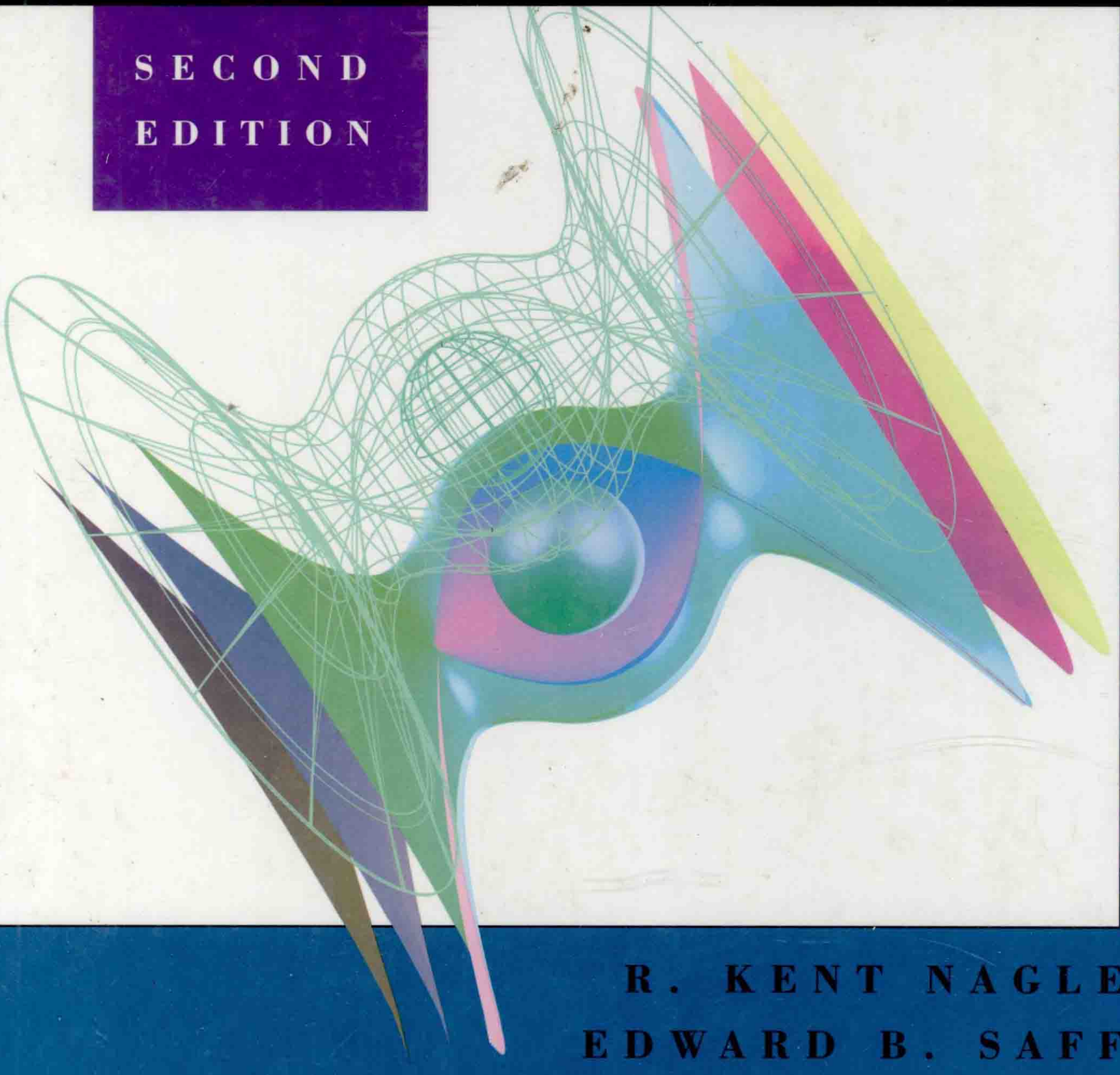
FUNDAMENTALS OF DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS

SECOND EDITION



R. KENT NAGLE
EDWARD B. SAFF

Fundamentals of Differential Equations and Boundary Value Problems

Second Edition

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UNIVERSITY OF SOUTH FLORIDA

with contributions by
A. D. Snider

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In Memoriam: R. Kent Nagle

He has left his imprint not only on these pages but upon all who knew him. He was that rare mathematician who could effectively communicate at all levels, imparting his love for the subject with the same ease to undergraduates, graduates, precollege students, public school teachers, and his colleagues at the University of South Florida.

