



# CALCULUS

FOURTH EDITION

MARVIN L. BITTINGER

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Indiana University—Purdue University at Indianapolis

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## CALCULATOR NOTE TO THE STUDENT

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Having a scientific calculator will enhance your study of this text. Be sure that it has a power key  $y^x$ , a natural logarithm key  $\ln$ , and possibly trigonometric keys. An  $e^x$  key would also be helpful.

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# PREFACE

Appropriate for a one-term calculus course, this text is an introduction to calculus as applied to business, economics, the behavioral sciences, the social sciences, biology, and medicine. For those who need a two-term text, *Applied Calculus*, by Marvin L. Bittinger and Bernard B. Morrel, covers all the material in this text and includes material on total differentials, differential equations, numerical methods, and Taylor polynomials and infinite series. A basic course in algebra is a prerequisite for the text, although Chapter 1 provides sufficient review to unify the diverse backgrounds of most students.

## Content Features

**Intuitive approach.** Although the word “intuitive” has many meanings and interpretations, its use here means “experience based.” Throughout the text, when a particular concept is discussed, its presentation is designed so that the students’ learning process is based either on their earlier mathematical experience or on a new experience presented by the authors before the concept is formalized.

- When maximum problems involving volume are introduced (see p. 195), a function is derived that is to be maximized. Instead of forging ahead with the standard calculus solution, the student is first asked to stop, compute the function values, graph the equation, and then estimate the maximum value. This experience provides students with more insight into the problem—recognition not only that different dimensions yield different volumes, but also that the dimensions yielding the maximum volume may be conjectured or estimated as a result of the calculations.

- The definition of the derivative in Chapter 2 is presented in the context of a discussion of average rates of change (see p. 97). This presentation is more accessible and realistic than the strictly geometric idea of slope.
- The concept behind the definition of the number  $e$  in Chapter 4 is explained both graphically and through a discussion of continuously compounded interest (see p. 239).

**Applications.** Relevant and factual applications drawn from a broad range of fields are included throughout the text as applied problems and exercises and in application sections. These have been updated and expanded in this edition.

- An early discussion of compound interest, cost and profit, and supply-and-demand functions in Chapter 1 sets the stage for later applications. The notions of total revenue and cost and profit, together with their derivatives (marginal functions), are threads that run throughout the text (see pp. 43, 132, and 193).
- All techniques of differentiation, including the Extended Power Rule, the Chain Rule, and material on higher derivatives are covered in Chapter 2 before applications of differentiation are introduced in Chapter 3. This allows for many interesting applications of differentiation to be presented in Chapter 3, which includes separate applications sections for business and biology, as well as a section on implicit differentiation and related rates (see pp. 204–231).
- When the exponential model is studied in Chapter 4, other applications, such as continuously compounded interest and the demand for natural resources, are also considered (see pp. 264–286). Growth and decay are covered in separate sections in Chapter 4 to allow room for the many worthwhile applications that relate to these concepts.
- Applications of integration are covered in a separate chapter and include a thorough coverage of probability (see pp. 367–429).

**Coverage.** Many aspects of the treatment and coverage in this text have been commended by reviewers. The following are a few examples.


- Trouble areas in algebra are integrated into coverage of traditional precalculus topics in Chapter 1.
- Relative maxima and minima and absolute maxima and minima are covered in separate sections in Chapter 3 so that students always understand which they are studying (see pp. 157–189).

- Chapter 5 on techniques of integration includes a section on integration by tables, which will prepare students for using calculus in real-life situations (see pp. 351–354).
- This text also includes a section on differential equations, coverage of probability, and a chapter on trigonometric functions.

## Pedagogical Features

**Interactive approach.** As each new section begins, its objectives are stated in the margin. These can be spotted easily by the student, and when the typical question, “What material am I responsible for?”, arises, the objectives provide an answer. They may also help take the fear out of the word “calculus.” In the margins on each page, sample, developmental, and exploratory exercises are placed near the related text material. As students work through the material, they are encouraged to do the margin exercises. This involves them actively in the development of each topic and gives them a deeper understanding of the material they are studying. All margin exercises have answers in the text. It is recommended that students work out all these problems, stopping to do them when the text so indicates (see pp. 146 and 454).

## Variety of Exercises

- *Calculator exercises.* Exercises and examples geared to the use of a calculator are included throughout the text, although students who do not have a calculator can still achieve their goals. Calculator exercises are highlighted by the symbol  (see pp. 206 and 240).
- *Extension exercises.* **Extension** exercises are included in most exercise sets and in all chapter tests. They require students to go beyond the immediate objectives of the section or chapter, and are designed both to challenge students and to make them think about what they are learning (see pp. 149 and 346).
- *Exploratory exercises.* These are extended exercises that are included in the exercise sets or in the margin exercises. They emphasize analysis of data produced from sets of equations, home experiments, or theoretical situations, and are designed to build students’ mathematical experience and conceptual understanding (see pp. 280 and 314).
- *Applications.* A section of applied problems is included in most exercise sets. These give the students extensive practice in applying the material they are learning to real-life situations (see pp. 200 and 273).

- *Computer software exercises.* Many exercises throughout *Calculus* can be done on the computer using the software utilities package that accompanies the text, *Cactusplot*, Student Edition. Step-by-step procedures for working out exercises designated with the symbol **CSS** are contained in the *Cactusplot* Student Supplement.

