Biomedical Ultrasonics

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BIOMEDICAL ULTRASONICS

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1977

Academic Press
London · New York · San Francisco
A Subsidiary of Harcourt Brace Jovanovich, Publishers

ACADEMIC PRESS INC. (LONDON) LTD
24–28 Oval Road
London NW1 7DX

U.S. Edition published by
ACADEMIC PRESS INC.
111 Fifth Avenue
New York, New York 10003

Library of Congress Catalog Number: 77–071842
ISBN: 0–12–742940–9

Printed in Great Britain by
Adlard & Son Ltd., Bartholomew Press, Dorking, Surrey

PREFACE

The importance of biomedical ultrasonics has increased greatly during the six years which I have spent working on this book. The impact which ultrasonic methods now have on health care exceeds all early expectation. As an academic pursuit, the study of biomedical ultrasonics has attracted many accomplished scientists. I have the privilege of being acquainted with many of the doctors, physicists, biologists and engineers whose work I have tried to describe. I hope that *Biomedical Ultrasonics* will contribute to the development of the subject by serving as the primary source of reference for everyone interested in the basic principles and applications of ultrasonic energy in medicine and biology.

February 1977 P. N. T. Wells

ACKNOWLEDGEMENTS

Many people contributed, wittingly or unwittingly, to the preparation of *Biomedical Ultrasonics*. Amongst those to whom I am especially grateful are Jim Aller, Peter Atkinson, Tom Brown, Maurice Bullen, Geoffrey Dawes, Gilbert Devey, Ian Donald, Ken Evans, John Fleming, Douglas Follett, Özgen Feizi, Herbert Freundlich, Ed Guibarra, Angus Hall, Michael Halliwell, Kit Hill, David Hughes, Jack Angell James, Ciaran McCarthy, Ken McCarty, Steve Morris, Dick Mountford, Robin Poore, Alan Read, Jack Reid, Frank Ross, Bob Skidmore, Shelley Stuart, Annette Campbell White, Denis White and John Woodcock. Anne Davies, Lily Davies, Rosemary Cardwell and Josephine Reynolds skilfully typed the manuscript. It is a pleasure to acknowledge both the help and the courtesy of the staff of Academic Press, who have worked hard and patiently to publish this book.

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1. WAVE FUNDAMENTALS*

1.1 SIMPLE HARMONIC MOTION

Consider the situation represented in Fig. 1.1. A particle of mass m is supported on a surface free from friction, and attached to a weightless spring of compliance ζ. If the particle is displaced from its equilibrium position, a restoring force F acts on the particle, given by, according to Hooke's law:

$$F = -u/\zeta \tag{1.1}$$

where u is the displacement amplitude. The restoring force acts on the particle in such a direction as to return it to its equilibrium position. The magnitude of this force is proportional to the amplitude of the displacement (i.e. the distance between the position of the particle and its equilibrium position). This *direct proportionality* is the feature of simple harmonic motion which distinguishes it from other, more complicated vibrations.

In simple harmonic motion there is always an equilibrium situation at which the oscillating system could remain at rest. Although the oscillations dealt with in this Section are of the mechanical type, analogous oscillations occur in electrical circuits (see Appendix 1).

Applying Newton's second law of motion to Eqn. 1.1:

$$-u/\zeta = ma = m\frac{d^2u}{dt^2} \tag{1.2}$$

where a is the acceleration of the particle at time t. Equation 1.2 may be rearranged thus:

$$\frac{d^2u}{dt^2} + \frac{1}{\zeta m}u = 0 \tag{1.3}$$

The dimensions of $1/\zeta\, m$ are $[T^{-2}]$. In a vibrating system, the particle completes one cycle of oscillation in a period τ, which has dimensions $[T]$; the frequency, f (i.e. the number of cycles in unit time) $= 1/\tau$. The dimen-

* Generally the results presented in this Chapter are derived from first principles. Where results are quoted without derivation or reference to other works, a published text on acoustics may be consulted. These texts include those of Blitz (1963), Gooberman (1968), Hueter and Bolt (1955) and Kinsler and Frey (1962).

