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# Advances in Gamma Ray Resonant Scattering and Absorption

Long-Lived Isomeric Nuclear States



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Long-Lived Isomeric Nuclear States

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# Preface

This book was written by an experimenter. It summarizes the results of my fifty-year work at A.I. Alikhanov Institute of Theoretical and Experimental Physics on problems of gamma ray interaction with nuclei. I tried to reveal the physical meaning of these results, making the exposition as simple as possible and sometimes resorting to arguments and derivations that could seem insufficiently strict, at least to orthodox theorists. The main part of the book addresses the problem of studying resonant gamma ray absorption and scattering by nuclei. These processes, which are essentially the simplest nuclear reactions, permit, if studied profoundly, revealing very interesting special features that are inherent in phenomena of gamma ray emission and absorption by nuclei, and which are seemingly of a general character. It is noteworthy that the concepts of the nature of the photon that are prevalent among the physics community are inaccurate in many respects, even sometimes erroneous. In particular, the assignment of a well-defined frequency  $\nu$  to a photon of energy  $E = h\nu$  is an approximation because a monochromatic harmonic oscillation is infinite in time, but by no means does a photon, which is produced at specific instant, exist limited during time, ending up in absorption inside a detector or in some substance. This means that the Fourier frequency spectrum of a photon must have a finite width. Also, opinions on the particle-wave duality of the photon differ widely. Recently, an article of the present author where resonant gamma ray scattering on nuclei was considered and where a photon was shown to manifest a spatial and a time extent in this process was rejected by an authoritative Russian physics journal on the basis of reviewer's evaluation. The argument of the reviewer was that the photon is a particle because it experiences photo-absorption even in very finely dispersed powders, and therefore cannot have extensive dimensions. Of course, the statement of the reviewer that, in processes like the photoelectric effect, photons behave as almost quasi point objects, not displaying wave properties, is correct. The same reveals in the behavior of photons in Compton scattering by electrons. However, the other processes exist in which the photon interacts with matter behaving itself like a wave of macroscopic size, not showing any particle property. In the monograph by Robert Wood "Physical Optics" [1], there is a description of an experiment where one observes light diffraction at a grating 3 cm

long and measures the resolution of the grating. After covering half the grating with a screen, the resolution becomes lower by a factor of two. Since a stationary diffraction pattern arises owing to the interference of a photon with itself (the interference between two photons cannot lead to a stationary pattern because of a stochastic character of the phase difference), this means that, under conditions of the experiment being discussed, photons have a size not smaller than 3 cm. Bragg scattering in crystals is yet another process of this type, but, here, it is gamma ray photons rather than optical photons behave as extended waves. In this process, each photon interacts with all crystal atoms within its absorption length, exhibiting no particle properties. A very convincing example is provided by an experiment of a group headed by V.K. Voitovetsky [2], where gamma rays of the  $^{181}\text{Ta}$  nuclide were transmitted between the cogs of a rotating gear, the shape of the detected gamma line being measured with the aid of a Mössbauer spectrometer. It was found that, at a large number of gear revolutions per unit time such that the gap between the cogs traverses the gamma beam within 0.1 of the mean lifetime of source nuclei in the excited state, the measured width of the Mössbauer gamma line was much larger than that in the case of a very slow rotation of the gear. This obviously indicated that the gear cogs interrupted the spatially extended wave train of a photon because wave trains shorter than natural ones corresponded to gamma lines of width larger than the natural width. We would like to emphasize that, in no physics process, a photon demonstrates its wave and particle properties simultaneously—either the former or the latter. After being involved in Bragg scattering in a crystal, a photon is recorded by a detector in an event of photo-absorption or Compton scattering; that is, the photon behaves as a particle that lost completely the wave properties that it has just revealed. However, this does not mean that the wave transformed into a particle immediately after Bragg scattering. If, instead of a detector, one places a second crystal on the path of a photon that experienced Bragg scattering, and if the Bragg conditions hold in this crystal, then the photon would be able to undergo Bragg scattering once again with a sizable probability—that is, to exhibit anew its wave properties. At the same time, a photon that has shown particle properties in an event of Compton scattering in a detector can thereupon interact in a wave manner with a crystal (under Bragg conditions other than those in the first case, because the photon energy changed after scattering), transforming from a particle into a wave again. The question of how and why such transformations occur is one of the most mysterious in modern physics.

