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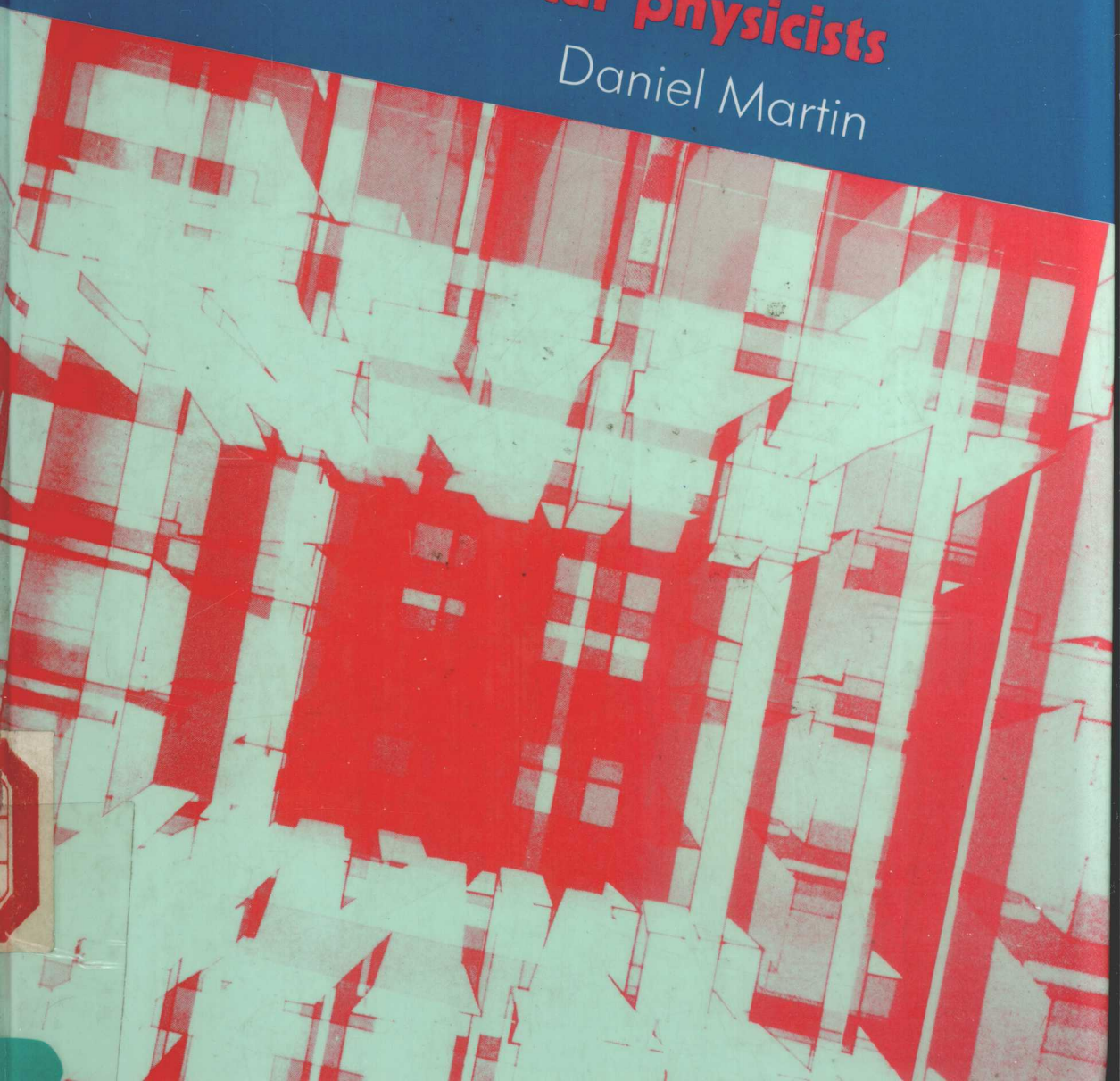
an introduction for mathematical physicists

Daniel Martin



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An Introduction for Mathematical Physicists

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To the memory of my mother Rose Martin

Preface

The aim of this book, which is based on courses of lectures given over many years to first-year research students in theoretical physics in this university, is to give a comprehensive account of basic manifold theory suitable for students who have completed an honours degree in physics or in mathematics and are about to begin postgraduate work in theoretical physics, in particular, relativity. The treatment is rigorous but less condensed (and therefore I hope easier to follow) than that given in some of the books written primarily for pure mathematicians and demanding considerable mathematical maturity on the part of the reader. For various reasons the proofs of a few theorems, especially among those in the latter part of the book have been omitted. Since the prerequisites for the study of manifold theory include a knowledge of certain parts of linear algebra not normally included in a first course on that subject either for mathematicians or for physicists, the book begins with a chapter dealing with these matters. Likewise, two appendices on topology have been included for the benefit of those readers who have not been exposed to an undergraduate course on the subject. Throughout the book numerous examples both worked and unworked have been provided. Any student who has conscientiously worked through the book should be well prepared for a study of the larger and more specialist books on manifold theory and related subjects.

In writing my lecture notes I was, of course, greatly influenced by the standard books on the subject and I acknowledge here my indebtedness to them, and, indeed, to all but a few of the books listed in the Bibliography. Where I have leaned heavily on a particular book, I hope that I have made that clear in the text. Finally, I wish to thank my colleagues Dr E. A. McHarg and Dr J. B. Hickey for valuable comments on certain chapters of the book, and Mr M. Shardlow of Ellis Horwood Ltd for the care that he has taken with the manuscript.

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