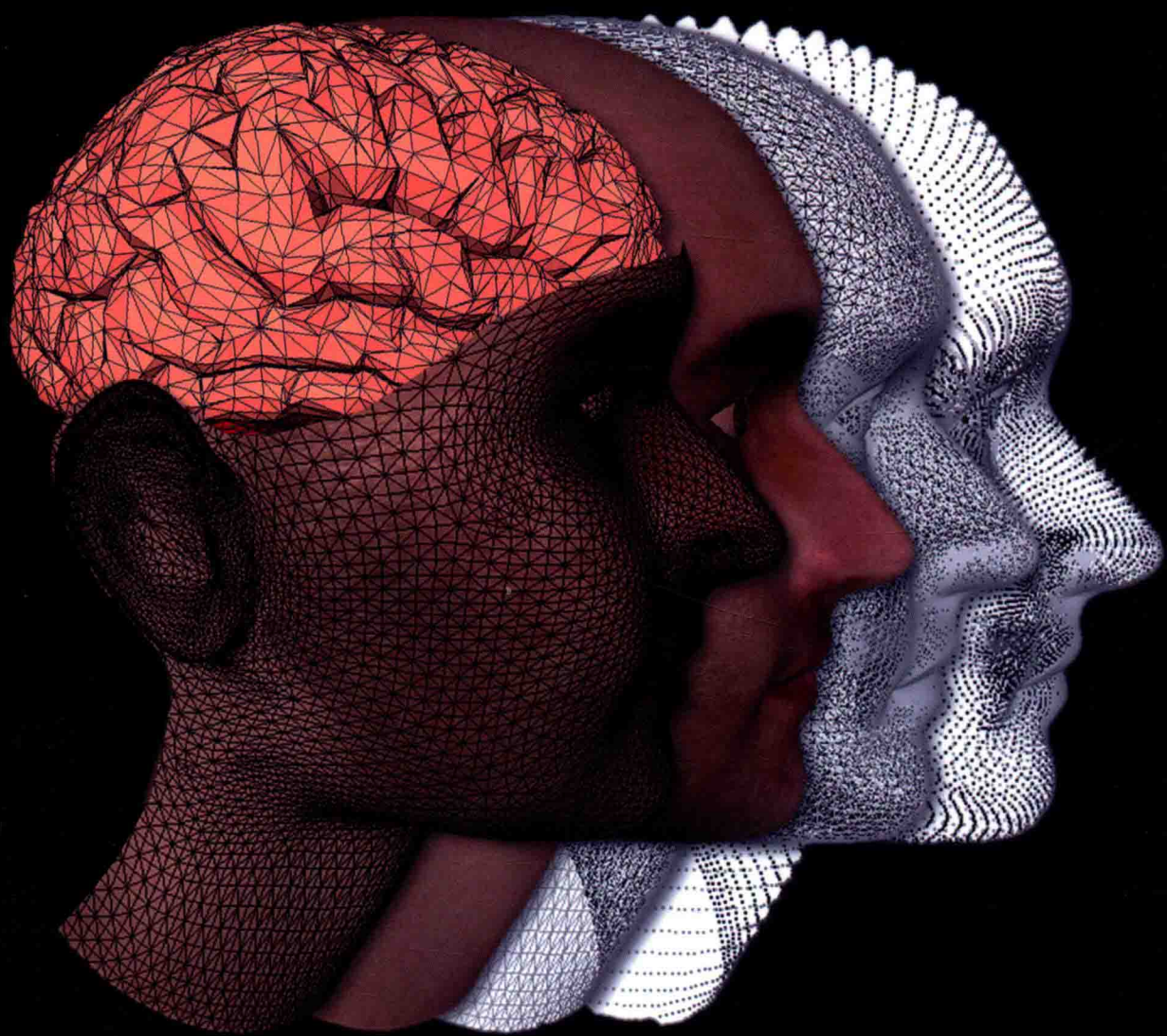


ALY A. FARAG

Biomedical Image Analysis

Statistical and Variational Methods



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ALY A. FARAG

University of Louisville



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Biomedical Image Analysis

Statistical and Variational Methods

Ideal for classroom use and self-study, this book explains the implementation of the most effective modern methods in image analysis, covering segmentation, registration, and visualization, and focusing on the key theories, algorithms and applications that have emerged from recent progress in computer vision, imaging, and computational biomedical science.

- Structured around five core building blocks – signals, systems, image formation, and modality; stochastic models; computational geometry; level-set methods; and tools and CAD models – it provides a solid overview of the field.
- Mathematical and statistical topics are presented in a straightforward manner, enabling the reader to gain a deep understanding of the subject without becoming entangled in mathematical complexities.
- Theory is connected to practical examples in X-ray, ultrasound, nuclear medicine, MRI and CT imaging, removing the abstract nature of the models and assisting reader understanding, whilst computer simulations, online course slides, and a solution manual provide a complete instructor package.

