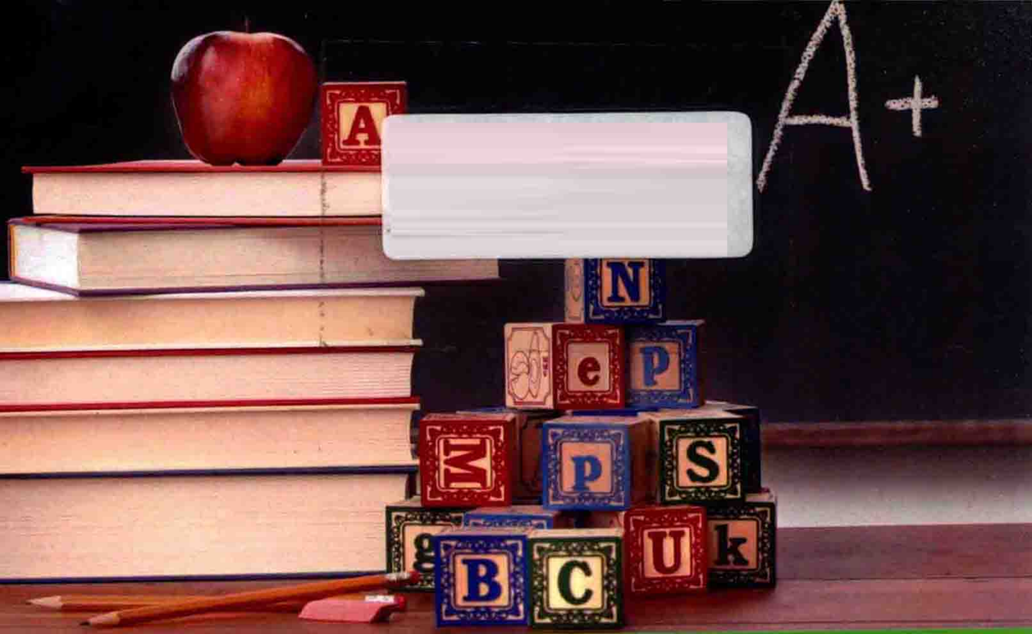

Prof. Dr Satyanarayana Bhavanari
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DIMENSION OF N-GROUPS AND FUZZY IDEALS IN GAMMA NEARRINGS (Monograph)

Nearring, Finite Goldie Dimension, N-group, Matrix Nearring, Hypercube, IFP Ideal, Fuzzy Ideal

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Impressum/Imprint (nur für Deutschland/ only for Germany)
Bibliografische Information der Deutschen Nationalbibliothek: Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.d-nb.de abrufbar.

Coverbild: www.ingimage.com

Verlag: VDM Verlag Dr. Müller GmbH & Co. KG
Dudweiler Landstr. 99, 66123 Saarbrücken, Deutschland
Telefon +49 681 9100-698, Telefax +49 681 9100-988
Email: info@vdm-verlag.de
Zugl.: Nagarjuna Nagar, Acharya Nagarjuna University, 2000

Herstellung in Deutschland:
Schaltungsdienst Lange o.H.G., Berlin
Books on Demand GmbH, Norderstedt
Reha GmbH, Saarbrücken
Amazon Distribution GmbH, Leipzig
ISBN: 978-3-639-36838-3

Imprint (only for USA, GB)
Bibliographic information published by the Deutsche Nationalbibliothek: The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Cover image: www.ingimage.com

Publisher: VDM Verlag Dr. Müller GmbH & Co. KG
Dudweiler Landstr. 99, 66123 Saarbrücken, Germany
Phone +49 681 9100-698, Fax +49 681 9100-988
Email: info@vdm-publishing.com

Printed in the U.S.A.
Printed in the U.K. by (see last page)
ISBN: 978-3-639-36838-3

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CONTENTS

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(Monograph)

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VDM VERLAG, GERMANY
2011

CONTENTS

DIMENSION OF N-GROUPS AND FUZZY IDEALS IN GAMMA NEARRINGS

(Monograph)