Kent was at peace in life—a peace that emanated from the depth of his understanding of the human condition and the strength of his beliefs in the institutions of family, religion, and education. He was a research mathematician, an accomplished author, a Sunday school teacher, and a devoted husband and father.

Kent was also my dear friend and my jogging partner who has left me behind still struggling to keep pace with his high ideals.

E. B. Saff

Preface

OUR GOAL

Fundamentals of Differential Equations and Boundary Value Problems is designed to serve the needs of a one- or two-semester course in basic theory as well as applications of differential equations. For this purpose, we have augmented our original text by including chapters on Eigenvalue Problems and Sturm-Liouville Equations, Stability of Autonomous Systems, and Existence and Uniqueness Theory. We have striven to create a flexible text that affords the instructor substantial latitude in designing a syllabus (we provide some sample syllabi later in this preface), in course emphasis (theory, applications and techniques, numerical methods), and in using commercially available computer software and/or the software utility programs specifically designed for this text.

IMPROVEMENTS IN THIS EDITION

In response to requests of users and reviewers, and recognizing recent developments in teaching and learning, we offer the following:

Focus on Analytical, Graphical, and Numerical Methods Certainly these three aspects strengthen conceptual learning. Moreover, current trends in education increasingly reflect the ideology that all three are essential pedagogical ingredients that are best utilized in combination with one another. With this in mind, we have substantially augmented our discussions of theory and methods with many new graphical illustrations and have enhanced both graphical and numerical techniques by providing the reader with a software utility package that is keyed to the text. With these new features, students are free of the burden of computations and graphing and can focus more immediately on interpretative aspects. Also, more Group Projects have been included (see discussion below) since they provide the student with a vehicle for blending analytical, graphical, and numerical methods in the framework of a challenging joint problem-solving activity. At the same time, we have retained a substantial number of practice exercises that are essential for building basic skills.

New Group Projects Twenty percent of the group projects are new. As a result of a national search sponsored by Addison-Wesley, instructors contributed their own most successful classroom-tested group projects (see acknowledgments below). Group projects appear at the end of each chapter and relate to the material covered in the chapter. They often involve a more challenging application, delve deeper into theory, or introduce more advanced topics. New projects include:

Chapter 2	*Torricelli's Law of Fluid Flow* *Equilibrium Points of Autonomous First Order Equations*
Chapter 3	*Aircraft Guidance in a Crosswind* *Bang-Bang Controls*
Chapter 5	*Designing a Landing System for Interplanetary Travel* *Things That Bob* *A Growth Model for Phytoplankton—Part I*
Chapter 9	*Undamped Second Order Systems*
Chapter 12	*Ecosystem on Planet Glia-2* *A Growth Model for Phytoplankton—Part II*

Revised Exercises Twenty percent of the exercises are new, including computer implementation problems as well as exercises that require students to interpret solutions, building conceptual knowledge.

New Figures To help students visualize concepts, dozens of new illustrations have been added, including direction field plots (Section 1.3), graphs of power series solutions (Chapter 8), and three-dimensional plots of solutions to partial differential equations (Chapter 10).

Software New exploratory software that is pedagogically tied to the text is available. It is student friendly, functional, and easy to use. A complete description appears under "Supplements" later in this preface.

World Wide Web Support System Useful supplements to the text and course will be made available on the web, including instructors' suggestions and comments, and a communication link to the authors. Point your web browser to http://www.aw.com/he/Math.

CONTENT CHANGES

- **Direction Fields** Now a separate section, 1.3, that includes analytical examples and exercises dealing with population growth and motion of particles. Graphs illustrate the basic existence-uniqueness theorem.
- **Introduction to Systems of First Order Equations** The concept is introduced much earlier, in Section 4.10, following the basic theory and techniques for second order linear equations. The material is presented in a manner that does not require background in linear algebra.
- **Introduction to Phase Plane Analysis** This section, 5.7, is presented earlier. It introduces the study of trajectories of autonomous systems, critical points, and stability. Chapter 12 contains an in-depth treatment of these topics.

- **Introduction to Laplace Transforms: A Mixing Problem** This new section, 7.1, emphasizes advantages of the Laplace method in solving linear equations when the forcing function is discontinuous. It includes a schematic comparing the Laplace method with the standard method.
- **Chapter 2** The section on linear equations is now presented before exact equations.

