

# Oskar Lange

## Introduction to Econometrics

4th Edition, revised and enlarged



Pergamon Press  
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# INTRODUCTION TO ECONOMETRICS

by  
**OSKAR LANGE**

Prepared with the collaboration of  
Antoni Banasiński  
on the basis of lectures delivered  
at the University of Warsaw

*Fourth edition*  
*Revised and enlarged*

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## PREFACE TO THE FIRST EDITION

This book is the outcome of lectures given at the Faculty of Political Economy of the University of Warsaw. My lectures were systematically noted by A. Banasiński, M. A., who then helped me to reproduce them, edit them stylistically and supplement them with some examples, and I wish to express here my heartfelt thanks to him.

I was induced to issue these lectures in book form by the conviction that econometric methods can be applied in the socialist economy as well, and that they constitute a necessary instrument for efficient planning and management of the socialist economy. This is especially true of the methods described in Chapter III which treat of the theory of programming. The approach presented in this chapter is, to a great extent, the result of my own work, aiming to link the theory of programming with Marxist political economy and with national economic planning. For these reasons, I have devoted particular attention to dynamic programming and to efficiency of investment. I first presented this approach in 1956 in lectures delivered at the Indian Statistical Institute in Calcutta and published under the title *Some Observations on Input-Output Analysis* in "Sankhya, Indian Journal of Statistics", 1957. (A French translation appeared in "Cahiers de Science Economique Appliquée", no. 49, Paris 1957; a Russian translation will soon be published in "Ucheniye zapiski po statistiky"). In this book that paper has been extended considerably and sup-

plemented with new results, particularly on the subject of efficiency of investment.

Chapter II, which deals with market analysis, treats of the new classical methods of studying supply and demand. I believe that these methods are applicable in the socialist economy as well, especially as far as analysis of the elasticity of demand is concerned.

Chapter I, on the study and forecasting of business cycles, refers to questions of the capitalist economy. I spoke of these questions in the lectures since they constitute part of general economic education. Moreover, we can employ a number of statistical methods, such as studies of seasonal fluctuations, which were developed in connection with studies of business cycles.

I hope that this book will prove useful in connection with our endeavours to improve the planning and management of our national economy.

*Oskar Lange*

Warsaw, July 31, 1957.

## PREFACE TO THE SECOND EDITION

Since the first edition was published four years ago this book has aroused widespread interest. First an English translation appeared (Państwowe Wydawnictwo Naukowe and Pergamon Press), then Italian and Serbo-Croat. French and Japanese versions are in preparation.

Similar interest has been shown in the paper *Some Observations on Input-Output Analysis* which, for the first time, presented the basic ideas of Chapter III of this book.

Besides the French translation of this paper mentioned in the Preface to the First edition, this paper has since appeared in Russian, Slovak and Spanish. The Russian translation was issued in the collection *Primenenie Matematiki w Ekonomicheskikh Issledovaniakh (Application of Mathematics in Economic Research)*, edited by Academician V. S. Nemchinov, which was published in Moscow in 1959. The Spanish translation has been issued in Mexico.

This interest is unquestionably linked with the growing interest in economic planning. And the main idea in this book is the conviction that econometric methods are a useful instrument of economic planning and, hence, can be especially useful in a socialist economy.

At the time when this book was being prepared (in 1956 and 1957) there were many objections to this idea. Today, to a great extent, these objections and doubts have disappeared. This is indicated by the appreciable number of econometric and mathematical-economic studies appearing

at present in the Soviet Union and in a number of People's Democracies, particularly in Hungary. Electronic mathematical machines have played a big role in this matter as they make possible the practical application of econometric methods to the planning and management of socialist economy.

As I have stated in my book, an aspect of econometric and mathematical economics which is of particular importance to socialist economy is the theory of programming. The first conceptions of linear programming, it turns out, were born in the Soviet Union in 1939, their author being Professor L. V. Kantorovich. Today, Professor Kantorovich's methods hold an important place in Soviet economic literature.

The second edition of the *Introduction to Econometrics* does not differ much from the first. The changes are primarily editorial and are intended to increase the clarity and precision of the exposition (e.g. the explanation of the fundamentals of the least squares method). Chapter III has the largest number of changes of this type. The terminology in this Chapter has been subjected to quite a far-reaching change and certain formulations have been made more precise. Several errors have also been corrected.

This present edition also has a number of additions. Among them there are tables of Polish input-output relations in 1957, 1958 and 1959, drawn up by the Central Statistical Office. As an appendix I have added an expanded version of my article *The Economic and Technological Foundations of the Efficiency of Investment*, published in „*Ekonomista*”. This appendix develops further the idea of Chapter III of this book and is a supplement to it. A Russian version was published in the second volume

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of *Primenienye matematyki v ekonomicheskikh issledovaniakh*, Moscow 1961. Another appendix is *On the Dependence between Investment and Growth of Production*, written by Krzysztof Porwit. This appendix was included in the first English edition of this book. It constitutes an attempt to determine numerical values for some formulae presented in the book. Dr Zbigniew Pawłowski aided me in editing and proof-reading the English version of the present edition for which I want to express my thanks.

The first Polish edition of this book was sold out quickly and so was the first English edition.

I hope that the present edition will also meet with the approval of readers.

*Oskar Lange*

Warsaw in April 1962



## INTRODUCTION

### THE OBJECT OF ECONOMETRIC STUDIES

The purpose of these lectures is to review the principal econometric problems and the methods employed to solve them. Econometric studies today cover widespread fields of economic life and embrace a variety of different problems. In this short course we shall deal only with some selected aspects of econometrics, making the selection according to two criteria. Firstly, we shall discuss problems (in the development of which econometrics has played a major role), which have entered professional literature and which constitute a component part of the general training in this field. Secondly, we shall pay particular attention to those econometric problems and methods of research which may be of practical application in the planning and management of the socialist economy.

What is econometrics and what is the object of its research? To put it in a nutshell we can say that *econometrics is the science which deals with the determination by statistical methods of concrete quantitative laws occurring in economic life.*

It follows from this definition that econometrics differs from both the theory of economics and economic statistics.

Econometrics differs from economic theory in that it tries to picture statistically, with the aid of concrete quantitative relations, those laws with which economic



theory deals in a general and schematic manner. Let us elucidate this with two examples.

Political economy studies, among others, the laws of the process of reproduction and determines the laws (relations) connecting certain economic quantities. It establishes, for instance, the relation between the rates at which the means of production and the means of consumption increase. Econometrics, on the other hand, tries to determine the quantitative character of these relationships in specific economic conditions, e.g. Poland in 1957.

Similarly, economic theory studies the impact of changes in the national income or the price of a given commodity on the demand. It finds that when national income rises, the demand for most commodities also rises. The demand for a given commodity, in general, also increases when its price is reduced. In connection with these studies there is introduced the concept of elasticity of demand with respect to national income or to the price of the given commodity and general relations are determined between the quantities under study.

Econometrics tries to establish the relations which exist between the quantities mentioned in the second example quantitatively and concretely in the given economic situation. We can by econometric methods, for instance, study the influence of the changes made in 1957 in the prices of periodicals in Poland on the number of copies sold or the influence of the growth in the national income planned for 1958 on the demand for meat.

Problems of econometrics are solved by means of statistical research and methods. Economic statistics, therefore, provides econometrics with factual material which enables it to determine the quantitative laws linking together