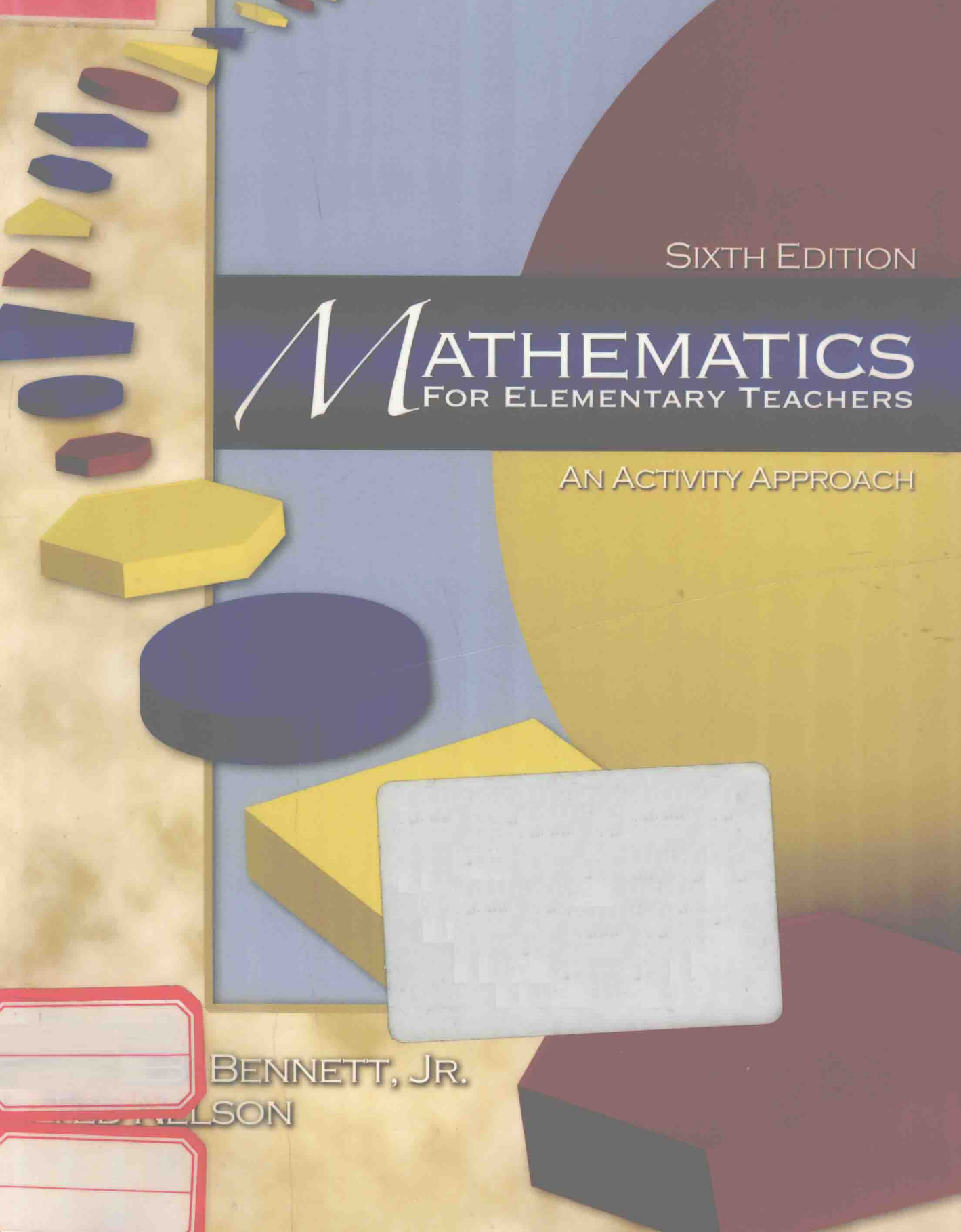
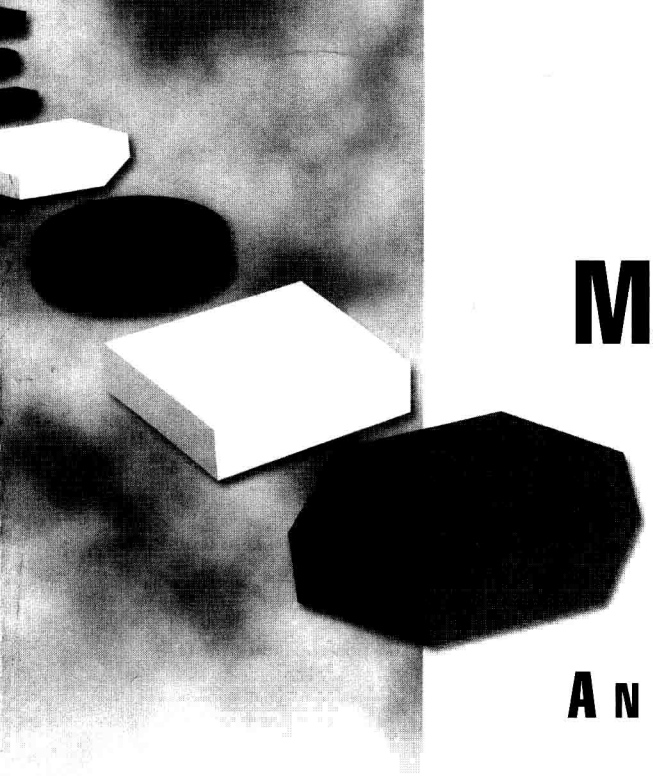