The ensuing exposition is organized as follows. In the first chapter, we consider theoretically the process of resonant gamma ray scattering by nuclei. We are interested in a question of how the angular distribution of resonantly scattered gamma rays depends on the perturbing action of magnetic fields. Solving this particular and seemingly trivial problem, we arrive at conclusions that give sufficient grounds to take a fresh look at some special features of processes involving gamma ray emission and absorption by nuclei. In the second chapter, we describe experiments performed by our group and devoted to measuring unperturbed and magnetic-field-perturbed angular distributions (ADs) of resonantly scattered gamma rays of  $^{182}\text{W}$  and  $^{191}\text{Ir}$ . Those experiments confirmed the prediction of the theory

that the result of perturbing ADs depends on the width of the spectrum of gamma rays incident to a resonant gamma ray scatterer. At the end of this chapter, we show that important conclusions follow from the theoretical and experimental data described in it: the mean lifetime of nuclei in an excited state depends on the mode of its excitation, and processes of gamma ray emission and absorption by nuclei have a protracted character. In the third chapter, we consider in detail the problem of gamma resonant excitation of long-lived isomeric states of nuclei. Experimental investigations of this problem revealed a glaring contradiction between present-day theoretical predictions, which require, among other things, that the Mössbauer gamma line emitted in the decay of  $^{109}\text{Ag}$  nuclei that were in the isomeric excited state characterized by an energy of 88.03 keV and a mean lifetime of 57 s must be broadened by five to six orders of magnitude in relation to the natural width, and the experimental results of three research groups (including ours), which obtained data indicating that the relative broadening of this gamma line does not exceed one to two orders of magnitude. So small a broadening of the Mössbauer gamma line of the  $^{109\text{m}}\text{Ag}$  isomer permitted implementing the idea of a gravitational gamma spectrometer and directly measuring the profile of the Mössbauer gamma resonance in this isomer. The use of a traditional Mössbauer spectrometer for this purpose is technically impossible because this would require creating a device capable of moving a gamma source with respect to the absorber at a velocity of about  $10^{-12}$  cm/s; that is, it would be necessary to push it forward over a distance per second nearly equal to the diameter of the silver-atomic nucleus, and to measure simultaneously this velocity by some method. The principle of operation of the gravitational gamma spectrometer based on the  $^{109\text{m}}\text{Ag}$  isomer is described in the fourth chapter. Its resolution is about eight orders of magnitude higher than the resolution of usual Mössbauer spectrometers employing gamma rays of the  $^{57}\text{Fe}$  nuclide. In the next chapter, we describe our experiments devoted to exploring the resonant scattering of annihilation photons by nuclei, whereupon (in the last chapter) we show how one can use this phenomenon to study the shape of Fermi surfaces in metals.

Some other experiments performed by our group with gamma rays are discussed at the end of this book along with the ideas of experiments that have yet to be conducted.

Some of the experiments described here were performed by methods that seem obsolete from the modern point of view, but I deemed it necessary to tell about them because they were an inalienable link in the chain of experiments that led to important conclusions both in what is concerned with the dependence of the mean lifetime of nuclei in an excited state on the method of excitation and in what is concerned with the duration of nuclear radiative processes.

One comment on the notation used is in order. Vector quantities appearing in some equations are printed in boldface.



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It is my pleasant duty to record here the benefit of working over many tens of years side by side with Yury Denisovich Bayukov, Yury Nikolaevich Isaev, and Mark Mikhailovich Korotkov, who are members of our research group and who made an invaluable contribution to the implementation of the experiments described in this book. I am also indebted to my son Professor V.A. Davydov for valuable advice and help in solving some mathematical problems, and to my second son Anton, my wife Nina Mikhailovna, and to the scientist from our group Yuri B. Novozhilov for their very valuable help in the preparation of this book for printing.

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I will always mourn the untimely death of the collaborators of our group Vladilen Grigor'evich Alpatov, Gavriil Romanovich Kartashov, Vadim Mikhailovich Samoylov, Galina Eugen'evna Bizina, Mikhail Georgievich Gavrilov, Gennadiy Victorovich Rotter, and Yury Ivanovich Nekrasov and cherish memory of their selfless work, which ensured the success of our experiments.

I recall with gratitude my first supervisor Professor N.A. Burgov, who introduced me in the realms of resonant gamma ray scattering, and the first director of ITEP Academician A.I. Alikhanov, whose permanent attention to our work and support were invaluable. I nourish warmest recollections of Professor A.L. Suvorov, who was ITEP's director until his untimely death in 2005. His support of our investigations was a great help to us all, and his attitude to me personally was highly benevolent.