Aly A. Farag is Professor of Electrical and Computer Engineering, and the founding Director of the Computer Vision and Image Processing Laboratory, at the University of Louisville. His research interests center around object modeling with biomedical applications, and his more recent biomedical inventions have led to the development of improved methods for tubular object modeling, virtual colonoscopies, lung nodule detection and classification based on CT scans, real-time monitoring of vital signs from thermal imaging, and image-based reconstruction of the human jaw. He is a Fellow of the IEEE.

“This is a comprehensive book on the topic of biomedical image analysis. It covers both statistical and variational approaches as well as some of the foundations of image acquisition. The individual chapters and sections also include practical examples, meaningful exercises, and computer labs. The book is an outstanding and thorough introduction to the field of biomedical image analysis and is suitable both for classroom use and self-study. A well curated bibliography provides starting points for additional study.”

Ron Kikinis
Harvard Medical School

**To my dear wife *Salwa* – thank you for love, support and dedication
to our blessed family!**

Preface

About two decades ago, I worked with the University of Louisville College of Engineering and the Office of the Vice President for Research to establish the Computer Vision and Image Processing Laboratory (CVIP Lab – www.cvip.uofl.edu) as a multidisciplinary environment for research, teaching, and training in computational image analysis. Over the years, the CVIP Lab has been home to researchers in engineering, medicine, dentistry, mathematics, and psychology who are interested in imaging. The support of the University of Louisville administration and colleagues at various units literally made the CVIP Lab a place that I miss whenever I am away from it, even for an enjoyable vacation.

At the CVIP Lab we pushed agendas for imaging research, from basics to applications, and in the process established immediate and auxiliary but essential infrastructure. Among the auxiliary infrastructure has been high-speed networking to link the University to what has become known as Internet 2, an initiative funded by the National Science Foundation, and to link the main campus (Belknap) to the Health Science Campus (HSC) a few miles away. The auxiliary infrastructure included supercomputers and immersive visualization. The essential hardware included high-end computing and graphics workstations, object scanners, and various laboratory benches for electronic design and testing. The laboratory has been visited by researchers, potential engineering students, faculty candidates in engineering, dentistry and medicine; its research activities have been showcased on national and local media, and the University President (John Shumaker) and the Dean of Engineering (Thomas Hanley) recorded advertisements there as the University pushed to promote biomedical research, and to establish a biomedical engineering department, during 1996–2002. Today the CVIP Lab is well recognized by colleagues elsewhere. Research at the laboratory has been funded by the National Science Foundation (NSF), the National Institutes of Health (NIH), the Department of Defense (DoD), the Department of Homeland Security (DHS), Norton and Jewish Hospitals, and various government and industrial organizations.

At the CVIP Lab I have had the privilege and pleasure of coaching some of the most brilliant students from around the world, and have supervised and hosted a large number of postdoctoral researchers and researchers who have spent sabbaticals and short visits here. The laboratory has three main areas of focus: computer vision, biomedical imaging, and biometrics. Students and researchers at the laboratory have worked on theoretical, algorithmic and practical domains of these three focal areas. A number of courses, seminars, and presentations have been created over the years to train researchers and promote research at the laboratory. This book is one such result of the activities at the

CVIP Lab. It offers both basic background and sample research problems on the subject of biomedical image analysis.

As the CVIP Lab has been the nest of so many brilliant researchers and students with whom I have worked with over the years, I will list only a few who have made an impact on me. First and foremost is Dr. Darrel Chenoweth, the Chairman of the ECE Department from 1994 to 2004. Darrel gave me unconditional support and encouragement; no words are enough to thank him for his impact on me. Thomas Hanley, Dean of J.B. Speed School of Engineering during 1992–2004, was a visionary who gave me freedom to think and never hesitated to provide support. Without him and Darrel, the CVIP Lab would have not been established. Dr. Nancy Martin, Vice President for Research during 1996–2006, kept the CVIP Lab on her radar and provided support whenever asked. Former Dean Mickey Wilhelm and current Dean Neville Pinto have maintained this trend of support, as did Dr. James Graham, ECE Department Chairman during 2006–2013.

More than 50 colleagues have collaborated with me at the CVIP Lab, so I will just mention a few: Dr. Christopher Shields, former Chairman of Neurological Surgery, Dr. Allan Farman, Professor of Dental Radiology, Dr. Thomas Starr, Associate Dean for Research, Dr. Manuel Casanova, Professor of Psychiatry, Dr. Edward Essock, Professor of Psychology and Brain Sciences, and Dr. Robert Falk, Director of Medical Imaging at Jewish Hospital, have been longstanding collaborators and friends with whom I have enjoyed working, and their support has been crucial to whatever I have achieved at the CVIP Lab.