PREREQUISITES

While some universities make linear algebra a prerequisite for differential equations, many schools (especially engineering) only require calculus. With this in mind, we have designed the text so that only Chapter 6 (Theory of Higher-Order Linear Differential Equations) and Chapter 9 (Matrix Methods for Linear Systems) require more than high school linear algebra. Moreover, Chapter 9 contains a review section on matrices and vectors as well as specific references for the deeper results used from the theory of linear algebra. We have also written Section 4.10 and Chapter 5 so as to give an introduction to systems of differential equations—including methods of solving, applications, numerical procedures, phase plane analysis, and Poincaré maps—that does not require a background in linear algebra.

SAMPLE SYLLABI

As a rough guide in designing a two-semester syllabus related to this text, we provide two samples that can be used for a sequence of two 15-week courses that meet three hours per week: the first emphasizes applications and computations including phase plane analysis; the second is designed for courses that place more emphasis on theory. As in the original text, Chapters 1, 2, and 4 provide the core for the first-semester course. The rest of the chapters are, for the most part, independent of each other.

	Methods, Computations, and Applications	**Theory and Methods**		
	SEMESTER 1			**SEMESTER 2**
Week	Sections	Sections	Week	Sections
1	1.1, 1.2, 1.3	1.1, 1.2, 1.3	1	9.2, 9.3
2	1.4, 2.2	2.2, 2.3	2	9.4, 9.5
3	2.3, 2.4	2.4, 3.2, 3.4	3	9.6, 9.7
4	3.2, 3.4, 3.5	4.2, 4.3	4	12.1, 12.2
5	3.6, 4.2	4.4, 4.5, 4.6	5	12.3, 12.4
6	4.3, 4.5	4.7, 4.8	6	12.5, 12.6

(continued)

SEMESTER 1			SEMESTER 2	
Week	Sections	Sections	Week	Sections
7	4.6, 4.7, 4.8	4.9, 4.10	7	12.7, 10.2
8	4.9, 4.10	5.7, 6.1	8	10.3, 10.4
9	5.1, 5.2, 5.3	6.2, 6.3	9	10.5, 10.6
10	5.4, 5.6, 5.7	7.2, 7.3	10	10.7, 11.2
11	7.2, 7.3	7.4, 7.5	11	11.3, 11.4
12	7.4, 7.5	7.6, 7.7	12	11.5, 11.6
13	7.6, 7.7	8.2, 8.3	13	11.7, 11.8
14	8.2, 8.3	8.5, 8.6, 8.7	14	13.1, 13.2
15	8.5, 8.6	8.8, 8.9	15	13.3, 13.4

RETAINED FEATURES

Flexible Organization Most of the material is modular in nature to allow for various course configurations and emphasis (theory, applications and techniques, and concepts).

Optional Use of Computer Software The availability of computer packages such as MATHEMATICA®, DERIVE®, and MAPLE® provides an opportunity for the student to conduct numerical experiments and tackle realistic applications that give additional insights into the subject. Consequently, we have added several exercises and projects throughout the text that are designed for the student to employ available software in phase plane analysis, eigenvalue computations, and the numerical solutions of various equations.

Choice of Applications Because of syllabus constraints, some courses will have little or no time for sections (such as those in Chapters 3 and 5) that exclusively deal with applications. Therefore, we have made the sections in these chapters almost completely independent of each other. To afford the instructor even greater flexibility, we have built in a variety of applications in the exercises for the theoretical sections. In addition, we have included many projects that deal with such applications.

Technical Writing Exercises Communication skills are, of course, an essential aspect of professional activities. Yet few texts provide opportunities for the reader to develop these skills. Thus we have added at the end of most chapters a set of clearly marked technical writing exercises which invite students to make documented responses to questions dealing with the concepts in the chapter. In so doing, students are encouraged to make comparisons between various methods and to present examples that support their analysis.

Historical Footnotes Throughout the text historical footnotes are set off by colored daggers (†). These footnotes typically provide the name of the person who developed the technique, the date, and the context of the original research.

Motivating Problem Most chapters begin with a discussion of a problem from physics or engineering that motivates the topic presented and illustrates the methodology.

Chapter Summary and Review Problems All of the main chapters contain a set of review problems along with a synopsis of the major concepts presented.

Computer Graphics Most of the figures in the text were generated on a microcomputer. Computer graphics not only ensure greater accuracy in the illustrations, they demonstrate the use of numerical experimentation in studying the behavior of solutions.

