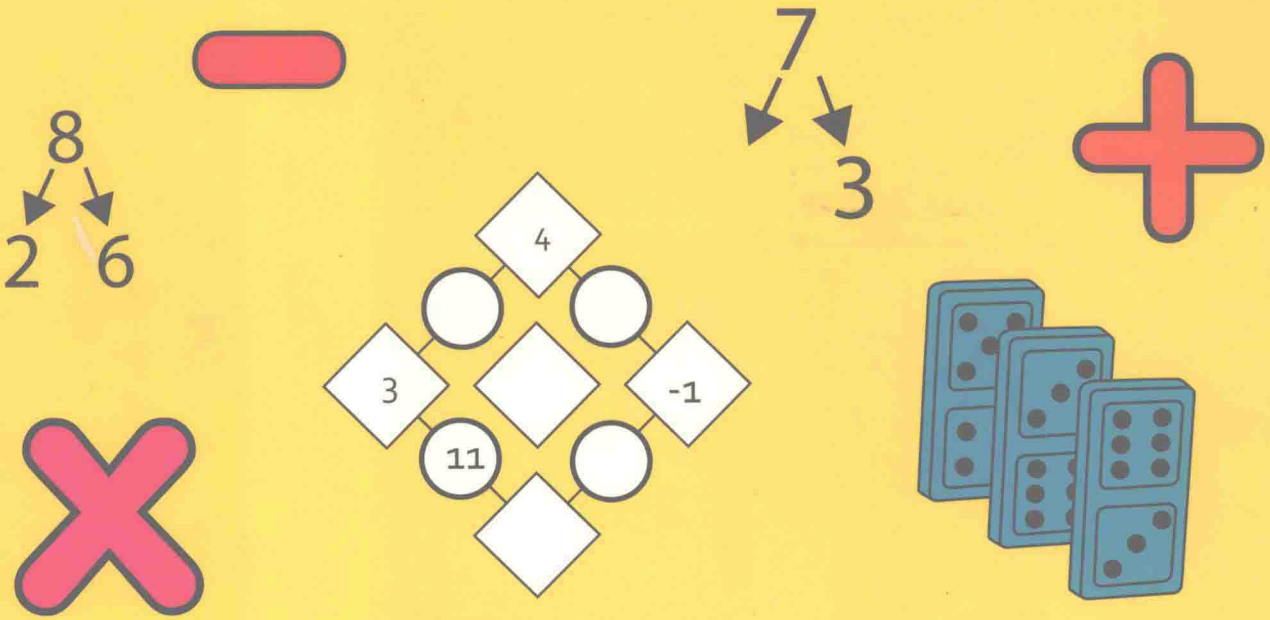


Overcoming **Difficulties** with Number

Ronit Bird

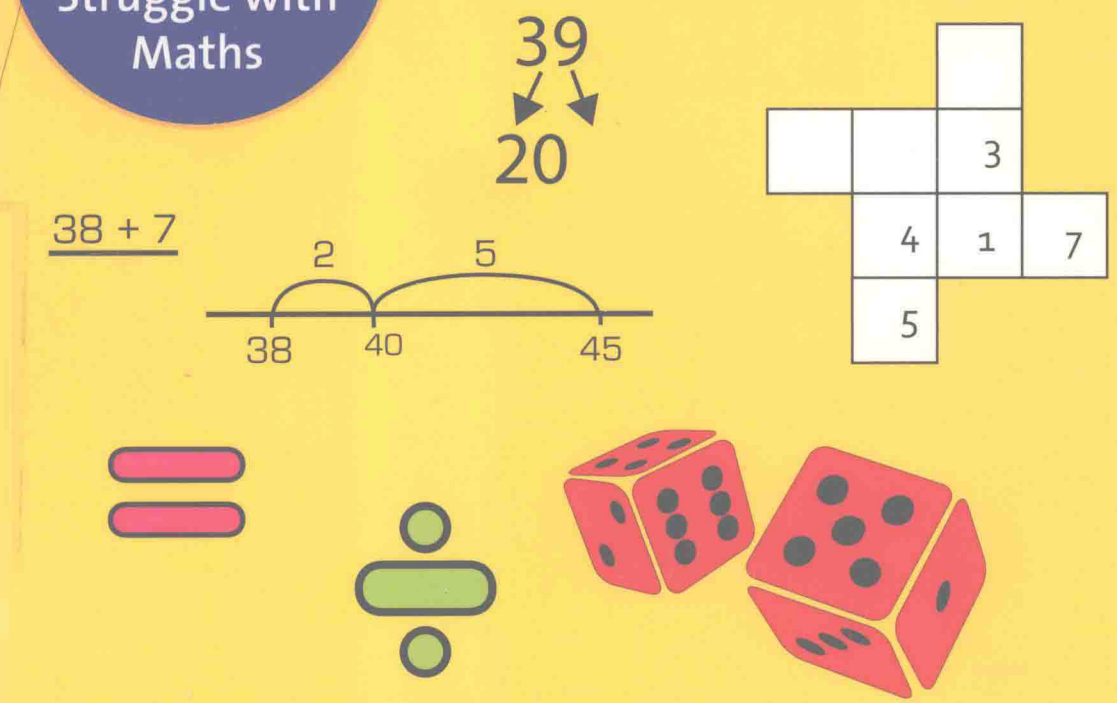
Supporting Dyscalculia and Students who Struggle with Maths