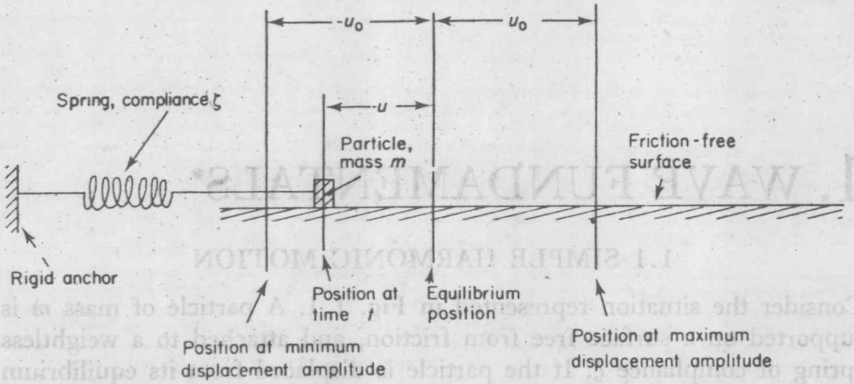

FIG. 1.1 Spring-particle system in simple harmonic motion.

sions of $1/\zeta m$ are therefore the same as those of f^2. Hence, Eqn. 1.3 may be rewritten:

$$\frac{d^2u}{dt^2}+(bf)^2u=0 \tag{1.4}$$

where $(bf)^2=1/\zeta m$, b being a constant, and f being the frequency at which the particle oscillates. There are two possible solutions to Eqn. 1.4: these are:

$$u=A\cos bft=A\cos \omega t \tag{1.5}$$

and

$$u=B\sin bft=B\sin \omega t \tag{1.6}$$

These solutions satisfy Eqn. 1.4 because, from Eqn. 1.5;

$$\frac{d^2u}{dt^2}=-A\omega^2\cos \omega t$$

and, from Eqn. 1.6;

$$\frac{d^2u}{dt^2}=-B\omega^2\sin \omega t$$

where A and B are constants with the same dimensions as u, and $\omega=bf=2\pi f$, where ω is defined as the angular frequency of the system.

The general solution to Eqn. 1.4 is given by superposition of the values of u in Eqns. 1.5 and 1.6:

$$u=A\cos \omega t+B\sin \omega t \tag{1.7}$$

Equation 1.7 becomes, if A is rewritten as $u_0\sin\phi$, and B, as $-u_0\cos\phi$, where $u_0=(A^2+B^2)^{1/2}$, and ϕ is a constant:

$$u=u_0\sin\phi\cos\omega t-u_0\cos\phi\sin\omega t$$
$$=u_0\sin(\omega t-\phi) \tag{1.8}$$

Expressed in words, Eqn. 1.8 means that the displacement amplitude u of a particle in simple harmonic motion from its equilibrium position is equal, at any time t, to the product of its peak displacement amplitude u_0, and the sine of a time-varying angle $(\omega t-\phi)$ in which ω is the angular frequency $2\pi f$ of the oscillation of the particle, and ϕ is the phase angle; ϕ defines the position in the cycle of oscillation at $t=0$.

Velocity is equal to rate of change of position. Hence the particle velocity may be found by differentiating Eqn. 1.8:

$$v=\frac{du}{dt}=u_0\omega\cos(\omega t-\phi) \tag{1.9}$$

Similarly, particle acceleration may be found by differentiating Eqn. 1.9;

$$a=\frac{dv}{dt}=-u_0\omega^2\sin(\omega t-\phi) \tag{1.10}$$

The significance of the negative sign in Eqn. 1.10 is that the particle is decelerating as it moves away from its equilibrium position.

