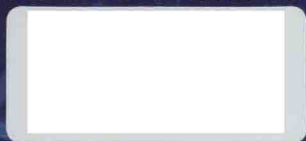


Paul I. Palmer



Essential Maths for Geoscientists

An Introduction



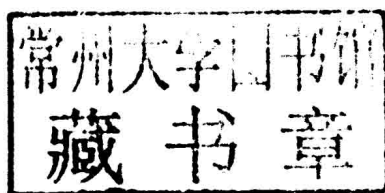
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Essential Maths for Geoscientists

An Introduction

Paul I. Palmer

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Essential Maths for Geoscientists

To A.L.M., L.M.P., and J.S.P.

Preface

This book includes the lectures and problem sets from the one-semester course 'Earth Modelling and Prediction' that I teach at the University of Edinburgh. The course is aimed at first-year geoscience undergraduates who want to understand the Earth and its evolving climate but do not have the necessary quantitative skills to move beyond qualitative studies. My primary and most ambitious objective for this course was to help students overcome the psychological barrier of *applying* mathematics to problems associated with the Earth. It is this barrier that artificially limits students' ability to gain a deeper understanding of the underlying science. My second objective was to show that the relatively simple mathematics covered in this course could be applied to learn something relevant to current areas of scientific research.

The focus of the book is the application of mathematics to scientifically relevant problems. Rather than being comprehensive, the material should be seen as providing a background for more advanced geoscience courses, which practise the application of mathematics and introduce the students to additional mathematics. I support the use of real data in teaching and so in recent years I have included progressively more exercises that involve the analysis of real measurements, many of which form the backdrop to a major news story in that year, for example, increased/decreased tropical deforestation rates or the reduction in the spatial extent of Arctic sea ice. I hope to include in future editions more varied data analysis problems that reflect the breadth of geoscience research.

I thank Patience Cowie, Roger Scrutton, and Roger Hipkin for recognizing the need for this course and for helping me to establish it at Edinburgh. For helping to teach topics within the course over the years I thank Patience Cowie, Godfrey Fitton, Gabriele Hegerl, Roger Hipkin, Ian Main, Chris Merchant, Mark Parrington, Simon Tett, and Thorvaldur Thordarson. I thank all the tutors who helped to make the course work well: Amber Annett, Louise Barron, Dave Bell, Anthony Bloom, Matthew Brolly, Iain Cameron, Ruth Carley, Craig Duguid, Leon Kapetas, Simon King, Jack Lonsdale, Malcolm McMillan, Simone Morak, Heather Nicolson, Katie Noak, Luke Ridley,

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Paul I. Palmer

University of Edinburgh

May 2013

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1

How Do You Know that Global Warming Is Not a Hoax?

The title of this introductory chapter is the question I pose at the start of my course in Edinburgh. It seems like a ridiculous question to ask a bunch of bright young students, especially ones who have chosen to study the Earth system. But up until walking through the doors of the university many students have not had the resources, inclination, and/or ability to *question* what they are told; the key to being an effective scientist is to ask the right questions, ones that probe at the very heart of the problem being studied. I provide the student with four possible choices to answer the question and ask for a show of hands:

1. popular media (internet, TV, radio, newspapers);
2. rigorous scientific reasoning and/or debate;
3. (blind) faith in scientists; or
4. other.

Typically, choice 1 represents the vast majority of hands. Why? Because we are bombarded with scientific and political coverage of climate change. Why is this dangerous? Because companies need to sell newspapers and to get people to watch TV, and politicians are invariably biased in their opinions. Much of the coverage is accurate but some programmes are biased, loosely based on fact, with a damaging effect on the science education of the general public. Sensationalism about Earth's climate (particularly looking to the future) is

rife, but some aspects of Earth's climate are *genuinely* remarkable and awe-inspiring. So how do you know what to believe?

Choice 2 often represents the second highest show of hands, but a much smaller proportion than choice 1. This is fine up to a point. Scientists are some of the biggest sceptics around and are generally very careful about what they say. For instance, we see later in this chapter that the wording used in the Intergovernmental Panel on Climate Change (IPCC) report¹ has very strict statistical interpretation that is difficult to misinterpret. But you only learn from the scientists what they tell you. How did they reach their conclusions? Could they have approached the problem from a different perspective and reached a different conclusion? With the renewed call for transparency in science, particularly related to climate, most data used to draw conclusions about Earth's climate are online and freely available to download. Often the only barrier to pursuing option 2, given that data are now freely available, is the confidence to understand and interrogate quantitative data. The aim of this book is to increase that confidence.

This mix of responses is reasonably similar to the general public response to the question 'How well do you feel you understand the issue of global warming?' that has been asked frequently by Gallup (www.gallup.com) for the past quarter century (Figure 1.1). For this admittedly crude comparison I have equated 'Great deal' with 'Rigorous scientific reasoning', 'Fair amount' with 'Popular media', and 'Only a little' with '(Blind) faith in scientists'.

How can mathematics help? In simple terms, mathematics (at this level) is a tool that allows us to move far beyond what we can learn from descriptive analysis. How much has sea ice changed? If we use the current rate of change, how long will it be before the Arctic is free of ice? These are simple example questions that cannot be answered without mathematics.

The Earth system: how do we know what we know?