CD Rom

Age Range 9-16

40 Games

Structured Teaching Activities



Overcoming Difficulties with Number

Supporting Dyscalculia and Students who Struggle with Maths

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About the author

Ronit Bird is a teacher whose interest in pupils with specific learning difficulties began with a focus on dyslexia. She qualified as a teacher at London University and subsequently gained a further qualification as a specialist teacher. While working with dyslexic pupils in a mainstream school, Ronit started to develop strategies and teaching activities to help support the learning of pupils who were experiencing difficulties in maths.

Ronit has taught in both primary and secondary settings, and has worked as a special educational needs coordinator (SENCO) in both the independent and state sectors. She currently works as a teacher and as a contributor to professional development courses. Ronit also runs training courses on dyscalculia for Harrow subject leaders, teachers and teaching assistants as part of the Harrow Dyscalculia Project, and works in an advisory capacity with the participating schools.

How to use this book

This book is for anyone working with learners who have not yet gained a secure understanding of the key ideas behind addition, subtraction, multiplication and division. It is particularly targeted at older learners who could feel humiliated or embarrassed by having to work with material obviously intended for young children but who still need help with some of the fundamental numeracy concepts.

The book identifies a small core of key strategies for numeracy and provides a detailed and practical guide to teaching them, taking into account the prerequisite skills that underpin each strategy. The key strategies addressed are:

- Bridging through 10, and through multiples of 10.
- Subtraction as complementary addition, so that pupils are always working up (forwards).
- Empty number lines for both addition and subtraction, not only as a way of working but also as a way of supporting visualisation strategies.
- Visualisation techniques to support mental calculation.
- The area model of multiplication and division.
- Multiplication and division taught side by side, with pupils always working up (forwards).
- Using logic and reasoning to extend knowledge and proficiency.

In developing the approach featured in this book, I have deconstructed the essential teaching points of the numeracy strategies listed above and teased out a structured and logical sequence for teaching and learning. The resulting series of teaching activities provides for a systematic and cumulative progression in very small incremental steps, frequently reinforced. In this way, pupils can be introduced to a single new idea at every step, without being prematurely exposed to problems beyond their level of understanding. Pupils are therefore able to experience success while developing their mathematical understanding.

I make no attempt to devise a formal or prescriptive teaching programme. Instead, I have tried to record, in as detailed and accessible a manner as I can manage, ideas that I have found to be successful with my own pupils in combating a range of common misconceptions and difficulties. The suggestions that I offer are presented as a structured series of consecutive activities and the work is predominantly practical and oral. I prefer to avoid worksheets because I suspect that having pupils work through pages of written examples will only result in the pupils reinforcing the same inefficient strategies and bad habits that have contributed to their lack of progress. I strongly believe that the teacher is each pupil's most important resource and that the

teaching focus should always be on developing logical thinking and mathematically sound cognitive models.