In this simple analysis, it has been assumed that the total energy stored in the oscillating system remains constant. (The effect of energy dissipation in such a system is discussed in Section 1.4.) The energy is stored entirely in the kinetic energy of the particle when the spring is not stressed (i.e. when $u=0$); likewise, the energy is stored entirely in the potential energy of the spring when the displacement of the particle is maximum or minimum (i.e. when $v=0$). At other times during the cycle, the energy is shared between the kinetic and potential stores. At any time t,

potential energy stored in spring

$$=\int_0^u u/\zeta\,du=u^2/2\zeta \tag{1.11}$$

kinetic energy stored in mass

$$=mv^2/2 \tag{1.12}$$

total energy stored in spring and mass

$$=u_0^2/2\zeta=m(u_0\omega)^2/2=e \tag{1.13}$$

1.2 THE WAVE EQUATION

A wave is a disturbance, the position of which in space changes with time.

1.2.a Transverse Waves

For example, consider a long thin string (which is really a chain of particles), fixed at one end. The other end is attached to a vibrator, so that it moves with simple harmonic motion along a line perpendicular to the undisturbed position of the string. The vibrations move along the string: in this way, energy is transmitted at a finite velocity. This type of vibration is called a *travelling wave*. The situation is illustrated in Fig. 1.2. Figure 1.2(a) represents the variation in displacement in space (where z is the distance) at any instant in time t. The *wavelength*, λ, is the z-distance between consecutive particles where the displacement amplitudes are identical. Figure 1.2(b) represents the variation in displacement in time at any particular position in space z. The *period*, τ, is the time which is required for the wave to move forward a distance λ. During the time τ, the wave completes one cycle of oscillation. The *frequency*, f, of the wave is equal to the number of cycles which pass through a given point in space in unit time. Thus:

$$f = 1/\tau \tag{1.14}$$

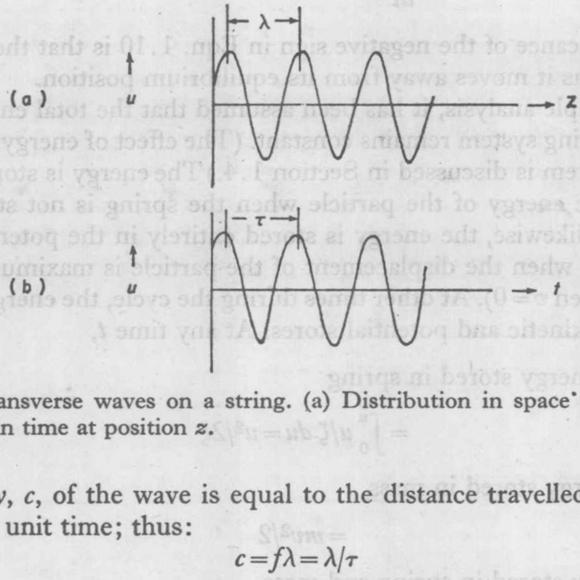

FIG. 1.2 Transverse waves on a string. (a) Distribution in space at time t; (b) distribution in time at position z.

The *velocity*, c, of the wave is equal to the distance travelled by the disturbance in unit time; thus:

$$c = f\lambda = \lambda/\tau \tag{1.15}$$

The kind of wave which occurs on a string is called a *transverse wave*, because the particles oscillate in a direction normal to the direction in which the wave travels.

Next, consider the displacement of a very short section of the string, as illustrated in Fig. 1.3. The section is under constant tension ψ, and has a length δl given by

$$\delta l^2 = \delta z^2 + \delta u^2$$

hence

$$\delta l = \left\{1 + \left(\frac{\delta u}{\delta z}\right)^2\right\}^{1/2} \delta z$$

and if $\delta u/\delta z$ is very small, $\delta l = \delta z$. The force acting in the u-direction on the element of string is equal to $\psi\{\sin(\theta+\delta\theta) - \sin\theta\}$; and, if θ is very small, $\sin\theta = \tan\theta = (\delta u/\delta z)_z$, where the subscript indicates the point at which the corresponding gradient is evaluated. Hence, the force is equal to

$$\psi\left\{\left(\frac{\partial u}{dz}\right)_{z+\delta z} - \left(\frac{\partial u}{dz}\right)_z\right\} \xrightarrow[\lim \delta z = 0]{} \psi\frac{\partial^2 u}{\partial z^2}\delta z$$

because the difference between the two terms in the bracket on the left-hand side of the expansion defines the differential coefficient of the gradient $\partial u/\partial z$ times the space interval δz.

