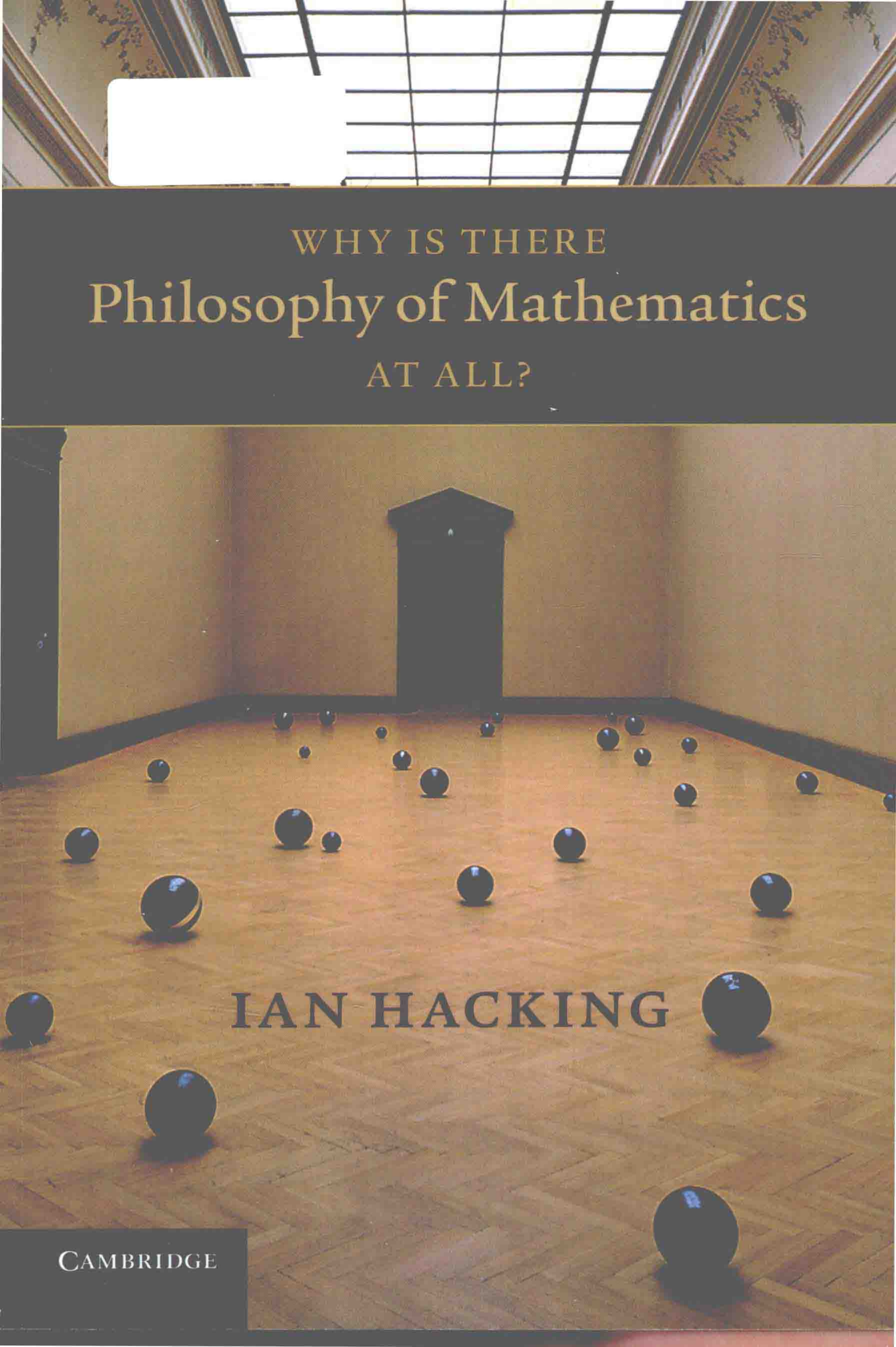
WHY IS THERE

Philosophy of Mathematics

AT ALL?