The activities and ideas in this book are all ready to use with a minimum of preparation. The only equipment needed is what is commonly available in the classroom or easily purchased: dominoes, dice, playing cards and number cards, Cuisenaire rods and base-ten blocks, paper and pencil. Various other resources, including printable game boards and information about Cuisenaire rods, can be found on the accompanying CD  .

The contents of the book are organised into four parts:

- Part I – How to help pupils stop counting in ones.
- Part II – The bridging technique.
- Part III – The area model of multiplication and division.
- Part IV – Reasoning strategies.

Each chapter begins with an overview of the subject, putting the teaching points into context. Each chapter also displays a summary of the individual steps that are later expanded into consecutive teaching activities.

Part I addresses one of the most common obstacles to struggling pupils' progress in numeracy, namely, their tendency to rely on counting in ones. A wealth of different ideas for activities and games are designed to promote component work and to help learners climb out of the 'counting trap'.

Part II contains a detailed step-by-step guide to teaching the bridging technique for both addition and subtraction. Following an analysis of the requisite pre-skills together with suggested activities for teaching these pre-skills to pupils for whom the concepts are not yet secure, two further chapters are devoted to the teaching of bridging through 10, and then through multiples of 10. These chapters advocate the teaching of subtraction as complementary addition, and explore how pupils can move from the concrete stage to the purely abstract stages of mental calculation.

Part III contains a detailed step-by-step guide to teaching the area model for multiplication and division. The first two chapters address both operations together and include an analysis of the requisite pre-skills complete with suggested activities for teaching these pre-skills to pupils for whom the concepts are not yet secure. Two further chapters are devoted to the teaching steps though which pupils can learn to manage the transition from the concrete stage to the abstract stage of understanding the standard written algorithms.

Part IV focuses on reasoning strategies. Pupils with difficulties in maths are rarely flexible in their thinking and must be explicitly taught how to use the few facts they know to derive new facts.

The CD accompanying this book contains a short introduction to Cuisenaire rods as well as printable resources for games, puzzles and activities  .

Contents

Background

Pupils underachieve in maths for all sorts of reasons. One of those reasons might be teaching methods that are not well matched to the pupils' learning needs; another might be the existence of learning disabilities. The Williams Review into maths teaching and learning in UK primary schools (DCSF 2008) gives a figure of approximately 6 per cent of pupils who have fallen significantly behind (i.e. who fail to achieve level 3 of the National Curriculum) by the end of Key Stage 2, the stage at which children transfer from primary to secondary school, and points out that this percentage has remained roughly constant for almost a decade.[1] Thus, despite the introduction of the National Numeracy Strategy in the late 1990s and the numerous government initiatives directed at improving maths teaching and learning in schools over the past decade, there is still a small number of learners who have significant and persistent difficulties with learning basic numeracy.

At the same time, there is growing evidence of specific learning difficulties affecting maths. Professor Brian Butterworth, Britain's leading authority on dyscalculia, estimates that 4–6 per cent of the population have dyscalculia, a disorder that is neurological in origin and that results in difficulties in learning about number and arithmetic.[2] Those working in the field of dyslexia estimate that 4–10 per cent of the general population are dyslexic;[3] of these roughly 50–60 per cent are thought to have associated difficulties with aspects of mathematics.[4] These types of specific learning difficulties are found in learners of any cognitive ability.

These different sets of figures are consistent with one another. They suggest that teachers should expect to find one or two pupils in every average class for whom maths will be a struggle. Whatever the cause of their difficulty, pupils who have not mastered basic numeracy concepts will find it impossible to make satisfactory progress at secondary school without specialised teaching to address their particular problems.

Specific learning difficulties affecting numeracy

Dyscalculia

Developmental dyscalculia was first recognised by the Department for Education and Skills (DfES) in 2001[5] and defined as:

> a condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers,

and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.

