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Theory of Complex Functions

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Reinhold Remmert

Theory of Complex Functions

Translated by Robert B. Burckel

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Preface to the English Edition

Und so ist jeder Übersetzer anzusehen, dass er sich als Vermittler dieses allgemein-geistigen Handels bemüht und den Wechseltausch zu befördern sich zum Geschäft macht. Denn was man auch von der Unzulänglichkeit des Übersetzers sagen mag, so ist und bleibt es doch eines der wichtigsten und würdigsten Geschäfte in dem allgemeinen Weltverkehr. (And that is how we should see the translator, as one who strives to be a mediator in this universal, intellectual trade and makes it his business to promote exchange. For whatever one may say about the shortcomings of translations, they are and will remain most important and worthy undertakings in world communications.) J. W. von GOETHE , vol. VI of *Kunst und Alterthum*, 1828.

This book is a translation of the second edition of *Funktionentheorie I*, Grundwissen Mathematik 5, Springer-Verlag 1989. Professor R. B. BURCKEL did much more that just produce a translation; he discussed the text carefully with me and made several valuable suggestions for improvement. It is my great pleasure to express to him my sincere thanks.

Mrs. Ch. ABIKOFF prepared this TEX-version with great patience; Prof. W. ABIKOFF was helpful with comments for improvements. Last but not least I want to thank the staff of Springer-Verlag, New York. The late W. KAUFMANN-BÜHLER started the project in 1984; U. SCHMICKLER-HIRZEBRUCH brought it to a conclusion.

Lengerich (Westphalia), June 26, 1989

Reinhold Remmert

Preface to the Second German Edition

Not only have typographical and other errors been corrected and improvements carried out, but some new supplemental material has been inserted. Thus, e.g., HURWITZ's theorem is now derived as early at 8.5.5 by means of the minimum principle and Weierstrass's convergence theorem. Newly added are the long-neglected proof (without use of integrals) of Laurent's theorem by SCHEEFFER, via reduction to the Cauchy-Taylor theorem, and DIXON's elegant proof of the homology version of Cauchy's theorem. In response to an oft-expressed wish, each individual section has been enriched with practice exercises.

I have many readers to thank for critical remarks and valuable suggestions. I would like to mention specifically the following colleagues: M. BARNER (Freiburg), R. P. BOAS (Evanston, Illinois), R. B. BURCKEL (Kansas State University), K. DIEDERICH (Wuppertal), D. GAIER (Giessen), ST. HILDEBRANDT (Bonn), and W. PURKERT (Leipzig).

In the preparation of the 2nd edition, I was given outstanding help by Mr. K. SCHLÖTER and special thanks are due him. I thank Mr. W. HOMANN for his assistance in the selection of exercises. The publisher has been magnanimous in accommodating all my wishes for changes.

Lengerich (Westphalia), April 10, 1989

Reinhold Remmert

Preface to the First German Edition

> Wir möchten gern dem Kritikus gefallen: Nur nicht dem Kritikus vor allen. (We would gladly please the critic: Only not the critic above all.) G. E. LESSING.

The authors and editors of the textbook series "Grundwissen Mathematik"[1] have set themselves the goal of presenting mathematical theories in connection with their historical development. For function theory with its abundance of classical theorems such a program is especially attractive. This may, despite the voluminous literature on function theory, justify yet another textbook on it. For it is still true, as was written in 1900 in the prospectus for vol. 112 of the well-known series *Ostwald's Klassiker Der Exakten Wissenschaften*, where the German translation of Cauchy's classic "Mémoire sur les intégrales définies prises entre des limites imaginaires" appears: "Although modern methods are most effective in communicating the content of science, prominent and far-sighted people have repeatedly focused attention on a deficiency which all too often afflicts the scientific education of our younger generation. *It is this, the lack of a historical sense and of any knowledge of the great labors on which the edifice of science rests.*"

The present book contains many historical explanations and original quotations from the classics. These may entice the reader to at least page through some of the original works. "Notes about personalities" are sprinkled in "in order to lend some human and personal dimension to the science" (in the words of F. KLEIN on p. 274 of his *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert* — see [H_8]). But the book is not a history of function theory; the historical remarks almost always reflect the contemporary viewpoint.

Mathematics remains the primary concern. What is treated is the material of a 4 hour/week, one-semester course of lectures, centering around

[1]The original German version of this book was volume 5 in that series (translator's note).

Cauchy's integral theorem. Besides the usual themes which no text on function theory can omit, the reader will find here

- RITT's theorem on asymptotic power series expansions, which provides a function-theoretic interpretation of the famous theorem of É. BOREL to the effect that any sequence of complex numbers is the sequence of derivatives at 0 of some infinitely differentiable function on the line.
- EISENSTEIN's striking approach to the circular functions via series of partial fractions.
- MORDELL's residue-theoretic calculations of certain Gauss sums.

In addition *cognoscenti* may here or there discover something new or long forgotten.

To many readers the present exposition may seem too detailed, to others perhaps too compressed. J. KEPLER agonized over this very point, writing in his *Astronomia Nova* in the year 1609: "Durissima est hodie conditio scribendi libros Mathematicos. Nisi enim servaveris genuinam subtilitatem propositionum, instructionum, demonstrationum, conclusionum; liber non erit Mathematicus: sin autem servaveris; lectio efficitur morosissima. (It is very difficult to write mathematics books nowadays. If one doesn't take pains with the fine points of theorems, explanations, proofs and corollaries, then it won't be a mathematics book; but if one does these things, then the reading of it will be extremely boring.)" And in another place it says: "Et habet ipsa etiam prolixitas phrasium suam obscuritatem, non minorem quam concisa brevitas (And detailed exposition can obfuscate no less than the overly terse)."

K. PETERS (Boston) encouraged me to write this book. An academic stipend from the Volkswagen Foundation during the Winter semesters 1980/81 and 1982/83 substantially furthered the project; for this support I'd like to offer special thanks. My thanks are also owed the Mathematical Research Institute at Oberwolfach for oft-extended hospitality. It isn't possible to mention here by name all those who gave me valuable advice during the writing of the book. But I would like to name Messrs. M. KOECHER and K. LAMOTKE, who checked the text critically and suggested improvements. From Mr. H. GERICKE I learned quite a bit of history. Still I must ask the reader's forebearance and enlightenment if my historical notes need any revision.

My colleagues, particularly Messrs. P. ULLRICH and M. STEINSIEK, have helped with indefatigable literature searches and have eliminated many deficiencies from the manuscript. Mr. ULLRICH prepared the symbol, name, and subject indexes; Mrs. E. KLEINHANS made a careful critical pass through the final version of the manuscript. I thank the publisher for being so obliging.

Lengerich (Westphalia), June 22, 1983 Reinhold Remmert

Notes for the Reader. Reading really ought to start with Chapter 1. Chapter 0 is just a short compendium of important concepts and theorems known to the reader by and large from calculus; only such things as are important for function theory get mentioned here.

A citation 3.4.2, e.g., means subsection 2 in section 4 of Chapter 3. Within a given chapter the chapter number is dispensed with and within a given section the section number is dispensed with, too. Material set in reduced type will not be used later. The subsections and sections prefaced with * can be skipped on the first reading. Historical material is as a rule organized into a special subsection in the same section were the relevant mathematics was presented.

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