PREFACE

The algebraic systems with binary operations of addition and multiplication satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition, called "Nearrings". The set of all mappings of an additive group (not necessarily abelian) into itself with pointwise addition and the composition of mappings becomes a nearring. In chapter 0, we present some definitions and results from the literature which are used in the sequel. In chapter 1, we present the concepts: l.i.u-elements, E-direct, and S-inverse systems and prove some structure results on Linearly independent elements in N-groups with Finite Goldie Dimension. We also present some fundamental results on finite Goldie dimension, and finite spanning dimension. In chapter 2, we start with the well known concept: matrix nearrings. We present the proof that the Goldie dimension of an N-group is invariant under the Goldie dimension of $M_n(N)$-group under consideration. In chapter 3, we begin with the notion of spanning dimension in N-groups. We present the concepts: direct systems, E-direct systems, inverse systems, S-inverse systems, and the results on these concepts.

In chapter 4, we present the insertion of factors property for an N-group and related results. In chapter 5, we present the notion of directed hypercube of dimension n and observed four properties of a directed graph; eventually we prove that a directed graph is isomorphic to a directed n-cube if and only if it satisfies these four properties. In chapter 6, we deal with an equivalence relation on the class of all complement ideals of an N-group with finite Goldie dimension 'n' and present a proof that a graph constructed on the set of equivalence classes forms a directed hypercube of dimension 'n'. In chapter 7, we give the concept of fuzzy, the study of vagueness and uncertainity. We present the results on the fuzzyness of algebraic system gamma nearring, and the cosets and prime ideals in Gamma Nearrings.

The authors wish to express thanks to Prof. P.V. Arunachalam (Former Vice-Chancellor, Dravidian University, Andhra Pradesh), Prof. Dr Richard Wiegandt, Prof. Dr Lazlo Marki (Hungarian Academy of Sciences), Dr. Babushri Srinivas (Manipal University, Manipal, Karnataka), Mr. Mohiddin Shaw Sk. (Acharya Nagarjuna University), Dr D. Nagaraju (Hindustan University, Chennai) for their co-operation and continuous help. Also thank Mr. Bhavanari Mallikarjun, B.Tech., for his help in editing this monograph.

The first author expresses his deep sense of gratitude and appreciation to his family members Koteswara Rao (father), Jayalakshmi (mother), Anitha (wife), and Bhanu Shashank (son), Narayana Rao Bodla (Rtd. Head Master, Govt. High School) (father-in-law), and other family members for their inspiration; without whose constant encouragement, it would not have been possible for him to pursue higher studies in Mathematics. He also thanks to the Manipal University and the faculties in the Department of Mathematics for their constant encouragement. Further, he extends thanks to Dr VRC Murty (HOD, Bio-Tech. MIT, Manipal Univ.), and Dr M. Srinivasulu (Asso. Professor, Dept. Chemistry, MIT, Manipal Univ.) for their right support in all his endeavors.

The second author place on record his deep sense of gratitude to his parents: Bhavanari Ramakotaiah (a teacher in an elementary school at the village named Madugula) (Father), and Bhavanari Anasuryamma (house hold) (Mother), without whose constant encouragement and help it would not have been possible for him to pursue higher studies in Mathematics. Also he thank his wife: Bhavanari Jaya Lakshmi, and his children: Mallikharjun B.Tech., Satyasri (IV Year MBBS, China), and Satya Gnyana Sri (Student, 10+) for their constant patience with him and helping in bringing out better output.

