Structural Analysis and Design of Multivariable Control Systems

An Algebraic Approach

多变量控制系统的结构分析与设计[英]

《一种代数方法》

Y. T. Tsay, L.-S. Shieh, S. Barnett

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Preface

Progress in system theory over the last two decades can be broadly categorized into two main streams:

(1) Algebraic System Theory - Study of basic notions and fundamental concepts of both algebra and system theory.

(2) System Analysis and Design Methods - Study of potential design techniques to analyze the characteristics of systems and to design controllers for satisfying various specifications and performance criteria.

Thousands of papers have been published in both areas in the last two decades. Systemic presentations in book form can be found, for example, in [1-5] for the former, in [6-10] for the latter, and in [11-16] for both. From this literature, we find that many elegant theories still cannot be employed to analyze/design the physical systems with ease. In other words, work is still needed to fill the gap between algebraic system theory and practical system analysis/design techniques. This provides the main motivation for our monograph.

The development of our work is based upon state-space representations and matrix fraction descriptions as the mathematical models for physical systems. A unified approach characterizing the dynamics of a system is presented through the formulation of the characteristic λ-matrix (also known as the matrix polynomial) of the system. Applications in pole assignment design, modal control design for multivariable systems, parallel realizations, and cascade realizations of multiport networks are illustrated. A detailed guide to the content of the monograph is provided in the last section of Chapter I.

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TABLE OF CONTENTS

CHAPTER I INTRODUCTION

In this introductory chapter, state-space representations and matrix fraction descriptions of multivariable linear systems are reviewed in Section 1.1. Some basic definitions on λ-matrices, which are the main mathematical tools used in our work, are summarized in Section 1.2, and Section 1.3 gives a guide to the content of the monograph.

1.1 State-Space Representations and Matrix Fraction Descriptions of Multivariable Systems

An m-input, p-output linear time-invariant system σ can be described by state equations as follows:

$$\lambda X(t) = AX(t) + Bu(t) \tag{1.1a}$$

$$y(t) = CX(t) + Du(t) \tag{1.1b}$$

where $X(t)\in\mathcal{X}\subseteq\mathcal{C}^n$, $y(t)\in\mathcal{Y}\subseteq\mathcal{C}^p$, $u(t)\in\mathcal{U}\subseteq\mathcal{C}^m$ are state, output, and input vectors, respectively; $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{U}$ are state, output and input spaces of σ, respectively; A,B,C,D are matrices of appropriate dimensions. For continuous-time systems, λ is a differential operator and $t\in\mathcal{R}$, while for discrete-time systems, λ is a forward shift operator and $t\in\mathcal{Z}$.

Equations (1.1a) and (1.1b) are referred to as the <u>state-space representation</u> of the multivariable system σ. A,B,C, and D can be treated as linear maps:

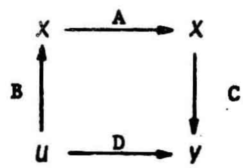
System map A: $\mathcal{X}\to\mathcal{X}$

Input map B: $\mathcal{U}\to\mathcal{X}$

Output map C: $\mathcal{X}\to\mathcal{Y}$

Forward map D: $\mathcal{U}\to\mathcal{Y}$ (1.2)

From Eq. (1.2), σ can be described by the following diagram:



(1.3)

which is not commutative.

The diagram in Eq. (1.3) is useful in studying the structural aspects of the system σ.

From Eq. (1.1), the input-output relationship of the system σ can be represented as

$$y(t) = G(\lambda)u(t) \tag{1.4a}$$

where

$$G(\lambda) = C(\lambda I_n - A)^{-1}B + D \in C^{p\times m}(\lambda) \tag{1.4b}$$

In Eq. (1.4b) $C^{p\times m}(\lambda)$ denotes the set of p×m matrices with elements being rational functions of λ over the complex field C. $G(\lambda)$ is called the <u>transfer function matrix</u> of the system σ. It has been shown in [1,13] that $G(\lambda)$ can be represented as the "ratio" of two matrix polynomials:

$$G(\lambda) = D_\ell^{-1}(\lambda)N_\ell(\lambda) \tag{1.5a}$$

$$= N_r(\lambda)D_r^{-1}(\lambda) \tag{1.5b}$$

where $D_\ell(\lambda)\in C^{p\times p}[\lambda]$, $N_\ell(\lambda)$, $N_r(\lambda)\in C^{p\times m}[\lambda]$, $D_r(\lambda)\in C^{m\times m}[\lambda]$; $C^{p\times p}[\lambda]$, $C^{p\times m}[\lambda]$ and $C^{m\times m}[\lambda]$ are sets of matrix polynomials of λ with coefficients in $C^{p\times p}$, $C^{p\times m}$, and $C^{m\times m}$, respectively. Combining Eqs. (1.4) and (1.5), yields

$$y(t) = D_{\ell}^{-1}(\lambda)N_{\ell}(\lambda)u(t) \qquad (1.6a)$$

$$= N_r(\lambda)D_r^{-1}(\lambda)u(t) \qquad (1.6b)$$

Equations (1.6a) and (1.6b) are referred to as left matrix fraction descriptions (LMFD) and right matrix fraction descriptions (RMFD) of the system σ, respectively.