SIXTH EDITION
MATHEMATICS
FOR ELEMENTARY TEACHERS
AN ACTIVITY APPROACH
BENNETT, JR.



Sixth Edition

MATHEMATICS FOR ELEMENTARY TEACHERS

AN ACTIVITY APPROACH

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MATHEMATICS FOR ELEMENTARY TEACHERS: AN ACTIVITY APPROACH
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To Prospective Teachers

In this course you are the student. However, this is an important time for you to begin thinking like the teacher you will become. This is the time to begin thinking about the nature of mathematics and about teaching and learning mathematics. You can do this by monitoring your own thoughts and feelings as you explore and investigate, and by making notes about how you might change or adapt an activity for use in an elementary or middle school classroom.

This book was written to help you experience some of the mathematical ideas you will soon be teaching and to provide methods for teaching them to your students. You will construct models, use manipulatives, work cooperatively, explore, investigate, discover, make conjectures, and form conclusions. You will find that the models and manipulatives that accompany this book can be used with children. When you have opportunities we encourage you to try appropriate activities with children and, by observing and asking questions, try to understand how they are thinking about mathematics. Eleanor Duckworth used her observations of children learning in her book, *The Having of Wonderful Ideas and Other Essays on Teaching and Learning,* where she makes these comments:[1]

> So what is the role of teaching, if knowledge must be constructed by each individual? In my view, there are two aspects to teaching. The first is to put students into contact with phenomena related to the area to be studied—the real thing, not books or lectures about it—and to help them notice what is interesting; to engage them so they will continue to think and wonder about it. The second is to have the students try to explain the sense they are making, and, instead of explaining things to students, to try to understand their sense. These two aspects are, of course, interdependent: When people are engaged in the matter, they try to explain it and in order to explain it they seek out more phenomena that will shed light on it.

[1]New York: Teachers College Press, Columbia University, 1987, p. 123.

The primary purpose of *Mathematics for Elementary Teachers: An Activity Approach* is to engage prospective elementary and middle school teachers in mathematical activities that will enhance their conceptual knowledge, introduce them to important manipulatives, and model the kind of mathematical learning experiences they will be expected to provide for their students.

The National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* ("Standards 2000") and its predecessor, *Curriculum and Evaluation Standards for School Mathematics,* strongly assert that students learn mathematics well only when they construct their own mathematical thinking. Information can be transmitted from one person to another but mathematical understanding and knowledge come from within the learner as that individual explores, discovers, and makes connections.

The National Council of Teachers of Mathematics' *Professional Standards for Teaching Mathematics* presents a vision of mathematics teaching that "redirects mathematics instruction from a focus on presenting content through lecture and demonstration to a focus on active participation and involvement. Mathematics instructors do not simply deliver content; rather, they facilitate learners' construction of their own knowledge of mathematics."

This book contains activities and materials to actively engage students in mathematical explorations. It provides prospective elementary and middle school teachers the opportunity to examine and learn mathematics in a meaningful way. It provides the instructor with the resources to make students' mathematical activity the focus of attention.