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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

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www.cambridge.org
Information on this title: www.cambridge.org/9781107658158

First published 2014
Reprinted 2014

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-05017-4 Hardback
ISBN 978-1-107-65815-8 Paperback

WHY IS THERE PHILOSOPHY OF MATHEMATICS AT ALL?

This truly philosophical book takes us back to fundamentals – the sheer experience of proof, and the enigmatic relation of mathematics to nature. It asks unexpected questions, such as 'What makes mathematics mathematics?', 'Where did proof come from and how did it evolve?', and 'How did the distinction between pure and applied mathematics come into being?' In a wide-ranging discussion that is both immersed in the past and unusually attuned to the competing philosophical ideas of contemporary mathematicians, it shows that proof and other forms of mathematical exploration continue to be living, evolving practices – responsive to new technologies, yet embedded in permanent (and astonishing) facts about human beings. It distinguishes several distinct types of application of mathematics, and shows how each leads to a different philosophical conundrum. Here is a remarkable body of new philosophical thinking about proofs, applications, and other mathematical activities.

IAN HACKING is a retired professor of the Collège de France, Chair of Philosophy and History of Scientific Concepts, and retired University Professor of Philosophy at the University of Toronto. His most recent books include *The Taming of Chance* (1990), *Rewriting the Soul* (1995), *The Social Construction of What?* (1999), *An Introduction to Probability and Inductive Logic* (2001), *Mad Travelers* (2002), and *The Emergence of Probability* (2006).

In memory of the first reader of this book, 1960
Paul Whittle 1938–2009

For mathematics is after all an anthropological phenomenon.
Wittgenstein (1978: 399)

Mathematical activity is human activity . . . But mathematical activity produces mathematics. Mathematics, this product of human activity, 'alienates itself' from the human activity which has been producing it. It becomes a living, growing organism.

(Lakatos 1976: 146)

The birth of mathematics can also be regarded as the discovery of a capacity of the human mind, or of human thought – hence its tremendous importance for philosophy: it is surely significant that, in the semilegendary intellectual tradition of the Greeks, Thales is named both as the earliest of the philosophers and the first prover of geometric theorems.

(Stein 1988: 238)

A square can be dissected into finitely many unequal squares, but a cube cannot be dissected into finitely many unequal cubes. *Proof of the latter:*

In a square dissection the smallest square is not at an edge (for obvious reasons). Suppose now a cube dissection does exist. The cubes standing on the bottom face induce a square dissection at that face, and the smallest of the cubes on that face stands on an internal square. The top face of this cube is enclosed by walls; cubes must stand on this top face; take the smallest – the process continues indefinitely.

(Littlewood 1953: 8)

Foreword

This is a book of philosophical thoughts about proofs, applications, and other mathematical activities.

Philosophers tend to emphasize mathematical 'knowledge', but as G. H. Hardy said on the first page of his *Apology* (1940), 'the function of a mathematician is to *do* something, to prove new theorems, to add to mathematics'. I have emphasized the 'do'. Hardy was writing not only an *Apologia pro vita sua*, but also a mathematician's *Lament* that he was now too old to create much more mathematics. He also, notoriously, wanted to keep mathematics pure, whereas I believe that the uses, 'the applications', are as important as the theorems proved. Neither proof nor application is, however, as clear and distinct an idea as might be hoped.

To reflect on the doing of mathematics, on mathematics as activity, is not to practise the sociology of mathematics. Happily that is now a burgeoning field, from which one can learn much, but what follows is philosophizing, moved by old-fashioned questions – to which I add my title question, why *do* these questions arise perennially, from Plato to the present day?

This book began as the René Descartes Lectures at Tilburg University in the Netherlands, in early October, 2010. (I started writing out the talks on the summer solstice of that year.) The format was three lectures, each followed by comments from two different scholars. The original intention was that the lectures and comments would be published immediately.