Let $T \epsilon C^{n \times n}$ be a nonsingular matrix, and from Eq. (1.1) define

$$\hat{A} = TAT^{-1},\ \hat{B} = TB,\ \hat{C} = CT^{-1},\ \hat{D} = D \qquad (1.7a)$$

and

$$\hat{X}(t) = TX(t) \qquad (1.7b)$$

Then the state equations for the system $\hat{\sigma}$ are as follows:

$$\lambda\hat{X}(t) = \hat{A}\hat{X}(t)+\hat{B}u(t) \qquad (1.8a)$$

$$y(t) = \hat{C}\hat{X}(t)+\hat{D}u(t) \qquad (1.8b)$$

For the same set of inputs u(t), σ in Eq. (1.1) and $\hat{\sigma}$ in Eq. (1.8) will generate the same set of outputs y(t) for $t \geq 0$ if $\hat{X}(0) = TX(0)$. The difference between the state vectors X(t) and $\hat{X}(t)$ in the system σ and $\hat{\sigma}$, respectively, is therefore not apparent if only the input-output relationships are considered. Thus, we say that σ and $\hat{\sigma}$ are equivalent systems. Formally, we have the following definition:

Definition 1.1 The system in Eq. (1.1) and the system in Eq. (1.8) are equivalent if and only if the states are related by:

$$\hat{X}(t) = TX(t)$$

We will call this equivalence relation similarity equivalence (SE). □

Let $U_\ell(\lambda)\varepsilon C^{p\times p}[\lambda]$, and det $U_\ell(\lambda) = K_\ell$ which is a nonzero constant (i.e. $U_\ell(\lambda)$ is unimodular). Define

$$\hat{D}_\ell(\lambda) = U_\ell(\lambda)D_\ell(\lambda) \tag{1.9a}$$

$$\hat{N}_\ell(\lambda) = U_\ell(\lambda)N_\ell(\lambda) \tag{1.9b}$$

and

$$\hat{G}(\lambda) = \hat{D}_\ell^{-1}(\lambda)\hat{N}_\ell(\lambda) \tag{1.9c}$$

which is an LMFD of a system $\hat{\sigma}$:

$$y(t) = \hat{G}(\lambda)u(t) \tag{1.10}$$

From Eqs. (1.6a) and (1.10), σ in Eq. (1.6a) and $\hat{\sigma}$ in Eq. (1.10) will generate the same set of y(t) for t≥0 if the same set of u(t) is used as inputs, and σ and $\hat{\sigma}$ both have the same set of initial conditions y(t), t<0. Thus, we say that σ and $\hat{\sigma}$ are equivalent systems. Similar reasoning can be applied for RMFDs. We reach the following definitions:

Definition 1.2 Two systems with LMFDs $G(\lambda) = D_\ell^{-1}(\lambda)N_\ell(\lambda)$ and $\hat{G}(\lambda) = \hat{D}_\ell^{-1}(\lambda)\hat{N}_\ell(\lambda)$ are equivalent if and only if

$$\hat{D}_\ell(\lambda) = U_\ell(\lambda)D_\ell(\lambda)$$

and

$$\hat{N}_\ell(\lambda) = U_\ell(\lambda)D_\ell(\lambda)$$

where $U_{\ell}(\lambda)$ is unimodular. Similarly, two systems with RMFDs $G(\lambda) = N_r(\lambda)D_r^{-1}(\lambda)$ and $\hat{G}(\lambda) = \hat{N}_r(\lambda)\hat{D}_r^{-1}(\lambda)$ are equivalent if and only if

$$\hat{D}_r(\lambda) = D_r(\lambda)U_r(\lambda)$$

and

$$\hat{N}_r(\lambda) = N_r(\lambda)U_r(\lambda)$$

where $U_r(\lambda)$ is unimodular. We will call this kind of equivalence relations <u>unimodular equivalence</u> (UE). □

It can easily be verified that both SE and UE satisfy the basic properties of an equivalence relation: transitivity, symmetry and reflexivity [13]. Since a system can be represented via state-space equations or matrix fraction descriptions, we have:

<u>Lemma 1.1</u> Denote SE or UE by ~; σ_x, σ_y and σ_z are systems. Then, we have

(1) Transitivity: $\sigma_x \sim \sigma_y$ and $\sigma_y \sim \sigma_z$ implies $\sigma_x \sim \sigma_z$.