Reasons for Mathematical Activities

It is well known that effective teachers have a good understanding of the mathematics they teach and are skillful in choosing and using a variety of appropriate instructional techniques. The importance of using manipulatives in mathematics teaching and learning is well documented and amply illustrated in the pages of the *Principles and Standards for School Mathematics.* Interviews with teachers successfully using manipulatives in classrooms revealed the following commonalities: the teachers had all received training for using manipulatives; they designed their own lessons and worked through the lessons with manipulatives themselves; and they prepared for classroom use of the manipulatives by anticipating how the class would react to the activity.[1]

Mathematics for Elementary Teachers: An Activity Approach is a collection of activities for prospective teachers that involves inductive and deductive reasoning. The activities allow students to think deeply about how manipulatives and visual models contribute to understanding mathematical concepts. Students experience mathematics directly by using models that embody concepts and promote mathematical thinking. This book represents the belief that:

- Prospective teachers who learn mathematics through appropriate use of manipulatives, models, and diagrams are more likely to develop a solid conceptual basis and a deeper understanding of the mathematics they will teach.
- Prospective teachers who learn mathematics by being actively involved in doing mathematics will be more likely to teach in the same manner.

[1]Mary K. Stein and Jane W. Bovalino, "Manipulatives: One Piece of the Puzzle," *Mathematics Teaching in the Middle School* 6 (February 2001): 356–359.

- Prospective teachers who use manipulatives effectively in their learning will experience how manipulatives assist understanding and be more likely to use them effectively in their teaching.
- Becoming familiar with manipulatives and models in structured activities will help prepare prospective teachers to develop similar lessons for their students.
- A concrete approach diminishes the mathematical anxiety often accompanying a more abstract approach.
- Tactile and visual approaches provide mental images that, for some students, can be easily retained to provide understanding for symbolic representations.

Features

Mathematics for Elementary Teachers: An Activity Approach contains 11 chapters with 34 activity sets and accompanying materials to provide a self-contained mathematics laboratory. Here are the special features of this book.

Active Learning Each activity set uses physical materials and/or visual models to provide a context for understanding. The questions and activities in each activity set are sequentially developed to encourage discovery and to provide depth into the topic.

Cooperative Learning The activity sets can be done individually or in small groups. In particular, Activity Set 2.3, Logic Problems for Cooperative Learning Groups, is designed for small-group interaction.

Individual Reflections Throughout the activity sets, students are asked to describe patterns, discuss their thinking, and write explanations of their reasoning.

Manipulative Kit and Material Cards The Manipulative Kit contains 9 common manipulatives on color card stock together with storage envelopes. The Activity Book also has 40 additional material cards with manipulatives, models, grids, templates, game mats, and other materials to be used in the activities.

Pedagogy The activity sets demonstrate ways that manipulatives and visual models can be used in classrooms to promote conceptual understanding and mathematical thinking. There are concrete or visual models for teaching the following topics:

reasoning	patterns and sequences	least common multiple
numeration	algebraic expressions	geometric relationships
whole numbers	indirect measurement	length, area, and volume
fractions	proportions	probability
integers	even and odd numbers	statistics
decimals	factors and primes	algebra word problems
percents	greatest common divisor	metric measurements

Just for Fun Each activity set is followed by a Just for Fun activity. These are related to the topics of the activity sets and often are recreational or artistic.

Classroom Ideas At the end of each chapter a Suggested Classroom Activity is given for students to try with children. In addition, any of the activities in the book can be adapted for use with children.

Readings for More Classroom Ideas There is a selected list of readings at the end of each chapter. Some of the articles were written by classroom teachers about activities they have successfully used. Also, there are references to articles

and books that discuss reform issues for teachers. Additional sources can be found on the Online Learning Center (described below).

Puzzlers Brain teasers are interspersed throughout the text to add variety and provide practice in problem solving.

Answer Section Answers to all puzzlers and selected (★) questions from the activity sets are in the back of the Activity Book.

Supplements

Activity Book Instructor's Manual (ISBN 0-07-282146-9) The *Instructor's Manual for Mathematics for Elementary Teachers: An Activity Approach* contains answers for all activity sets, puzzlers, and Just for Fun activities. There is a set of sample test questions, with answers, for each chapter of the activity book. There are blackline masters of various grids used throughout the book.

Companion Text (ISBN 0-07-253294-7) The text, *Mathematics for Elementary Teachers: A Conceptual Approach,* Sixth Edition, is a companion volume to this activity book. Like the activity book, the text contains 11 chapters and 34 sections. Each of these sections corresponds to an activity in the activity book. The text also contains a one-page math activity at the beginning of each section that uses the same Manipulative Kit as the activity book.