I define the Earth system as the land, ocean, and atmosphere, all the physical, chemical, biological, and social processes and their interactions (Figure 1.2). This is a big unwieldy interconnected system that is coupled on a wide spectrum of spatial and temporal scales. To minimize the risk of discussing current science results that might be superseded by new data, I have decided to focus on *how* scientists generally know what they know about the Earth system and the recent role of human activity and not *what* they know:

- First, we have a basic physical understanding of the Earth. We know, for example, about the heat-trapping properties of gases in the atmosphere,

¹ A report prepared by a subset of leading climate scientists that summarizes the state of the science. The latest report can be found at www.ipcc.ch

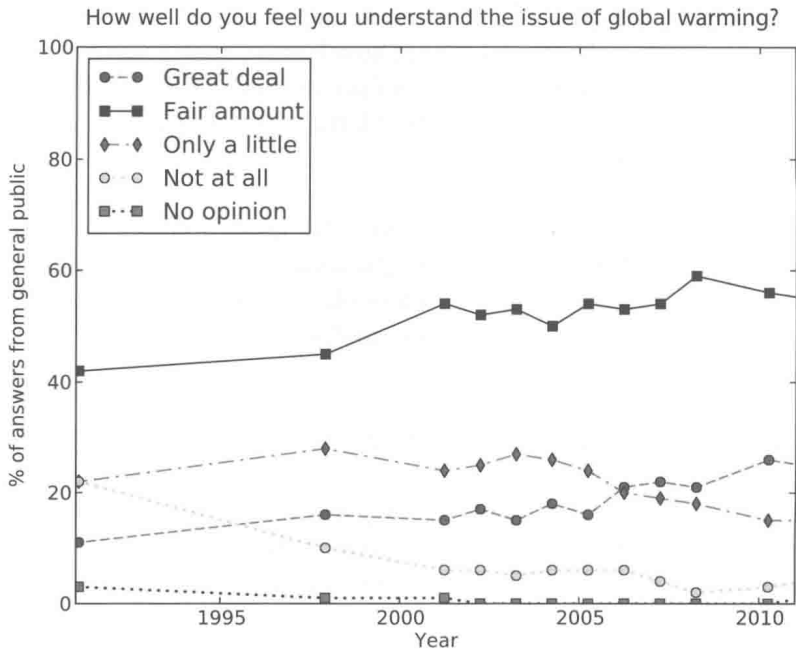


Figure 1.1 Results from a Gallup poll question ‘How well do you feel you understand the issue of global warming?’ that has been asked since 1989.

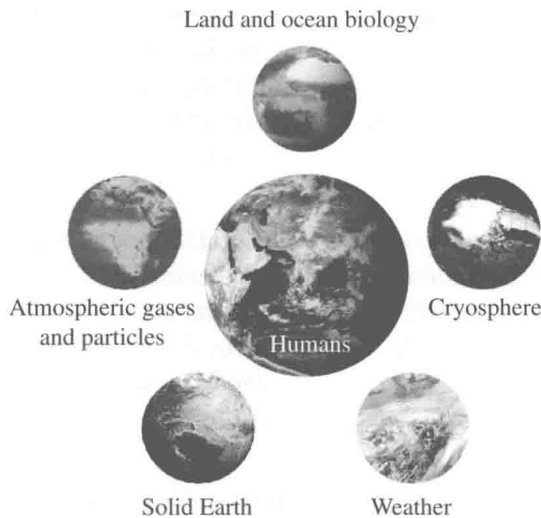


Figure 1.2 A schematic describing the broad-scale subcomponents of the Earth system. Graphics reproduced with permission from the UK/NERC National Centre for Earth Observation. (Image courtesy of NASA.)

based on work first started in the nineteenth century. Another example is continental drift, a theory describing how Earth's continents move relative to each other, which has been known since the twentieth century. These are well-established science theories that have stood up to decades/centuries of scientific scrutiny.

- Second, we have circumstantial evidence. We make qualitative connections between observations of disparate quantities and results from computer models² of the Earth system, for example, warming of oceans, lands, and the lower atmosphere, cooling of the middle atmosphere, and increases in water vapour.
- Third, we have palaeoclimate evidence. We can reconstruct past climate using a variety of data, for example, ice core, lake sediment core, coral reefs, pollen. This places contemporary warming trends in the longer-term context. Although there is debate about whether the past is any guide to the future, they do provide us a history of how Earth has behaved in the past.
- Finally, we have so-called 'fingerprint' evidence. The underlying philosophy is that individual (natural and human-driven) processes will leave their own unique signature (or fingerprint) on measurements of the Earth. By comparing these data that naturally include these signatures with computer models of climate with/without descriptions of the processes responsible for these signatures we can understand the importance of individual processes. This can also potentially identify the need for additional processes that are currently not present in the model.

It is important to acknowledge that several independent lines of inquiry are used to investigate phenomena and provide evidence to test a hypothesis. The IPCC is testing the overarching hypothesis that human activity has determined recent changes in climate. As we will see in the next chapter, the hypothesis is right at the crux of the *scientific method*. In successive IPCC reports the headline result has been stronger and stronger:

- **1995:** The balance of evidence *suggests* a discernable human influence on global climate.

² A model in this instance is a collection of interrelated equations, written in a computer language, that describe, for example, the physics, chemistry, and biology of the atmosphere and ocean. Without a computer, evaluating these equations would be an intractable task. In fact some of the fastest computers in the world are dedicated to studying Earth's climate.