As a teacher, you might suspect that you have a dyscalculic pupil in your class if an otherwise competent student has a surprising level of difficulty with ordinary numeric operations and relies on finger-counting, often for all four arithmetic operations, well beyond the age at which most of the others in the class have progressed to more efficient strategies. A dyscalculic learner stands out as having no 'feel for numbers' at all, no ability to estimate even small quantities, and no idea whether an answer to an arithmetic problem is reasonable or not. Memory weaknesses, both long-term and short-term, are a great handicap and result in a pupil with dyscalculia being unable to remember facts and procedures accurately, or consistently, no matter how many times they try to learn them by heart. Pupils who have dyscalculia simply cannot remember their times tables reliably, and you may find they can recall some facts one day but not the next. They are also likely to lose track of what they are doing when attempting any procedure that requires more than two or three steps. Even basic counting can be a problem for pupils with dyscalculia, especially counting backwards.

Indicators for dyscalculia are:

- An inability to subitise (perceive without counting) even very small quantities.
- An inability to estimate whether a numerical answer is reasonable.
- Weaknesses in both short-term and long-term memory.
- An inability to count backwards reliably.
- A weakness in visual and spatial orientation.
- Directional (left/right) confusion.
- Slow processing speeds when engaged in maths activities.
- Trouble with sequencing.
- A tendency not to notice patterns.
- A problem with all aspects of money.
- A marked delay in learning to read a clock to tell the time.
- An inability to manage time in their daily lives.

Dyslexia

A dyslexic pupil might show many of the same indicators as those mentioned above, because it is thought that at least half of all dyslexics also have difficulties with maths. Outside the maths classroom, you might suspect that pupils are dyslexic if they read and write much less willingly and fluently than you might expect, if they read and reread written material with little comprehension

and if their spelling is particularly weak, inconsistent or bizarre. Dyslexic learners show much greater ability and understanding when speaking than you could ever guess from looking at the scrappy and minimal amount of written work they produce. Other indicators are memory weaknesses, problems with processing auditory information, and difficulties with planning and organisation.

Dyspraxia

A typical dyspraxic pupil does not seem to have the same long-term memory problems as a dyslexic and so might be able to remember times tables facts with ease. Dyspraxia, also known as DCD (developmental coordination disorder), mainly affects motor control, which results in pupils being clumsy and uncoordinated, poor at planning and organisation, and unsuccessful at subjects like PE and sports that require balance and coordination. Dyspraxic pupils cannot process sensory information properly and are therefore forever tripping and falling, dropping and breaking things, and mislaying their belongings. In the maths classroom, dyspraxic pupils have particular difficulty handling equipment such as a ruler, a protractor or a set of compasses, and their written work is likely to be very messy and difficult to decipher.

Diagnosis

A quick and informal way of identifying pupils who need extra help, or further assessment, is to: (a) find whether pupils have difficulties counting backwards, (b) discover which pupils cannot remember times tables reliably, and (c) notice which pupils have no calculation strategies beyond counting in ones. A less subjective identification can be achieved by using Brian Butterworth's computer-based Dyscalculia Screener, published under the nferNelson imprint and obtainable through GL Assessment. The Screener, which is based on Professor Butterworth's neuroscientific research, can be administered to several pupils at once and produces a profile of each pupil that can provide evidence (or an absence of evidence) of dyscalculia. A formal diagnosis of dyscalculia can, like dyslexia or dyspraxia, only be given by a qualified educational psychologist after a thorough assessment.

How to help pupils who have difficulties with numeracy

The following principles summarise this book's approach to teaching and learning the key numeracy strategies.

- Start with concrete materials, making sure that the equipment you use is mathematically sound and is robust enough to model a wide range of numeracy topics. In my opinion, the best concrete resource is a collection of base-ten materials, such as Cuisenaire rods, supplemented by Dienes blocks.

- Allow the pupils to use the concrete materials themselves. Do not appropriate them solely for demonstration purposes.

- Never allow the concrete materials to be used mechanically, simply to find an answer. Their value lies in the way they can be used to support visualisation techniques and to build cognitive models.

- Target pupils who are using counting as their only calculation strategy. Before mathematical progress can be made, pupils must be helped out of the 'counting trap' by learning to think in terms of components, or chunks, for building or partitioning numbers.