A.V. Davydov

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# Chapter 1

## Theory of the Resonant Scattering of Gamma Rays by Nuclei in a Magnetic Field

### 1.1 Introduction

In this chapter, we consider the problem of magnetic-field-induced perturbation of the angular distribution of resonantly scattered gamma rays. Originally, interest in this problem arose in connection with the possibility of employing a magnetic field perturbing the angular distribution of resonantly scattered gamma rays (ADRSG) to determine magnetic moments of nuclei in excited states that are intermediate in resonant-scattering processes. At first glance, this method is similar to the widely used method for determining magnetic moments of excited nuclei by measuring magnetic-field-perturbed angular correlations of two photons sequentially emitted by a nucleus [1]. Indeed, functions that describe ADRSG and angular correlations for a two-photon cascade (ACG) are identical in the absence of a perturbing field, provided that the spins of initial, intermediate, and final states are identical for all transitions in the two cases and so are the parameters of multipole-mixing ratios. It seemed natural to extend the identity of the descriptions to the same processes occurring in a perturbing magnetic field. In particular, the results of the experiments reported in [2–4] were considered from this point of view. However, it was first shown in [5] that, in the particular case of a pure E2 transition between  $0^+$  and  $2^+$  levels, the result of the perturbation introduced by a magnetic field in ADRSG depends substantially on the hierarchy of the natural width of the excited nuclear level and the characteristic width of the spectral distribution of exciting (resonantly scattered) gamma rays. In [6], this result was generalized to the case of arbitrary mixed transitions. The final results of these two studies refer to the case where the hyperfine-interaction energy  $\mu H$  (here,  $\mu$  is the magnetic moment of the excited nucleus and  $H$  is the strength of the magnetic field affecting the nucleus) is small in relation to the natural width  $\Gamma$  of the excited nuclear state. The most general results, free from constraints on the hyperfine-interaction energy, were obtained by our group in [7] for the case where the magnetic field is perpendicular to the gamma-ray scattering plane. Also given there is an expression for the ADRSG function in the

case where magnetic-field directions are distributed chaotically over nuclei of the sample that scatters gamma rays [8].

Below, a derivation of expressions for ADRSG functions is given for the first two cases on the basis of the method that we used in [7, 8]. First, we follow the computations from [6] and then go over to deducing general expressions. Our line of reasoning is basically the same as that in [1]. Constant factors that appear in intermediate computations, but which do not affect the form of the angular distribution, will be discarded without mentioning this in each specific case.

## 1.2 ADRSG Function for the Case Where the Magnetic Field Is Perpendicular to the Plane of Gamma-Ray Scattering

We represent the time-dependent ADRSG function in the form

$$W(q_1, q_2, t) = \sum_{m_i m_f} A_{if}(q_1, q_2, t) A_{if}^*(q_1, q_2, t) \quad (1.1)$$

where  $q_1$  and  $q_2$  are the wave vectors of, respectively, the incident and the scattered photon and

$$A_{if}(q_1, q_2, t) = \sum_{m_a m_b} \langle I_f m_f | H_2 | I m_b \rangle \langle I m_b | \Lambda(t) | I m_a \rangle \langle I m_a | H_1 | I_i m_i \rangle. \quad (1.2)$$

The quantities appearing in the summand in (1.2) include the following:

$\langle I m_a | H_1 | I_i m_i \rangle$ , which is the matrix element of the operator  $H_1$  for the first transition in the resonant-scattering process (the transition that corresponds to photon absorption and in which the nucleus goes over from the state of spin  $I_i$  and magnetic quantum number  $m_i$  to the state where these quantum numbers are  $I$  and  $m_a$ , respectively);

$\langle I m_b | \Lambda(t) | I m_a \rangle$ , which is the matrix element of the nuclear transition from the excited state of spin  $I$  and magnetic quantum number  $m_a$  to the other sublevel of this state where the magnetic quantum number is  $m_b$ . This transition occurs under the effect of the perturbation operator  $\Lambda(t)$  which, in the case being considered, has the form

$$\Lambda(t) = e^{-\frac{iKt}{\hbar}}. \quad (1.3)$$

In the reference frame where the quantization axis is aligned with the magnetic-field strength vector  $\mathbf{H}$ , the Hamiltonian  $K$  appearing in (1.3) and representing the interaction of the nucleus with the magnetic field is given by

$$K = -(\mu\mathbf{H}) = -g\mu_N H m = \Omega m \hbar, \quad (1.4)$$

where  $\mu$  is the magnetic moment of the excited nucleus,  $g$  is the  $g$ -factor of the nuclear excited state,  $\mu_N$  is the nuclear magneton,  $m$  is the quantum number of the nuclear-spin projection onto the quantization axis  $z$ , and  $\Omega$  is the Larmor frequency of nuclear-spin precession in the magnetic field.