Online Learning Center (www.mhhe.com/bennett-nelson)

This new website created for the Sixth Edition of the activity book and its companion text offers a variety of resources to both instructors and students. From this site you can:

- Use the Digital Manipulative Kit to carry out various activities using colored manipulatives online.
- Access 11 interactive mathematics applets to learn key mathematical concepts.
- Obtain Writing/Discussion questions about teaching issues and mathematics concepts.
- Download color masters for transparencies of the manipulatives and black and white masters for a variety of grids and dot paper.
- Access an extended bibliography and list of Internet links.
- Explore 14 open-ended computer investigations with the browser-based Mathematics Investigator software to generate data.
- Download instructions and exercises for Network Graphs and Logo.

Class Formats

The familiar chapter headings in *Mathematics for Elementary Teachers: An Activity Approach* will be convenient for those who wish to use the activities with the companion text or another mathematics textbook. Many of the activity sets are independent and may be selected out of sequence.

Mathematics for Elementary Teachers: An Activity Approach can be effectively used in a variety of class formats:

- **A lab course** based on the activity sets with outside readings from a reference text or journals
- **A combination lab and recitation course** in which the activities are used as starting points, followed by discussions or lectures based on extensions of the ideas raised in the activity sets
- **A traditional lecture/recitation course** with the activity sets as supplemental and used for assignments

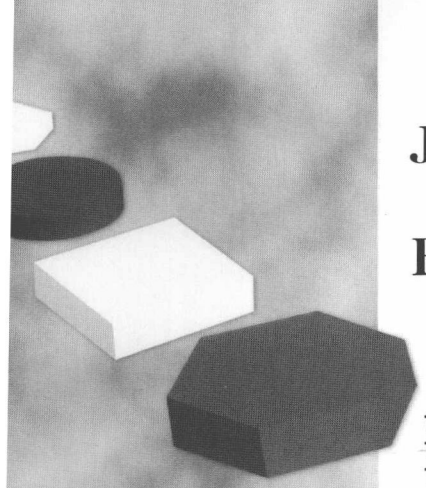
Acknowledgments

The authors would like to thank the students and instructors who have used the previous editions of this book, along with the instructors who reviewed this book and its companion text, *Mathematics for Elementary Teachers: A Conceptual Approach*. The authors are sincerely grateful for the helpful suggestions and continued support.

Material Cards

1. Rectangular Grid (2.2)
2. Isometric Grid (2.2)
3. Attribute-Game Grid (2.1)
4. Two-Circle Venn Diagram (2.1)
5. Three-Circle Venn Diagram (2.1, 9.1)
6. Pica-Centro Recording Sheet (2.3)
7. Coordinate Guessing and Hide-a-Region Grids (2.2)
8. Table of Random Digits (7.2, 7.3, 8.1)
9. Two-Penny Grid (8.1)
10. Three-Penny Grid (8.1)
11. Geoboard Recording Paper (9.1)
12. Grids for Game of Hex (9.2)
13. Perpendicular Lines for Symmetry (9.4)
14. Metric Measuring Tape (10.1)
15. Centimeter Racing Mat (10.1)
16. Pentomino Game Grid (10.2)
17. Attribute Label Cards (2.1)
18. Logic Problem Clue Cards and People Pieces (Problem 1) (2.3)
19. Logic Problem Clue Cards (Problems 2 and 3) (2.3)
20. Logic Problem Clue Cards (Problems 4 and 5) (2.3)
21. Object Pieces for Logic Problem 5 (2.3)
22. Mind-Reading Cards (3.1)
23. Decimal Squares* (6.1, 6.2)
24. Decimal Squares* (6.1, 6.2)
25. Decimal Squares* (6.1, 6.2)
26. Decimal Squares* (6.1, 6.2)
27. Rectangular Geoboard Template (6.4, 9.1, 10.2)
28. Algebra Pieces (1.3)
29. Algebraic Expression Cards (1.3)
30. Simulation Spinners (8.2)
31. Trick Dice (8.2)
32. Metric Ruler, Protractor, and Compass (9.1, 10.1, 10.3, 11.1, 11.2, 11.3)
33. Circular Geoboard Template (9.1)
34. Regular Polyhedra (9.3)
35. Regular Polyhedra (9.3)
36. Cube Patterns for Instant Insanity (9.3)
37. Pentominoes (10.2)
38. Prism, Pyramid, and Cylinder (10.3)
39. Hypsometer-Clinometer (11.3)
40. Interest Gameboard (6.3)