(2) Symmetry: $\sigma_x \sim \sigma_y$ implies $\sigma_y \sim \sigma_x$.

(3) Reflexivity: $\sigma_x \sim \sigma_x$. ■

From the idea of equivalent systems, both state-space representations and matrix fraction descriptions of multivariable systems are non-unique. In Chapter II, we will develop canonical forms, which are unique for a given system, for both state-space representations and matrix fraction descriptions.

1.2 Fundamental Properties of λ-Matrices

Since the MFD representations of a MIMO (multi-input, multi-output) system involve the ratio of two λ-matrices, and the results presented in the following chapters are closely related to λ-matrices, it is appropriate to review some definitions of λ-matrices in this section. Further details and properties can be found, for example, in [2] and [3]. Specifically we can define λ-matrices as follows [17,18]. Let F be an arbitrary field, and $F[\lambda]$ be the ring of polynomials over the field F. A λ-matrix, denoted by $A(\lambda)\varepsilon F^{p\times m}[\lambda]$ is a p×m matrix whose elements are in $F[\lambda]$. Let $A_{ij}(\lambda)$ be the (i,j)th element of $A(\lambda)$, then

$$A(\lambda) = (A_{ij}(\lambda)), \quad 1\le i\le p, \quad 1\le j\le m \tag{1.11a}$$

and

$$A_{ij}(\lambda) \triangleq \sum_{k=0}^{k_{ij}} a_{ijk}\lambda^{k_{ij}-k}, \quad a_{ijk}\varepsilon F \tag{1.11b}$$

where k_{ij} is the degree of the polynomial $A_{ij}(\lambda)$.

Let $r = \mathrm{Max}(k_{ij},\ 1\le i\le p,\ 1\le j\le m)$, then $A(\lambda)$ can be written as

$$A(\lambda) = \sum_{k=0}^{r} A_k\lambda^{r-k} \tag{1.12a}$$

where $A_k\varepsilon F^{p\times m}$, and the (i,j)th element of A_k is given by

$$(A_k)_{ij} = \begin{cases} a_{ijk} & \text{if } k\le k_{ij} \\ 0 & \text{otherwise} \end{cases} \tag{1.12b}$$

A λ-matrix $A(\lambda)\varepsilon F^{p\times p}[\lambda]$ is said to be <u>nonsingular</u> if $\det(A(\lambda)) \ne 0$, and <u>regular</u> if the matrix coefficient A_0 of the highest degree term (referred to Eq. (1.12a)) is nonsingular. A regular λ-matrix is <u>monic</u> if A_0 is an identity matrix.

Let $A(\lambda)$ be given by Eq. (1.11), and define

$$\nu_i = \mathrm{Max}(k_{ij},\ 1\le j\le m), \quad 1\le i\le p \tag{1.13}$$

Then, ν_i, denoted by $\nu_i = \partial_{r_i}(A(\lambda))$, is the row degree [12] of the ith row of $A(\lambda)$. Similarly

$$\kappa_j = \mathrm{Max}(k_{ij},\quad 1 \le i \le p), \quad 1 \le j \le m$$

denoted by $\kappa_j = \partial_{c_j}(A(\lambda))$, is the column degree [12] of the jth column of $A(\lambda)$. Define

$$A_{hr} = ((A_{hr})_{ij}), \ 1 \le i \le p, \ 1 \le j \le m \tag{1.15a}$$

where

$$(A_{hr})_{ij} = \begin{cases} a_{ij\nu_i} & \text{if } k_{ij} = \nu_i \\ 0 & \text{if } k_{ij} < \nu_i \end{cases} \tag{1.15b}$$

Then A_{hr} is called the leading row matrix of $A(\lambda)$. $A(\lambda)$ called is a row-reduced λ-matrix if p=m and A_{hr} is nonsingular [12]. Similarly, defining

$$A_{hc} = ((A_{hc})_{ij}), \ 1 \le i \le p, \ 1 \le j \le m \tag{1.16a}$$

where

$$(A_{hc})_{ij} = \begin{cases} a_{ij\kappa_j} & \text{if } k_{ij} = \kappa_j \\ 0 & \text{if } k_{ij} < \kappa_j \end{cases} \tag{1.16b}$$

then A_{hc} is called the leading column matrix of $A(\lambda)$, and if p=m and A_{hc} is nonsingular, $A(\lambda)$ is a column-reduced λ-matrix [12].

To analyze the structure of λ-matrices, it is convenient to transform a general λ-matrix to certain specific forms whose structures can be easily handled. The most commonly used transformations are those of equivalence [12,13].