The summand in (1.2) also involves the matrix element  $\langle I_f m_f | H_2 | I m_b \rangle$  of the operator  $H_2$  of the second transition occurring in the resonant-scattering process and corresponding to photon emission from the nucleus, whereupon the nucleus returns to the ground-state sublevel where the magnetic quantum number is  $m_f$ . Obviously,  $I_f = I_i$  in the resonant-scattering process.

Let us first consider the matrix element of photon emission. Since the photon-emission process in resonant scattering does not differ from the analogous process in cascade gamma-ray emission, one may borrow the expression for the matrix element of the final transition from the theory of angular correlations of sequentially emitted photons. This expression is given in [1] [Eq. (19.59)]. In our case, it has the form

$$\begin{aligned} \langle I_f m_f | H_2 | I m_b \rangle = & \sum_{L_2 \mu_2 M_2 \pi_2} (-1)^{-I_f + L_2 - m_b} \begin{vmatrix} I_f & L_2 & I \\ m_f & M_2 & -m_b \end{vmatrix} \langle 0 \sigma_2 | L_2 \mu_2 \pi_2 \rangle \\ & \times \langle I_f || L_2 \pi_2 || I \rangle D_{M_2 \mu_2}^{L_2} (z \rightarrow \mathbf{q}_2) \end{aligned} \quad (1.5)$$

Here, the factor  $\langle 0 \sigma_2 | L_2 \mu_2 \pi_2 \rangle$  is the eigenfunction corresponding to the eigenvalues  $L_2$ ,  $\mu_2$ , and  $\pi_2$  of the operators of, respectively, the angular momentum, its projection on the quantization axis, and parity in the reference frame where the quantization axis coincides with the direction of the photon wave vector  $\mathbf{q}_2$ . Upon the multiplication of this function by the rotation matrix  $D_{M_2 \mu_2}^{L_2*} (z \rightarrow \mathbf{q}_2)$ , it transforms into its counterpart in a reference frame where the quantization axis  $z$  has an arbitrary direction. The corresponding Euler angles are arguments of the  $D$ -functions. The factor  $\langle I_f || L_2 \pi_2 || I \rangle$  is the reduced matrix element of the transition operator. Further,  $M_2$  and  $\mu_2$  are the quantum numbers of the total-angular-momentum projections onto the quantization axes  $z$  and  $\mathbf{q}_2$ , respectively;  $\begin{vmatrix} I_f & L_2 & I \\ m_f & M_2 & -m_b \end{vmatrix}$  is the Wigner  $3j$  coefficient, which is determined by the quantum numbers appearing in it and representing the angular momenta and their projections;  $\pi_2$  is a parity of the radiation wave function; and  $\sigma_2$  is the photon spin.

The expression for the matrix element corresponding to photon absorption may be derived in following way. We represent the matrix element of our interest in the form

$$\begin{aligned} \langle I m_a | H_1 | I_i m_i \rangle &= \langle I m_a | H_1 | I_i m_i \mathbf{q}_1 \sigma_1 \rangle \\ &= \sum_{L_1 M_1 \pi_1} \langle L_1 M_1 \pi_1 | \mathbf{q}_1 \sigma_1 \rangle \langle I m_a | H_1 | I_i m_i L_1 M_1 \pi_1 \rangle \end{aligned} \quad (1.6)$$

where  $\sigma_1$  is the spin of the photon to be absorbed.

We first transform the second factor in the summand in (1.6) as in [9]

$$\langle I m_a | H_1 | I_i m_i L_1 M_1 \pi_1 \rangle = \langle I_i m_i L_1 M_1 \pi_1 | H_1 | I m_a \rangle$$

then isolate the reduced matrix element  $\langle I_i || L_1 \pi_i || I \rangle$  in it as

$$\langle I_i m_i L_1 M_1 \pi_1 | H_1 | I m_a \rangle = (-1)^{I_i+L_1-m_a} \sqrt{2I+1} \begin{vmatrix} I_i & L_1 & I \\ m_i & M_1 & -m_a \end{vmatrix} \langle I_i || L_1 \pi_1 || I \rangle \quad (1.7)$$

The eigenfunction  $\langle L_1 M_1 \pi_1 | \mathbf{q}_1 \sigma_1 \rangle$  associated with a reference frame featuring an arbitrary quantization axis  $z$  will be transformed [10] as

$$\langle L_1 M_1 \pi_1 | \mathbf{q}_1 \sigma_1 \rangle = \langle \mathbf{q}_1 \sigma_1 | L_1 M_1 \pi_1 \rangle^*.$$