- Allow plenty of time. This means allowing pupils as much time as they need to use concrete materials and to experiment with them. It means building into your teaching plenty of opportunities for recap and revision. It means pausing after asking a question so that a pupil has enough time to think about what the question means without feeling rushed, and yet more time to come up with a reasoned answer. It means addressing problems and misconceptions not at the end of the lesson or the end of a topic, but when there is still time enough for pupils to reconstruct their understanding. It also means removing the time pressure of having to get through a whole term's syllabus in a single term and instead allowing pupils to become thoroughly familiar and secure with one topic before moving on to the next.

- Engage in a lot of talk as you work. Encourage the pupils to talk about what they are doing at every stage. Get pupils into the habit of reflecting on what they do or see and on putting their thoughts into words. Use and teach a wide variety of appropriate vocabulary.

- Focus on practical activities and teaching games. Apart from the fun that is to be had from solving puzzles and playing games, let your pupils see that maths is something we do, not something we necessarily need to write down. Introduce written calculation as a way of recording only what the pupil has already done concretely and/or has already understood.

- Progress in very small steps. Break down the teaching and learning of every topic into tiny incremental steps and address only one new idea at a time.

- Aim to move pupils gradually from the concrete stage through the diagrammatic stage before moving to the purely abstract stage of calculation.

- Lighten the burden on pupils' working memory by encouraging them to minimise the number of steps in any calculation.

- Give more attention to how a solution is reached than to what the solution is.

- Let pupils make mistakes. Encourage them to see errors as not only inevitable but also a helpful part of the learning process.

- Teach each new strategy by building on a foundation of what is already known. Check that all the necessary pre-skills are secure. Make the connections explicit.

- Allow informal calculation strategies to replace standard written algorithms provided that pupils can consistently reach the correct solution in a reasonable amount of time.

- Minimise the amount that pupils are expected to commit to long-term memory by focusing on key strategies, i.e. those having the widest application. If there are several acceptable ways

of tackling or recording a calculation, do not expect your pupils to become familiar with them all. Instead, allow individual pupils to choose whichever method they are most comfortable with and encourage them to practise that method consistently.

- Teach reasoning strategies explicitly. Show pupils how to use logic and reasoning to extend their knowledge of facts and procedures.

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PART I

How to help pupils stop counting in ones

CHAPTER ONE

More than 50 ideas to help pupils stop counting in ones

Overview

Many pupils who struggle with arithmetic have a tendency to count in ones. What is a normal stage of development for most children becomes a crutch for pupils with poor number sense. Pupils who continue to rely on this unsophisticated and laborious strategy well beyond the stage at which counting is appropriate or efficient have fallen into the 'counting trap'.

The counting trap is the situation in which pupils know very few arithmetic facts for certain, and therefore have to calculate every new fact from scratch. They calculate by counting in ones, an arduous and long-winded process that puts so much strain on their already weak memories that the newly found answer becomes dissociated from the question and therefore cannot be added to the store of known facts. Which, in turn, means that very few number facts can be instantly recalled or relied upon, so that every new fact must be calculated afresh.

In order to help such pupils make progress, it is essential to teach them to replace their ones-based approach with chunking techniques. The aim is to minimise the number of calculation steps in order to increase a pupil's chances of achieving a correct solution in a reasonable amount of time and without putting any undue strain on working memory. Eddie Gray, writing in *Teaching & Learning Early Number*,[1] explains how a reliance on counting inhibits flexibility and puts forward the idea that in order for newly calculated facts to be laid down in long-term memory the counting process must be compressed.

Pupils who habitually count on their fingers need to be given lots of opportunities to work through targeted activities that help them become thoroughly familiar with the number bonds of the first ten whole numbers, and then, by extension, of all the whole numbers up to twenty. Pupils must also engage in activities that encourage them to partition numbers into suitable components and to manipulate the components, rather than collections of single units, as they work towards a solution. The objective is for pupils to replace their immature habits with more efficient methods. However, pupils will, understandably, be reluctant to relinquish their well-established counting habits before they feel absolutely secure about any new approach. It is only after plenty of practice, therefore, that pupils will begin to accept that working with components is better than working with ones.