Proofs While more pragmatic students may balk at proofs, most instructors regard these justifications as an essential ingredient in a textbook on differential equations. As with any text at this level, certain details in the proofs must be omitted. When this occurs, we flag the instance and refer readers either to a problem in the exercises or to another text. For convenience, the end of a proof is marked by the symbol ■ End.

Linear Theory We have developed the theory of linear differential equations in a gradual manner. In Chapter 4 (Linear Second Order Equations) we present the basic theory for linear second order equations and discuss various techniques for solving these equations. Higher-order equations are briefly mentioned in this chapter. A more detailed discussion of linear higher-order differential equations is given in Chapter 6 (Theory of Higher-Order Linear Differential Equations). For a beginning course emphasizing methods of solution, the presentation in Chapter 4 is sufficient and Chapter 6 can be skipped.

Numerical Algorithms Several numerical methods for approximating solutions to differential equations are presented along with program outlines that are easily implemented on a microcomputer. These methods are introduced early in the text so that teachers and/or students can use them for numerical experimentation and for tackling complicated applications. All algorithms discussed in the text are implemented on the software package available with this text.

Exercises An abundance of exercises is graduated in difficulty from straightforward, routine problems to more challenging ones. Deeper theoretical questions, along with applications, usually occur toward the end of the exercise sets. Throughout the text we have included problems and projects that require the use of a microcomputer. These exercises are denoted by the symbol . The software specifically designed for use with this text greatly facilitates the solutions to these numerical problems.

Optional Sections These sections can be omitted without affecting the logical development of the material. They are marked with an asterisk in the table of contents. As mentioned earlier, the sections in Chapters 3 and 5 are almost completely independent of each other.

Laplace Transforms We provide a detailed chapter on Laplace transforms since this is a recurring topic for engineers. Our treatment emphasizes discontinuous forcing terms and includes a section on the Dirac delta function.

Power Series Power series solutions is a topic that occasionally causes student anxiety. Possibly, this is due to inadequate preparation in calculus where the more subtle subject of convergent series is (not infrequently) covered at a rapid pace. Our solution has been to provide a thorough treatment of power series solutions that also includes a review of their properties as well as a discussion of real analytic functions. Unlike many texts, this one provides an extensive section on the *method of Frobenius* (Section 8.6) and two sections on the various methods for finding a second linearly independent solution.

While we have given considerable space to power series solutions, we have also taken great care to accommodate the instructor who only wishes to give a basic introduction to the topic. *An introduction to solving differential equations using power series and the method of Frobenius can be accomplished by covering the materials in Section 8.2, Section 8.3, and Section 8.6.*

Partial Differential Equations An introduction to this subject is provided in Chapter 10, which covers the method of separation of variables, Fourier series, the heat equation, the wave equation, and Laplace's equation. Examples in two and three dimensions are included.

SUPPLEMENTS

Student Solutions Manual By Anne Kusmierczyk and E. B. Saff. Containing complete, worked-out solutions to selected exercises, this provides students with an excellent study tool. ISBN 0-201-80877-3

Instructor's Resource Guide Contains short answers to all exercises and additional group projects. ISBN 0-201-80876-5

Software Available free to adopters, this DOS-based software is student-friendly, requiring little learning time. It is designed not only to aid the student in solving numerical problems in the text, but also as a convenient utility to encourage students to experiment. The software includes direction field plots, improved Euler's method, Runge-Kutta methods (also for systems), phase plane diagrams, eigenvalues and eigenvectors, computational methods such as Newton's method for solving equations, Simpson's rule, plots and tabulation of one or several functions, and many other useful algorithms. ISBN 0-201-82688-7

World Wide Web Support System Useful supplements to the text and course will be made available on the web, including instructors' suggestions and comments, and a communication link to the authors. Point your web browser to http://www.aw.com/he/Math.

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The newly developed software that accompanies this text was expertly developed by Emil Moskona and Spas Tashev of the Bulgarian Academy of Sciences. Their efforts not only enhance the usefulness of this text, but also provide a convenient utility for calculus/differential equations problem solving.

We are grateful for the encouragement we received from our students and staff here at the University of South Florida. Maria Carvalho, through her untiring efforts and devotion

to duty, somehow lent order to an otherwise chaotic existence during the creation of this edition.

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E. B. SAFF
TAMPA, FLORIDA

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*Denotes optional sections that can be omitted without compromising the logical flow.