Three scientists and engineers of the highest caliber and professionalism – Charles Sites, Mike Miller and Salwa Elshazly – made me anxious to come to the CVIP Lab; and if I go away, I can trust that it is safe in their hands. Chuck is a visionary; he and I wrote proposals that brought high-speed networking, computing, visualization and autonomous robotics to the University of Louisville. We have worked together since 1996. Salwa agreed to assist me at the Laboratory in 1997 and for 15 years has provided intellectual support and coordination of efforts that have been crucial for success at the laboratory. Mike joined the laboratory in 2006 after two decades of work in the industry. He showed unbounded dedication and rare talent in almost every aspect of computing and circuit design, and has handled the university regulations for biomedical data. Mike has been my right hand in student advising, documentation of research, and communications with the university as well as funding agencies. Many other technical staff have helped the CVIP Lab; too many to list, but I appreciate all their efforts and assistance.

As I mentioned before, I have been privileged to coach some of the most brilliant people from around the world for their Ph.D. and Masters research. They have worked on my funded projects and have excelled in executing the research plans and expanding them to frontiers that I could not imagine, or perform, alone. I owe a great deal of appreciation and thanks to each of them. In this book, I must acknowledge the following: Dr. Mohamed Sabry, Dr. Hossam Abdelmunim, Dr. Asem Ali, Dr. Rachid Fami, Dr. Shireen Elabian, Dr. Amal Farag, Dr. Ham Rara, Dr. Melih Aslan, Mr. Ahmed Shably, Dr. Mostafa Abdelrahman, Ms. Marwa Ismail, and Dr. Aly Abdelrahim. These 12 individuals have had a direct impact on this book and I owe them my deepest appreciation. In particular, Dr. Shireen Elhabian and Dr. Ahmed Shalby have shown

tenacity, intelligence and dedication in assisting me throughout the preparation of this book; I remain very grateful to both of them.

Funding from various organizations and support of the University of Louisville are gratefully acknowledged, as is a long stream of local, national and international collaborators with whom I have had the honor and pleasure to collaborate and interact over three decades.

I must state the obvious: all errors and mishaps in the book are mine. I shall be grateful for any hints from readers that might assist in improving the text in revised prints or new editions. Together with Cambridge University Press, I have a website for auxiliary material including teaching aids, newer homework problems and laboratories, solutions to problems, and codes for the implementations in the book.

Michelle Carey and Elizabeth Horne at Cambridge University Press have provided encouragement throughout this project, Lindsay Nightingale provided a most skilled review of the manuscript, and Christina Sarigiannidou managed the book production. They were very patient with me and worked around my schedule, despite my endless obligations to the CVIP Lab and derailment by circumstances, not the least of which has been the engagement of my mind and soul with the events in my beloved home country of Egypt. Since 2011, countless hours of thought have been spent engaging with my compatriots, family members and officials, pushing for the common good and towards peaceful democratic changes in a country that the entire world wishes to see peaceful and prosperous. I thank Michelle, Elizabeth, Lindsay and Christina for their help, and repeat my highest appreciation to the person most deserving of thanks and appreciation, my dear wife, collaborator, and friend, Salwa A. Elshazly.

Aly A. Farag

Nomenclature

The following conventions are used throughout the document.

Symbol	Description
\mathbf{x}	Point in 2D or 3D Cartesian space
\mathbb{R}	Set of real numbers
Ω	An open bounded subset of \mathbb{R}^n
$\Omega \setminus \Omega_0$	Complement of Ω_0 in Ω
\in	Element of
\subseteq	Subset of
$ \cdot $	Absolute value in \mathbb{R}
$\ \cdot\ $	Euclidean norm in a vector space
\mathcal{C}	A curve or family of curves in \mathbb{R}^2
ϕ	Level set function
Φ_S	Implicit representation of a given shape S
$\tilde{\phi}$	Implicit representation of a shape prior
$L(\cdot)$	Labeling function
\mathcal{A}	Rigid or affine transformation in \mathbb{R}^n
S	Scale matrix
\mathcal{R}	Rotation matrix
θ	Rotation angle
T	Translation vector
$\mathbf{u} = (u_i)_{1 \leq i \leq n}$	Displacement field in \mathbb{R}^n
F	Speed function
V	Vector distance function
$div(\cdot)$	Divergence of a vector field
$\mathcal{D}(\cdot)$	Dissimilarity measure
$\mathcal{D}^{MI}(\cdot)$	Mutual information dissimilarity measure
$\mathcal{D}^{SSD}(\cdot)$	Sum of squared differences dissimilarity measure
$\mathcal{R}(\cdot)$	Regularization term
\mathcal{M}	Manifold in \mathbb{R}^n
$d(\mathbf{x})$	Distance from $\mathbf{x} \in \mathbb{R}^n$ to a manifold \mathcal{M}
$\mathbb{D}(\mathbf{x})$	Squared distance from $\mathbf{x} \in \mathbb{R}^n$ to a manifold \mathcal{M}

