# Applications of Statistics

Paruchuri R. Krishnaiah Editor

# APPLICATIONS OF STATISTICS

PROCEEDINGS OF THE SYMPOSIUM HELD AT WRIGHT STATE UNIVERSITY, DAYTON, OHIO, 14-18 JUNE 1976

Edited by

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# APPLICATIONS OF STATISTICS

### PREFACE

Statistical techniques play an important role in drawing meaningful conclusions from non-deterministic data in many disciplines. Therefore, there is a great need to improve the interaction between statisticians and scientists in other disciplines who are interested in applying statistical methodology. A Symposium on Applications of Statistics was organized under the sponsorship of the Air Force Flight Dynamics Laboratory to increase such interaction and to stimulate research in statistical sciences. This symposium was held at Wright State University, Dayton, Ohio, during the period June 14-18, 1976. Professor J. Neyman, who has made many pioneering contributions to the field of statistics, delivered the Inaugural Address entitled "Experimentation with Weather Control and Statistical Problems Generated by It". The program consisted of sessions with invited papers and contributed papers as well as clinical sessions. In the clinical sessions scientists and engineers in the U. S. Air Force presented practical problems that require solution.

This book consists of the invited papers presented at the symposium. In these papers, distinguished scientists discuss the current state-of-the-art on a very broad spectrum of topics in the field of applications of statistics. Some of the areas of applications covered include acoustics, cancer research, cluster analysis, communication, econometrics, hydrology, meteorology, model building, pattern recognition, pharmacokinetics, psychometrics, reduction of dimensionality, reliability, stability of structures, and turbulence. Unfortunately, the contributed papers and the presentations made at the clinical sessions are not included in this book due to the limitation of space, but their titles are listed at the end of the book.

I wish to thank Dr. R. Kahal (Office of the Assistant Secretary, Research and Development, U. S. Air Force) for making the Opening Remarks. Also, I wish to express my appreciation to Dr. D. Zonars (Air Force Flight Dynamics Laboratory) and Dr. J. V. Murray (Wright State University) for welcoming the delegates. Thanks are due to Professors R. E. Bargmann, A. P. Basu, C. B. Bell, R. Elashoff, K. S. Fu, S. S. Gupta, T. Jayachandran, R. H. Jones, M. Kabrisky, G. Kallianpur, F. Kozin, C. Maneri, M. R. Paruchuri, M. L. Puri, A. Ramakrishnan, C. R. Rao, J. R. Van Ryzin, D. G. Shankland, N. Timm, C. P. Tsokos, V. M. R. Tummala and Drs. J. S. Bendat, S. Dubey, I. I. Gringorten, H. Hughes, R. Launer, W. R. Riggan and E. A. Robinson for presiding over different sessions. Thanks are also due to Professors R. E. Bargmann, L. Kanal, C. G. Khatri, S. Kotz, K. V. Mardia, F. Proschan, J. S. Rustagi, S. Saunders, S. Sclove, B. K. Shah and V. R. R. Uppuluri for refereeing the papers. I am grateful to Drs. H. L. Harter and I. N. Shimi for their help in the organization of the symposium and to Professors J. C. Lee and C. Maneri for their help in making the local arrangements. I wish to thank Colonel M. Duggins for his encouragement and Mrs. R. Kish for typing the manuscript patiently.

PREFACE

I am indebted to Professor J. Neyman for delivering the Inaugural Address. Special thanks are due to the contributors to this volume and to the North-Holland Publishing Company for their excellent cooperation. I wish to express my appreciation to the Department of Statistics at Carnegie-Mellon University, the Department of Mathematics and Statistics at University of Pittsburgh and the Air Force Flight Dynamics Laboratory for providing the necessary facilities to edit this volume. Last but not least, I wish to thank my wife, Indira, for her constant encouragement in the organization of the symposium.

P. R. Krishnaiah

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# EXPERIMENTATION WITH WEATHER CONTROL AND STATISTICAL PROBLEMS GENERATED BY IT\*

# JERZY NEYMAN

Statistical Laboratory, University of California
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- 4. Convincing Domestic Freezer Experiment
- First Disenchantment: "Report of the Advisory Committee on Weather Control"
- 6. Some Findings in the Theory of Statistics

# III. SECOND PERIOD OF MY INVOLVEMENT, 1964 TO PRESENT

- 7. Arrival of Seven Annual Reports on Grossversuch III
- 8. Special Weather Modification Conference at the Fifth Berkeley Symposium on Mathematical Statistics and Probability
- 9. Our Use of the Grossversuch III data
- 10. Difficulties in Using Old Experiments to Verify New Hypotheses
- 11. Verification of Generality of the Findings at Grossversuch III
- 12. Hypothetical Mechanisms of the Far Away Losses of Rain at the Arizona  $\mbox{\it Experiment}$

# I. INTRODUCTION

I feel honored by being asked to give this inaugural address at the present Symposium attended by so many distinguished scholars, some of them specialists in mathematical statistics, and others, specialists in the various domains of applications.