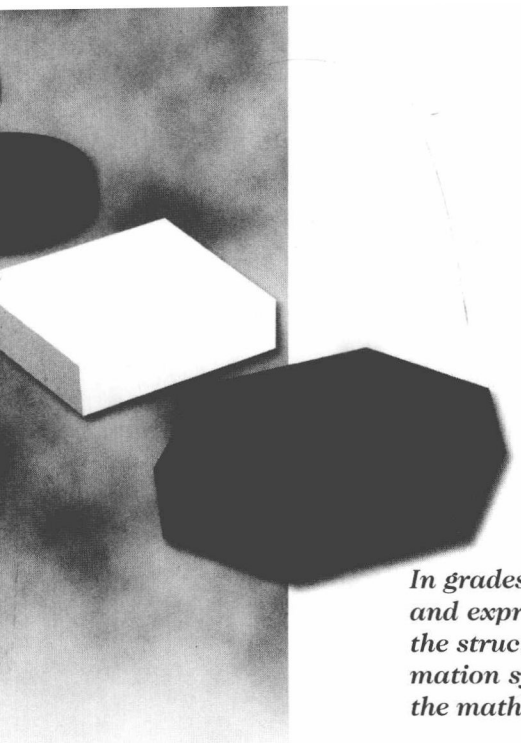
*Decimal Squares® is a registered trademark of Scott Resources.



JUST FOR FUN ACTIVITIES AND ELEMENTARY CLASSROOM IDEAS

CONTENTS

ACTIVITY SETS



CHAPTER 1

PROBLEM SOLVING

In grades 3–5, students should investigate numerical and geometric patterns and express them mathematically in words or symbols. They should analyze the structure of the pattern and how it grows or changes, organize this information systemically, and use their analysis to develop generalizations about the mathematical relationships in the pattern.[1]

ACTIVITY SET 1.1

SEEING AND EXTENDING PATTERNS WITH PATTERN BLOCKS

Purpose To recognize, describe, construct, and extend geometric patterns

Materials Pattern blocks from the Manipulative Kit

Activity Human beings are pattern-seeking creatures. Babies begin life's journey listening for verbal patterns and looking for visual patterns. Scientists in search of extraterrestrial intelligence send patterned signals into the universe and listen for incoming patterns on radio telescopes. Mathematics is also concerned with patterns. Many mathematicians and educators involved in reforming mathematics teaching and learning at the elementary and middle school levels are suggesting that the notion of mathematics as the study of number and shape needs to be expanded. Some suggest that "mathematics is an exploratory science that seeks to understand every kind of pattern."[2]

In this first activity set colored geometric shapes called pattern blocks will be used to recognize, study, and extend geometric patterns. The set of pattern blocks consists of six different figures: a green triangle, an orange square, a red trapezoid, a blue rhombus, a white rhombus, and a yellow hexagon.

[1]*Principles and Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 2000): 159.

[2]Lynn A. Steen, *On the Shoulders of Giants: New Approaches to Numeracy* (Washington, DC: National Academy Press, 1990): 1–8.

1. The pattern block figures shown here form the first five figures of a sequence. Use your green triangles to construct the sixth and seventh figures that you think extend the given pattern and sketch these figures.

★ **a.** Describe in writing at least three ways that the seventh figure in your sequence differs from the sixth figure.

★ **b.** Describe in writing what the 15th figure in this sequence would look like so that someone reading your description, who had not seen this sequence, could build the same figure.

2. Use your pattern blocks to construct the sixth figure of the sequence below and sketch that figure.

a. Describe in writing how new figures are created as this sequence is extended.

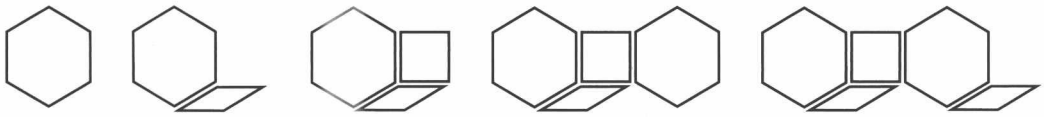
b. Will the 10th figure in an extended sequence have a green triangle or a blue rhombus on the right end? Explain your reasoning.

c. How many triangles and how many rhombuses are in the 25th figure of the extended sequence? Explain how you arrived at your answer.

d. Complete the following statement that will enable readers to determine the number of triangles and rhombuses in any figure they choose.

In the nth figure, where n is an even number, there will be n ÷ 2 triangles and n ÷ 2 rhombuses. If n is an odd number, the nth figure will contain _____ triangles and _____ rhombuses.