$\delta(\cdot)$	Dirac function
$H(\cdot)$	Heaviside function
α_i	Weight of shape energy
D	Space dimension, degrees of freedom
$W(\cdot)$	Transform on \mathbb{R}^D
\mathbf{t}	Translation vector
$L(\cdot)$	Linear transform on \mathbb{R}^D
\mathbf{L}	Matrix form for a linear transform $L(\cdot)$ on \mathbb{R}^3 with matrix elements ℓ_{ij}
s	Scaling parameter in the case of uniform scaling
s_x, s_y, s_z	Scaling parameters in the case of non-uniform scaling
\mathbf{S}	Matrix form for a scale transform on \mathbb{R}^3 with scale parameter(s) $s \in \mathbb{R}$ in the case of uniform scaling and $s_x, s_y, s_z \in \mathbb{R}$ in the case of non-uniform scaling
t_x, t_y, t_z	Translation parameters
\mathbf{T}	Matrix form for a translation transform on \mathbb{R}^3 with translation parameters $t_x, t_y, t_z \in \mathbb{R}$
$\mathbf{R}_{x,\alpha}$	Matrix form for a rotation transform on $\mathbb{R}^2 \setminus \mathbb{R}^3$ about the x -axis by angle of rotation α
(α, β, γ)	Euler rotation angles
$E(\alpha, \beta, \gamma)$	Euler transform
\mathbf{v}	(Eigen)vector in \mathbb{R}^D
\mathbf{A}	Square transformation matrix defined on $\mathbb{R}^{D \times D}$
λ	Eigenvalue of matrix \mathbf{A}
$ \cdot $	Matrix determinant
\mathbf{r}, \mathbf{q}	A quaternion
q_0, q_1, q_2, q_3	Elements of a quaternion \mathbf{q}
$\mathbf{V}_{\mathbf{q}}$	An ordinary vector defining the complex part of a quaternion \mathbf{q}
q_x, q_y, q_z	The imaginary parts of a quaternion \mathbf{q}
i, j, k	Basis defining the imaginary parts of a quaternion
\mathbf{R}	Orthogonal matrix defining the multiplication of two quaternions
$\bar{\mathbf{R}}$	Same as \mathbf{R} , except that the lower right-hand 3×3 submatrix is transposed
\mathbf{q}^*	Conjugate of a quaternion \mathbf{q}
\mathbf{I}	4×4 identity matrix
M	Model shape represented by a set of points $\{\mathbf{m}_i\}$
N_m	Number of points in the model shape
P	Model shape represented by a set of points $\{\mathbf{p}_i\}$
N_p	Number of points in the scene shape
$R(\cdot)$	Rotation operator
$d(\mathbf{p}, \mathbf{m})$	Euclidean distance between two points $\mathbf{p}, \mathbf{m} \in \mathbb{R}^3$
$d(\mathbf{p}_i, M)$	Euclidean distance between a scene point \mathbf{p}_i and the model point set M
Y	Closest point in the model set which yields the minimum distance
C	The closest point operator
\mathbf{e}_i	Residual error for each point pair
E	Sum of squares of the residual error for each point pair

μ_p	Centroids/origins of the shape defined by the point set P
\mathbf{p}'_i	Centered (zero-means) points
\mathbf{t}'	Translation vector of the centered points
$\mathcal{S}_y, \mathcal{S}_p$	Sums of the squares of the distances between points and their centroids
\mathbf{S}	The S-matrix whose elements are sums of products of coordinates measured in the scene shape with coordinates measured in the model shape
$p(x_k)$	Probability of the outcome x_k
$I(x_k)$	Information measure of the outcome x_k
$H(X)$	Entropy of the random variable X
$H(X, Y)$	Joint entropy of the random variables X and Y
$p(x_i, y_j)$	Joint probability density function of the two random variables, i.e. the probability of having both outcomes x_i and y_j occur together
$H(X Y = y_j)$	Conditional entropy of X given $Y = y_j$
$H(X Y)$	Conditional entropy of X given Y
$I(X, Y)$	Mutual information between X and Y
$p_X(x)$	Marginal probability mass function of the random variable X
$p_{XY}(x, y)$	Joint probability mass function of the random variables X and Y
T_Θ	A transformation with registration parameters Θ
R	Random variable denoting the intensities observed in the reference volume
F	Random variable denoting the intensities observed in the floating volume
$T_\Theta F$	Random variable denoting the intensities observed in the transformed floating volume
$h(\cdot)$	Normalized joint histogram

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