Dr Syam Prasad Kuncham
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Introduction

In recent decades interest has been arisen in algebraic systems with binary operations of addition and multiplication satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition. Such systems are called " Nearrings ". A natural example of a Nearring is given by the set M(G) of all mappings of an additive group G (not necessarily Abelian) into itself with addition and multiplication defined by

$(f + g)(a) = f(a) + g(a)$; and

$(fg)(a) = f(g(a))$ for all f, g $\in$ M(G), and a $\in$ G

This monograph deals with the concepts: l.i.u-elements; E-direct and S-inverse systems; fuzzy IFP ideal in N-groups where N is a zero-symmetric right nearring and establishes several interesting results related to these concepts. We present the concept: directed hypercude of dimension 'n' in Graph theory. We define an equivalence relation on the class of all complement ideals of an N-group with finite Goldie dimension 'n' and prove that a graph constructed on the set of equivalence classes forms a directed hypercube of dimension 'n'. We introduce the concept "fuzzy ideal" of a Γ-Nearring and study the fuzzy ideal theory in Γ-Nearrings in analogy with corresponding theory in rings and Nearrings.

In **Chapter-0** we present fundamental definitions and basic results which are used in the sequel. It is well known that the concept of finite Goldie dimension (FGD, in short) in modules over rings is a generalization of the dimension of finite dimensional vector spaces. This concept of FGD is generalized and studied to N-groups by Reddy & Satyanarayana [2]. The aim of the **Chapter-I** of the monograph is to introduce the concepts: "uniform element (u-element, in short), linearly independent element (l.i. element, in short), linearly independent u-element (u.l.i. element or l.i.u-element, in short)" in N-groups and to obtain structure theorems for N-groups which contains no infinite direct sum of non-zero ideals. In fact, we prove that

(i) If G has FGD and H is a non-zero ideal of G then there exist l.i.u-elements a_i, $1 \leq i \leq k$ such that the sum $< a_1 > + \ldots + < a_k >$ is direct and essential in H with k = dim H;

(ii) if G has FGD then K is complement ideal of G if and only if there exist l.i.u-elements u_i+K, $1 \leq i \leq m$ in G/K which spans G/K essentially where m = dim G–dim K; and

(iii) if G has FGD with dim G = n, k < n and u_i, $1 \leq i \leq k$ are l.i.u-elements in G then there exists $u_{k+1}, \ldots, u_n$ in G such that u_i, $1 \leq i \leq n$ are l.i.u-elements of G which spans G essentially. As a consequence, we obtain five equivalent conditions for an N-group G to have dimension n. The content of this chapter forms a paper entitled " ***Linearly independent elements in N-groups with Finite Goldie Dimension*** ", published in Bull. Korean Mathematical Society, 42 (3) 433-441, **2005**.

In **Chapter–II**, we continue the study of l.i. elements in N-groups. Meldrum and Van der Walt [1] introduced the concept of matrix near-ring $M_n(N)$. Several authors like Booth & Groenewald [1], Satyanarayana, Lokeswara Rao & Syam Prasad [1] and Van der Walt [1] studied different concepts in matrix Nearrings. In this chapter we observe some relations between l.i. elements (l.i.u-elements) in an N-group N and those of in $M_n(N)$-group N^n. In particular, we prove that the Goldie dimension of the N-group N is equal to that of the $M_n(N)$-group N^n. The content of this chapter forms a paper entitled "***On finite Goldie Dimension of $M_n(N)$-group N^n***", published in the Proceedings of the 18th International Conf. on Nearrings and Nearfields, (Editors: H. Kiechel, A. Kreuzer & M.J. Thomsen) (**Springer, Netherlands**) 301-310, **2005**.

In **Chapter–III**, we present the concepts E-direct and S-inverse systems in N-groups. E-direct and S-inverse systems in modules over rings were studied by Satyanarayana [1]. Reddy & Satyanarayana [3] extended the concept of finite spanning dimension (FSD, in short) of a module over rings to N-groups. We extend the concepts: Direct, Inverse, E-direct and S-inverse systems to N-groups. We present some interesting results indicating the relations between these concepts: finite Goldie dimension and finite spanning dimension. In particular we prove that

(a) for an N-group G, the conditions: (i) G has ACCI; (ii) for any ideal J of G, the condition: every direct system of ideals of G which are contained in J is bounded above by an ideal J* of G where $J^* \subsetneq J$; and (iii) every ideal J of G is finitely generated, are equivalent;

(b) for an N-group G, the conditions: (i) G has FGD; and (ii) every E-direct system of non-zero ideals of G is bounded above by a non-essential ideal of G, are equivalent. In Chapter- III we also introduced the concepts: S-inverse system, small-1, hollow-1 ideals, and FSD-1 in N-groups and proved some results. We obtain some examples relating to FSD and FSD-1.