3. The pattern block sequence started below uses three different types of pattern blocks. Use your pattern blocks to build and sketch the next figure in the extended sequence.



1st 2d 3d 4th 5th 6th

★ **a.** Describe in writing how new figures are created as this sequence is extended.

★ **b.** What pattern block will be attached to the right end of the 16th figure to obtain the 17th figure in this sequence?

★ **c.** Determine the number of hexagons, squares, and rhombuses in the 20th figure of the sequence. Explain how you thought about it.

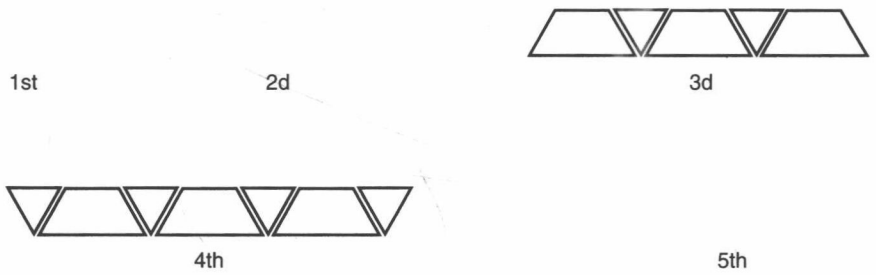
★ **d.** Repeat part c for the 57th figure in the sequence.

4. Use your pattern blocks to build and sketch the sixth figure of the sequence here.

a. Determine the number of triangles and hexagons in the 10th figure of the extended sequence. Do the same for the 15th figure.

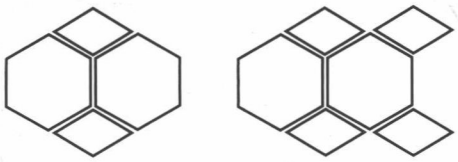
b. Any figure number that is a multiple of 3 has 1/3 that number of hexagons and 2/3 that number of triangles. Explain how you can determine the number of hexagons and triangles if the figure number is 1 more than a multiple of 3. One less than a multiple of 3.

5. The third and fourth figures of a sequence are given below. Use your pattern blocks to construct and sketch the first, second, and fifth figures in this sequence.



a. Describe how the odd-numbered figures differ from the even-numbered figures.

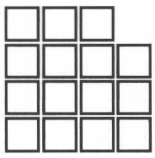
b. Sketch the missing figures for the sequence on the *next page*. Explain how you can determine the number of hexagons in any even-numbered figure of the sequence, then explain it for any odd-numbered figure.



1st 2d 3d 4th 5th 6th

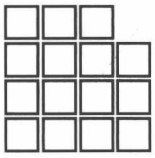
★ 6. The third term of a sequence is shown below. Create more than one sequence for which the given figure is the third term and sketch diagrams of the first, second, and fourth figures. Write a rule for extending each pattern you create so that the reader is able to build the next few figures in the sequence. (You may wish to use the colored tiles from your Manipulative Kit for this activity.)

Sequence I



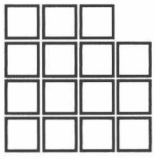
1st 2d 3d 4th

Sequence II



1st 2d 3d 4th

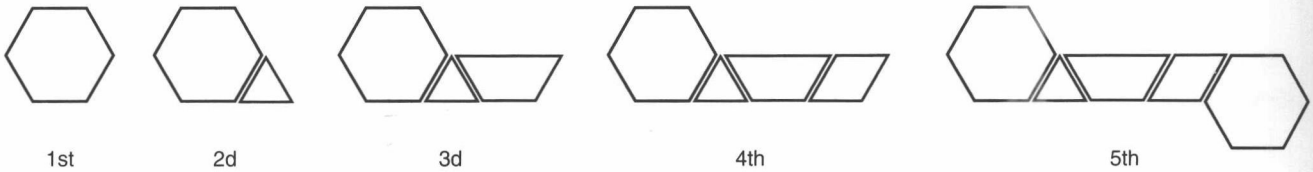
Sequence III



1st 2d 3d 4th

7. Sequences I and II begin repeating in the fifth figure and sequence III begins repeating in the sixth figure. Build and sketch the next figure in each sequence with your pattern blocks. For the 38th figure in each sequence determine which pattern block is at its right end and how many of each type of pattern block it contains. Describe how you reached your conclusion in each case.

Sequence I



1st 2d 3d 4th 5th

6th

★ *Sequence II*

1st 2d 3d 4th 5th 6th

Sequence III

1st 2d 3d 4th 5th 6th

7th