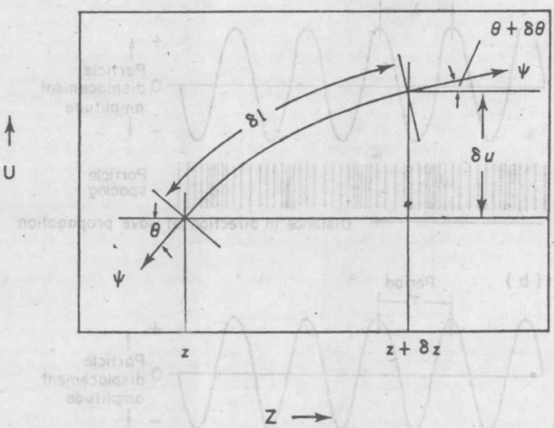

FIG. 1.3 An element of a string supporting a transverse wave.

If η is the mass per unit length of the string, then, according to Newton's second law of motion:

$$\psi\frac{\partial^2 u}{\partial z^2}\delta z = \eta\delta z\frac{\partial^2 u}{\partial t^2}$$

and hence

$$\frac{d^2 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{1.16}$$

where $c = (\psi/\eta)^{1/2}$, which has dimensions $[LT^{-1}]$ and is a velocity. Equation 1.16 is a *wave equation.* It relates the second differential of the participle displacement with respect to distance, to the acceleration of a simple harmonic oscillator. The significance of this is discussed in Section 1.3.

1.2.b Longitudinal Waves

Equation 1.16 was derived for transverse waves. Another *wave mode* occurs when the particles* in a medium supporting a wave oscillate backwards and forwards in the same direction as that in which the wave is travelling. This is called a *longitudinal wave*. The oscillation of the particles sets up periodic variations in pressure within the supporting medium and a pressure wave travels through the medium as neighbouring particles interact with each other, as illustrated in Fig. 1.4. Consider an unlimited liquid within which a plane is subjected to simple harmonic motion. (Non-viscous fluids cannot support shear stress, and so transverse waves cannot be generated.) The situation is illustrated in Fig. 1.5, which shows a small element of the liquid. In Fig. 1.5(a) the element is in equilibrium: it has

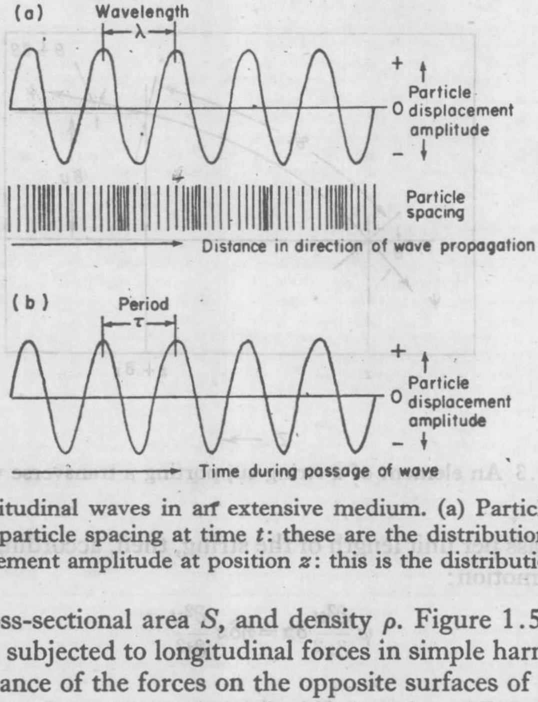

FIG. 1.4 Longitudinal waves in an extensive medium. (a) Particle displacement amplitude and particle spacing at time t: these are the distributions in space; (b) particle displacement amplitude at position z: this is the distribution in time.