The first part of this Chapter forms the paper entitled "A Result on E-direct systems in N-groups" and published in "Indian J. Pure & Appl. Math. 29 (1998) 285 – 287". The remaining part of this chapter forms a paper entitled 'On Direct and Inverse Systems in N-Groups' and published in "Indian Journal of Mathematics, 42 (2) 183-192, **2000**".

Chapter–IV concerned with IFP N-group, Fuzzy IFP ideal in nearrings. We introduce the notion IFP N-group and prove that if G is an IFP N-group such that every monogenic N-subgroup of G has ACCI, then there exists an element $g \in G$ such that $(0:g)$ is a prime left ideal of N. As a consequence, we get that under the above conditions, the N-group G contains a monogenic N-subgroup which is isomorphic to a prime near-ring N/P for some prime ideal P of N. We observe some properties of IFP $M_n(N)$-group $M_n(N)$ where $M_n(N)$ is the matrix near-ring. In Datta & Biswas [1], and Salah Abou-Zaid [1], introduced and studied different concepts in fuzzyness in Nearrings. In the remaining part of Chapter-IV, we introduce fuzzy IFP ideal of a nearring. We prove that a fuzzy ideal μ of N has IFP if and only if every level subset μ_t is an IFP ideal. We also construct some examples of IFP N-groups. The content of this chapter forms a paper "On IFP N-Groups and Fuzzy IFP Ideals", and published in "Indian J. Mathematics, 46 (1) 11-19, **2004"**.

Mulder [1] introduced the concept of interval in Graph theory which is an analogue of the notion of interval on the real line. Mulder characterized n-cube as an interval regular graph. Also Mulder [1] introduced the concept of median graph and obtained that a graph is an n-cube if and only if it is a median graph. In **Chapter-V**, we introduce the notion of directed hypercube of dimension n; we bringout four properties P_1, P_2, P_3 and P_4 of a directed graph; and eventually proved that a directed graph is isomorphic to a directed n-cube if and only if it satisfies P_1 - P_4. The content of this chapter forms the paper "***An Isomorphism theorem on Directed Hypercubes of Dimension n***" and published in "Indian J. Pure & Appl. Mathematics 34 (10)(2003) 1453-1457".

In **Chapter –VI**, we consider the set of all complement ideals of an N-group G with FGD; define an equivalence relation on this set, and obtain a directed graph on the set of all equivalence classes; and finally we prove that the graph obtained is a directed hypercube of dimension n, where n is the finite Goldie dimension of G. The content of this chapter forms a paper ***A Note on Complements in N-groups with FGD and related graph***, communicated to a journal for possible publication.

We consider the concept "Γ-Nearring", a generalisation of both the concepts near-ring and Γ-ring, introduced by Satyanarayana [6] and later studied by the authors like Satyanarayana [4], [5]; Booth [1], [2]; Booth & Godloza [1].

In **Chapter –VII**, we introduce the concept of fuzzy ideal in Γ-Nearrings and study several interesting results on these fuzzy ideals in Γ-Nearrings in analogy to the corresponding results in rings and Nearrings. The first part of this chapter forms the paper 'Fuzzy Cosets of Gamma Nearrings' published in "Turkish Journal of Mathematics, (29) 11-22, **2005".** The remaining part of this chapter forms the paper "On Fuzzy Prime ideal of a Gamma Nearring" published in "Soochow Journal Mathematics, 31(1) 121-129, **2005"**.