### Tests and Reviews

- *Summary and Review.* New to this edition, the Summary and Review section is found at the end of each chapter and is designed to provide students with all the material they need for successful review. The objectives of the chapter are summarized in boldface type and followed by a set of review exercises. Answers are at the back of the book, together with section references so that students can easily find the correct material to restudy if they miss an exercise (see pp. 153 and A-11).
- *Test.* Each chapter ends with a chapter test, which includes challenge questions. There is a cumulative review at the end of the text, which can also serve as a final examination; the answers to the chapter tests and the cumulative review are at the back of the book. Four additional forms of each of the tests appear, ready for classroom use, in the Instructor's Manual, and six additional chapter tests are available in the Printed Test Bank. The Instructor's Manual also includes four different forms of a very comprehensive final examination.

### What's New in the Fourth Edition?

The style, format, and approach of the third edition have been retained in this new edition, but the text has been polished in many places on the basis of extensive reviewer feedback. New applications have been added, and many of the former applications have been updated.

- A Summary and Review section has been added to the end of each chapter (see the description above).
- In Chapter 2, the discussion of limits has been split into two sections, and is now presented before the material on continuity.
- In Chapter 3, the material on determining absolute maxima and minima has been extensively revised, and is now presented in a section separate from the discussion on relative maxima and minima. Relative maxima and minima are now covered in Section 3.3, where such concepts are directly useful to graphing functions.

- Chapter 4 has been revised under the assumption that all students have calculators and no longer need tables to find values of the exponential and logarithmic functions.
- Chapter 5 now contains a complete definition of differential notation, together with exercises to reinforce the concept. Integration by tables is now covered in a separate section.
- In Section 6.2 the concept of continuous money flow replaces the finite process of determining the amount of an annuity.
- In Chapter 7, finding function values for functions of several variables has been separated from finding partial derivatives to ease the length of the first section.
- The trigonometry material in Chapter 8 has been related more directly to right triangles and degrees as well as to radians.

### Supplements for the Instructor

**Instructor's Manual.** The Instructor's Manual contains four alternative test forms with answers for each chapter test and four comprehensive final examinations. The tests have been revised and are now completely different from those for the first edition. The Instructor's Manual also has answers for the even-numbered exercises in the exercise sets, which can be copied and handed out to students. The answers to the odd-numbered exercises are at the back of the text.

**Printed Test Bank.** This contains over 1200 test items that have been generated from Addison-Wesley's computerized test generator, "AWTest." The test bank items are arranged in a convenient chapter-test format with six different tests for each chapter. Instructors can pick and choose test items to create their own tests or tear out ready-to-use chapter tests complete with answer sheets and answer keys.

**Videotapes.** Videotape reviews that cover important topics in the book have been prepared. John Jobe of Oklahoma State University speaks to students, works out examples, and provides lucid explanations. Although the videotapes do not provide an entire course on television, they have many uses, among which are to supplement lectures, to provide partial lectures, and to offer self-study opportunities for students.

### Software

- *Computerized testing.* "AWTest," Addison-Wesley's random-number test-generating system, is available with this text for the IBM PC.

Using AWTest, instructors can generate up to 99 variations of any particular test with a few keystrokes. They can also choose test items by number from a bank of over 200 test items or request tests to be printed out in chapter-test format. AWTest will support almost any completely compatible IBM PC printer.

- *Computer Software Supplement.* The Student Version of Cactusplot Software by John Losse of Scottsdale Community College is available exclusively with this text in both Apple II series and IBM PC versions. Cactusplot is an easy-to-use software utilities package that can perform many mathematical operations, such as graphing functions, processing tables, solving equations, and finding the area under curves. A site license to the Student Version of Cactusplot is available from Addison-Wesley and is free to qualifying adopters.

Cactusplot is accompanied by a student software supplement containing directions for solving several key exercises in each chapter using a computer. These exercises are designated in the text by the symbol **CSS**. The manual also gives the students additional activities for each exercise, such as changing the parameters of a graph, which are designed to strengthen their intuitive understanding of the concepts involved. Cactusplot will also print out graphs during the course of an exercise so that students can show their work to their instructor. The Software Supplement is for sale to students at a nominal cost.

The Student Version of Cactusplot is available only from Addison-Wesley. A professional version of Cactusplot with many enhanced capabilities is available directly from John Losse, The Cactusplot Company, 1442 N. McAllister, Tempe, AZ 85281 (602) 945-1667.

**Related Addison-Wesley Software Titles.** *The Calculus Toolkit* is a calculus utilities package available for the Apple II series and IBM PC. It is designed to accompany Thomas/Finney's *Calculus and Analytic Geometry*, Seventh Edition (Addison-Wesley, 1988), but can be used with any calculus text. *MathCAD, Student Edition* is an electronic scratch-pad for the IBM PC that allows you to write equations on the PC exactly as you would on paper. MathCAD automatically calculates and displays results as numbers or graphs. *Lotus 1-2-3, Student Version* is a fully functional version of Lotus 1-2-3 with a 64-column-by-256-row spreadsheet that includes all the database, calculating, and graphics capabilities of Lotus 1-2-3 with teaching materials and enhancements particularly suited for educational use. Contact your Addison-Wesley sales representative for more information about these titles.

## Supplements for the Student

**Student's Solutions Manual by Judith A. Beecher and Judith A. Penna** Complete worked-out solutions with extra hints and suggestions are provided in this booklet for all odd-numbered problems in the exercise sets. This supplement is available to instructors and is for sale to students.

## Acknowledgments

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