length δz, cross-sectional area S, and density ρ. Figure 1.5(b) shows the element when subjected to longitudinal forces in simple harmonic motion: the out-of-balance of the forces on the opposite surfaces of the element is represented by a force δF on the right-hand surface. This force produces a displacement of $u+\delta u$ in the z-position of the right-hand surface. There is a gradient of force across the element; this is approximately linear because the element is small, and it is equal to $\partial F/\delta z$. Therefore,

$$\delta F = \frac{\partial F}{\partial z}\,\delta z \tag{1.17}$$

*In this context, a *particle* is a volume element which is large enough to contain many millions of molecules, so that it is continuous with its surroundings; but it is so small that quantities variable within the medium (such as displacement amplitude) are constant within the particle.

According to Hooke's law,

$$F = KS\frac{\partial u}{\partial z} \tag{1.18}$$

where K is the bulk modulus of the liquid; so that

$$\frac{\partial F}{\partial z} = KS\frac{\partial^2 u}{\partial z^2} \tag{1.19}$$

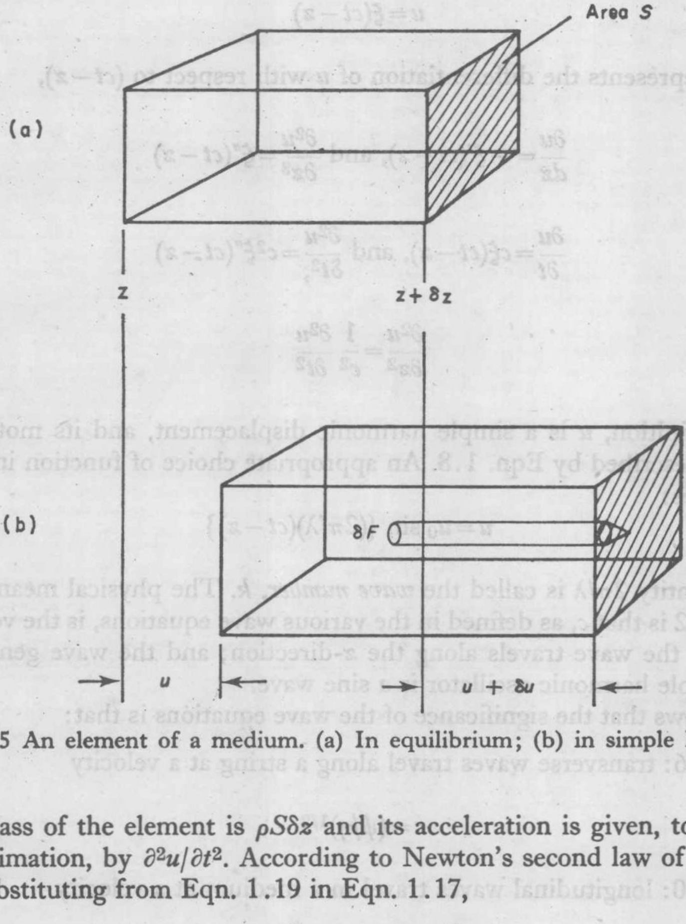

FIG. 1.5 An element of a medium. (a) In equilibrium; (b) in simple harmonic motion.

The mass of the element is $\rho S \delta z$ and its acceleration is given, to a close approximation, by $\partial^2 u/\partial t^2$. According to Newton's second law of motion, and substituting from Eqn. 1.19 in Eqn. 1.17,

$$KS\frac{\partial^2 u}{\partial z^2}\delta z = \rho S \delta z\frac{\partial^2 u}{\partial t^2}$$

and hence

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{1.20}$$

where $c=(K/\rho)^{1/2}$. Equation 1.20 is the wave equation for longitudinal waves; it is identical in form to the Eqn. 1.16 which is the wave equation for transverse waves. Its significance is discussed in Section 1.3.

