on either route, which highway should he take in order to arrive at his destination in the shortest time?



14. Refer to the figure shown in problem 13. Suppose a fleet of 100 automobiles are to be shipped from an assembly plant located in town A to town D. They may be shipped either by freight train along Route 1 at a cost of 11 cents per mile per automobile or by truck along Route 2 at a cost of $10\frac{1}{2}$ cents per mile per automobile. Which means of transportation minimizes the shipping cost? What is the net savings?