I must confess that the pleasure I experience by being so honored is augmented by a particular circumstance of personal character. While thinking of what I might present that would be likely to be interesting to the distinguished audience, I suddenly realized that the present year, 1976, marks the quarter of a century

<sup>\*</sup>Inaugural Address

2 JERZY NEYMAN

since the first report on weather modification has been issued by our Statistical Laboratory in Berkeley. In fact, this first report of ours [1] is dated December 15, 1951. The work we did was in response to the request of the Division of Water Resources, Department of Public Works of the State of California. Since that time, we worked on the subject of weather control almost continuously. Thus, to me personally, the present occasion is very much like a celebration of the silver jubilee of continuing efforts in an important domain of study!

During the past quarter of a century a large number of problems came to our attention. Quite a few of them were "substantive" in character, as was the question of the Division of Water Resources in Sacramento. This question was whether the enthusiastic claims of success voiced by the many commercial cloud seeding operators in California should be taken seriously. The specific question was whether reliable evidence exists that cloud seeding represents a realistic resource of water much needed in California, particularly in the southern part. Here, then, we had to deal with a "substantive" problem concerned with rainfall or snowfall.

However, as is almost invariably the case, the substantive problem relating to a natural phenomenon generated a number of theoretical statistical problems which we tried to solve. Then, there were repeated "feed-backs", etc. In this process, certain new statistical methodologies were developed and also certain substantive findings occurred. We believe, or should I say we have the illusion, that these substantive findings are quite important. To my personal regret, the cloud seeding community does not like our results, that is, apart from a few exceptional individuals. Just too bad!

It seems to me that a reasonable plan for the present paper might consist of a necessarily brief sketch of three distinct periods of our research, in a summary of our findings, and in pointing out certain regrettable phenomena occurring in the weather modification literature. Hopefully this plan would provide a reasonably comprehensive picture of the present state of the domain as seen from our Statistical Laboratory in Berkeley. Curiously, some of our substantive findings resulted not from some kind of systematic search, but unexpectedly. Then systematic work was needed to confirm the finding.

# II. FIRST DECADE, 1951 - 1960

1. Randomized Experiments vs. Non-Randomized Cloud Seeding Operations.

As I see it, the greatest achievement of R. A. Fisher consists in inventing, in developing and in propagandizing the idea that, when working with variable material, the randomization of experiments is a necessary condition of their reliability. A very interesting historical sketch of this subject was recently published by Cochran [2]. One of the interesting details is that some of the difficulties with

non-randomized trials were noted more than two centuries ago. The individual in question is described by Cochran as Arthur Young, "the great English experimentalist." The year given is 1764! One of the difficulties pointed out is that the experimentalists tend to have personal preferences to the treatments they study. The unavoidable results are first self-deception and later deception of others. As Cochran states, the general remarks of Arthur Young are as much relevant now as they were in 1764.

2. Efforts to Learn the New Field of Study. In the early 1950's, our efforts were directed towards learning the field of study which at the time was new to us, and towards convincing the authorities in Sacramento that, because of the lack of randomization of the commercial cloud seeding operations, the successes claimed by the operators should be taken with a degree of skepticism [3, 4].

It will be noticed that the reports quoted were authored or coauthored by several individuals who, at the time, were in early stages of their respective careers, but now hold responsible positions in various institutions. In alphabetical order they are: Arnold Court (now Professor at California State University, Northridge), Terry A. Jeeves (now with the Westinghouse Company), Lucien M. LeCam (perhaps the "most mathematical" leader in the field of mathematical statistics, deeply interested in applied fields such as immunotherapy of cancer), Elizabeth L. Scott (our former Department Chairman and a respected colleague), George Steck (now with Sandia Corporation), B. V. Sukhatme (now Professor at Iowa State University), and several others.

With the kind help of Edward M. Vernon, then Chief Regional Forecaster of the U. S. Weather Bureau in San Francisco, it did not take us too long to learn some of the background, sufficient to initiate certain studies of our own. The point was that, while the idea of the necessity of randomization as a basic prerequisite to reliability of experimentation was clear to me, as well as to my colleagues, the prospects of success in our efforts to convince the authorities depended very much on producing some "tangible" evidence invalidating the evaluations of the commercial cloud seeders.

The methodology of evaluating non-randomized cloud seeding operations used at the time, and also now, is the so-called "historical regression" methodology. Consider two not very distant areas, one being the area in which the operator contracted to increase the precipitation, called "target", and the other called "control". The presumption was, and continues to be, that cloud seeding over the target cannot possibly affect the precipitation over the control.

Using precipitation data available for a few years before the beginning of the cloud seeding era, one establishes the linear regression line of the target precipitation, say  $\underline{\mathbf{Y}}$ , on the control precipitation, say  $\underline{\mathbf{X}}$ . (Frequently, this is done with  $\underline{\mathbf{X}}$  and  $\underline{\mathbf{Y}}$  not expressed in inches of actual precipitation, but in terms of some