1.3 SOLUTION OF THE WAVE EQUATION

The wave equation, as expressed in Eqns. 1.16 and 1.20, is satisfied by a function ξ of the form

$$u=\xi(ct-z) \tag{1.21}$$

If ξ' represents the differentiation of u with respect to $(ct-z)$,

$$\frac{\partial u}{dz}=-\xi'(ct-z), \text{ and } \frac{\partial^2 u}{\partial z^2}=\xi''(ct-z)$$

also

$$\frac{\partial u}{\partial t}=c\xi(ct-z), \text{ and } \frac{\partial^2 u}{\delta t^2;}=c^2\xi''(ct-z)$$

hence

$$\frac{\partial^2 u}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

By definition, u is a simple harmonic displacement, and its motion at $z=0$ is described by Eqn. 1.8. An appropriate choice of function in Eqn. 1.21 gives:

$$u=u_0 \sin\{(2\pi/\lambda)(ct-z)\} \tag{1.22}$$

The quantity $2\pi/\lambda$ is called the *wave number*, k. The physical meaning of Eqn. 1.22 is that c, as defined in the various wave equations, is the velocity at which the wave travels along the z-direction; and the wave generated by a simple harmonic oscillator is a sine wave.

It follows that the significance of the wave equations is that:

Eqn. 1.16: transverse waves travel along a string at a velocity

$$c=(\psi/\eta)^{1/2} \tag{1.23}$$

Eqn. 1.20: longitudinal waves travel in a medium at a velocity

$$c=(K/\rho)^{1/2} \tag{1.24}$$

Some typical values of longitudinal wave velocity in non-biological materials are given in Table 1.1. The variation with temperature of the velocity in water is shown in Fig. 1.6.

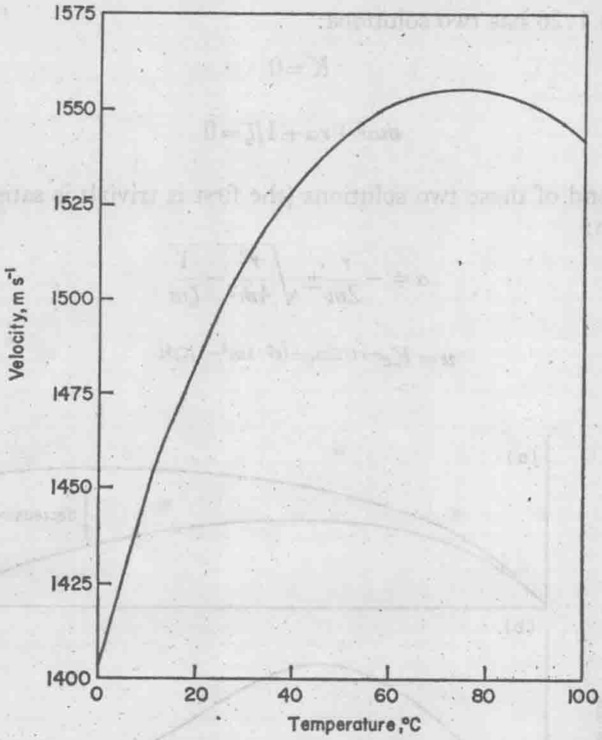

FIG. 1.6 Variation of propagation velocity with temperature in water. (Data of Grosso and Mader, 1972.)

1.4 DAMPED SIMPLE HARMONIC MOTION

In the lossless system discussed in Section 1.1, the peak displacement u_0 of the particle is constant. However, in practice such an ideal system cannot be realised, and energy is dissipated by processes such as friction, or imperfect elasticity. This results in an additional force, which is generally proportional to the velocity of the particle, so that Eqn. 1.3 must be modified thus:

$$\frac{d^2u}{dt^2}m+\frac{du}{dt}r+\frac{1}{\zeta}u=0. \qquad (1.25)$$

where r is a constant with the dimensions of force per unit velocity.

Equation 1.25 can be solved by putting $u=Ke^{\alpha t}$; it follows that $du/dt=\alpha Ke^{\alpha t}$, and $d^2u/dt^2=\alpha^2Ke^{\alpha t}$. Hence:

$$Ke^{\alpha t}(m\alpha^2+r\alpha+1/\zeta)=0 \qquad (1.26)$$