Definition 1.3 Two λ-matrices $A_1(\lambda)$ and $A_2(\lambda)$ are row equivalent, column equivalent, or equivalent, iff $A_1(\lambda) = U_L(\lambda)A_2(\lambda)$, $A_1(\lambda) = A_2(\lambda)U_R(\lambda)$, or $A_1(\lambda) = U_L(\lambda)A_2(\lambda)U_R(\lambda)$, respectively, where $U_L(\lambda)$ and $U_R(\lambda)$ are unimodular λ-matrices. □

The equivalence of nonsingular λ-matrices can be stated as follows:

Lemma 1.2 Any nonsingular λ-matrix is row equivalent, column equivalent, or equivalent to a row-reduced, a column-reduced, or a row- and column-reduced λ-matrix. ■

It is well known that equivalent row-reduced or column-reduced λ-matrices of a given nonsingular λ-matrix [12] are not unique. According to Definition 1.3, a regular λ-matrix is always equivalent to a monic λ-matrix, and the properties and applications of monic λ-matrices have been discussed by many authors [17-32]. We shall extend some known results on monic λ-matrices to row-reduced or column-reduced λ-matrices in the following chapters.

In the analysis and design of multi-input, multi-output (MIMO) systems, MFD representations of the systems are rational matrices over the complex field C. Therefore, we will set F = C in the following chapters whenever λ-matrices are involved.

1.3 Organization of Chapters

The material in this monograph can be regarded as being in two parts: The first part, which includes Chapters II, III and IV, is devoted to exploring the spectral decomposition theory of λ-matrices via the canonical structures of MIMO systems represented in state space equations and MFDs; the second part, which consists of Chapters V and VI, considers applications of the structure theory developed in the first part to the design and decomposition of MIMO systems. Illustrative numerical examples are presented throughout the book.

In Chapter II, the characteristic λ-matrices of multivariable control systems are defined. For a reachable system the characteristic λ-matrix can be constructed from the coefficients of the dependence equations for the column vectors of the reachability test matrix; on the other hand, for an observable system, the left characteristic λ-matrix can be constructed from the coefficients of the dependence equations for the row vectors of the observability test matrix. The controller and observer canonical state-space representations for reachable and observable MIMO systems, respectively, are formally defined. The canonical RMFDs and LMFDs for reachable and observable systems, respectively, are defined, and their properties are discussed based on the canonical controller and observer state-space representations. The characteristic λ-matrices, the canonical state-space forms, and the canonical MFDs are highly dependent on the Kronecker or observability indices of the system. Thus, we also present a numerical method using an orthogonalized projection scheme to compute the Kronecker and observability indices of MIMO system. This numerical algorithm is based on the so-called minimal nice selections.

Spectral analysis of general nonsingular λ-matrices is presented in Chapter III. Firstly, column-reduced and row-reduced canonical λ-matrices are defined; then the equivalent transformations of a nonsingular λ-matrices to a column-reduced or a row-reduced canonical λ-matrix are established. Consequently, the latent roots and latent structures of a general nonsingular λ-matrix can be studied in terms of its equivalent column-reduced or row-reduced canonical λ-matrix. The relationships between the latent structures of nonsingular λ-matrices and the eigenstructures of the system maps in their associated state-space minimal realization quadruples are investigated. As a result, the Jordan chains of nonsingular λ-matrices can be easily found from the input and output maps of their associated Jordan canonical minimal realization quadruples. The matrix roots, formally called solvents, of nonsingular λ-matrices are defined and briefly discussed.

Chapter IV is devoted to developing the theory of divisors and spectral factors of nonsingular λ-matrices. The state-space structures of canonical left and right divisors of nonsingular λ-matrices are extensively investigated via the so-called geometric approaches. Constructive proofs on the existence of the canonical divisors are provided, and some properties of left/right divisors of nonsingular λ-matrices are investigated. Also, the concepts of complete sets of canonical left/right divisors, which are extremely important in the applications to the design and decomposition of MIMO systems, are presented. For completeness, the structures of spectral factorizations used to factor a nonsingular λ-matrix into the product of lower degree canonical λ-matrices are also explored. Finally, computational algorithms for divisors and spectral factors based on block triangularization and block diagonalization of square matrices are discussed. A newly developed matrix sign algorithm is suggested for effective computation of divisors and spectral factors of nonsingular λ-matrices.

The applications of the theory begin in Chapter V, where state-feedback control designs of multivariable systems are studied. Properties of linear state-feedback controls are discussed first. The invariance property of the Kronecker indices of MIMO systems under linear state-feedback controls is an important guide in devising various control schemes. The characteristic λ-matrix and column-reduced λ-matrix assignments for the denominators of the closed-loop MFDs are derived. A study is then made properties of the closed-loop MFDs. For controlling the latent structure of the characteristic λ-matrices in the closed-loop system, we introduce the left/right latent structure assignment. For the purposes of closed-loop decomposition, the divisor assignment and decoupling design, via the notions of divisors, are also presented.

Decomposition theories and their applications to multivariable analysis and design are developed in Chapter VI. Parallel decomposition theory is derived