I began to realize at the end of the week the extent to which the material needed to mature. The commentators generously agreed to keep their comments. So my first duty is to thank them deeply for their hard work. Hard work? Typically they received, late in the day, some 20,000 words per lecture, of which only 7,000 would be spoken, and they did not even know which ones.

> For the first talk, 'Why Is There Philosophy of Mathematics?': Mary Leng and Hannes Leitgeb.

For the second talk, 'Meaning and Necessity – and Proof': James Conant and Martin Kusch.
For the third talk, 'Roots of Mathematical Reasoning': Marcus Giaquinto and Pierre Jacob.

Thank you all.

I originally proposed 'Proof' as the series title. That was the title of a thesis, which, together with some work in modal logic, was awarded a PhD by Cambridge University in 1962. It was dominated by my reading of Wittgenstein's recently published *Remarks on the Foundations of Mathematics*, although much influenced by what was to become Imre Lakatos' *Proofs and Refutations*, which was being completed in Cambridge as a doctoral dissertation when I began mine.

I have published very little about the philosophy of mathematics, but it has always been at the back of my mind, so the Descartes Lectures were a chance to finish the job. The title 'Proof' would give no idea of what the talks would be about, so Stephan Hartmann, the organizer of the events (to whom many thanks), and I hit on 'Proof, Calculation, Intuition, and A Priori Knowledge'.

Very soon after the Descartes Lectures, in late October 2010, I gave three similar talks at the University of California, Berkeley, beginning with the Howison Lecture, 'Proof, Truth, Hands and Mind'. Here is how I explained the title, after indulgently admiring my choice of words of one syllable:

> Why this title? First, because *proof* has been an essential part of Western mathematics ever since Plato. And Plato thought that mathematics was the sure guide to *truth*. I want also to think of how we do mathematics, in a material way that Plato would hardly have acknowledged. We think with our *hands*, our whole bodies. We communicate with one another not only by talking and writing but also by gesticulating. If I am thinking mathematically I may draw a diagram to take you through a series of thoughts, and in this way pass my thoughts in my *mind* over to yours.

After California I put this material aside while teaching on other topics at the University of Cape Town, and intensely experiencing all too little of that amazing land and its peoples. In January 2011 I did attend the annual meetings of the Philosophical Society of Southern Africa, and the corresponding Society for the Philosophy of Science, near Durban. There I presented, respectively, abridged forms of the first two Descartes/Howison Lectures (Hacking, 2011a, 2011b). I may mention also a contribution to a conference in Israel in honour of Mark Steiner, in December 2011, which began with

Pythagoras and ended with P. A. M. Dirac (Hacking, 2012b). Then in November 2012 I did part of the third Descartes Lecture as the Henry Myers Lecture for the Royal Anthropological Institute, London.

In March and April of 2012 I gave six Gaos Lectures at the National Autonomous University of Mexico, at the invitation of Carlos Lópes Beltrán and Sergio Martinez, to whom again many thanks. The title was *The Mathematical Animal*, but in fact the first five lectures covered only the first Descartes Lecture. And so it has come to pass that this book is not the entire set of lectures given in Tilburg, but only the first.

The connection between the present book and my dissertation of 1962 will not be obvious, but *plus ça change*. My title here is, *Why Is There Philosophy of Mathematics At All?* I was astonished, in preparing the present book for the press, to reread the brief preface to my dissertation of 1962: 'We must return to simple instances to see what is surprising, to discover, in fact, why there are philosophies of mathematics at all.' And I may mention that my choice of topics comes from the first edition of Wittgenstein's *Remarks on the Foundations of Mathematics* (1956). The two significant nouns most often used in that edition (to which I prepared my own index) are *Beweis* and *Anwendung*, 'proof' and 'application'.

I thank the Social Science and Humanities Research Council of Canada for awarding me its annual Gold Medal for Research. The cash coming with the medal is rightly dedicated to further research, and much of it was used in preparing this book. I thank James Davies in Toronto and Kaave Lajevardi in Teheran for a lot of help in the home stretch. The final threads were tied up in March 2013 during a blissful time at the Stellenbosch Institute for Advanced Study.

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