At the same time, we have

$$\langle \mathbf{q}_1 \sigma_1 | L_1 M_1 \pi_1 \rangle^* = \sum_{\mu_1} \langle 0 \sigma_1 | L_1 \mu_1 \pi_1 \rangle^* D_{M_1 \mu_1}^{L_1} (z \rightarrow \mathbf{q}_1) \quad (1.8)$$

From Eqs. (1.6)–(1.8), it follows that

$$\begin{aligned} \langle I m_a | H_1 | I_i m_i \rangle &= \sum_{L_1 M_1 \mu_1 \pi_1} (-1)^{-I_i+L_1-m_a} \begin{vmatrix} I_i & L_1 & I \\ m_i & M_1 & -m_a \end{vmatrix} \times \langle 0 \sigma_1 | L_1 \mu_1 \pi_1 \rangle^* \\ &\times \langle I_i || L_1 \pi_1 || I \rangle D_{M_1 \mu_1}^{L_1} (z \rightarrow \mathbf{q}_1) \end{aligned} \quad (1.9)$$

The hypothesis of parity conservation in strong and electromagnetic interactions does not contradict modern experimental data; therefore, we can retain only one term in the sums over  $\pi_1$  and  $\pi_2$  in expressions (1.5), (1.6) and (1.9) and henceforth avoid employing summation over the parity quantum number.

Let us perform the Fourier transform the functions  $A_{if}(\mathbf{q}_1, \mathbf{q}_2, t)$  in the reference frame where the quantization axis  $z$  coincides with the direction of the magnetic-field strength vector ( $\omega$  is the frequency of the photon to be absorbed):

$$\begin{aligned} U_{if}(\mathbf{q}_1, \mathbf{q}_2, \omega) &\sim \int_0^\infty A_{if}(\mathbf{q}_1, \mathbf{q}_2, t) e^{i(\omega - \frac{i}{2\hbar})t} dt = \int_0^\infty \left[ \sum_m \langle I_f m_f | H_2 | I m \rangle \langle I m | H_1 | I_i m_i \rangle e^{i(m\Omega + \omega + \frac{i}{2\hbar})t} \right] dt \\ &= i \sum_m \frac{\langle I_f m_f | H_2 | I m \rangle \langle I m | H_1 | I_i m_i \rangle}{(\omega + m\Omega + \frac{i}{2\hbar})}. \end{aligned} \quad (1.10)$$



In this reference frame, the magnetic quantum number of the intermediate state does not change under the effect of the magnetic-perturbation operator; therefore, we have

$$m_a = m_b = m.$$

The quantity  $\Gamma$  appearing in (1.10) is the natural width of the excited nuclear state. Following [6], one can represent the correlation function in the form

$$W(\mathbf{q}_1, \mathbf{q}_2) = \sum_{m_i m_f} \int_{-\infty}^{\infty} \left\{ |f_i(\omega)|^2 \sum_{\sigma_1 \sigma_2} |U_{if}(\sigma_1, \sigma_2)|^2 \right\} d\omega \quad (1.11)$$

We emphasize that we consider only the angular dependence of the correlation function, assuming that the polarization of gamma rays is not measured and that the initial gamma radiation is not polarized.

The function  $f_i(\omega)$  appearing in expression (1.11) describes the frequency distribution of radiation to be absorbed. Following (1.8), we set it to

$$f_i(\omega) = \frac{C_1}{\omega - \frac{s + \varepsilon_i}{h} + i \frac{\Delta}{2h}}, \quad (1.12)$$

where  $C_1$  is a dimensional normalization constant; below, we omit its numerical part. This frequency distribution corresponds to the Lorentzian gamma line form with a width  $\Delta$ . In (1.12),  $s$  is the summed isomeric and Doppler shifts of the gamma line, while  $\varepsilon_i$  is the energy of the hyperfine interaction of scatterer nuclei in the ground state. The photon energy determined by the frequency  $\omega$  is reckoned from the position of the non split resonance.

Omitting, as usual, constant factors, which do not affect the form of the angular distribution, we represent the correlation function as

$$\begin{aligned} W(\mathbf{q}_1, \mathbf{q}_2) = & S_1 S_2 \sum_{m_i m_f m m'} \int_{-\infty}^{\infty} |f_i(\omega)|^2 \\ & \times \frac{\langle I_f m_f | H_2 | I m \rangle \langle I m | H_1 | I_i m_i \rangle \langle I_f m_f | H_2 | I m' \rangle^* \langle I m' | H_1 | I_i m_i \rangle^*}{(\omega + m\Omega + \frac{i\Gamma}{2h})(\omega + m'\Omega - \frac{i\Gamma}{2h})} d\omega \end{aligned} \quad (1.13)$$

The symbols  $S_1$  and  $S_2$  denote summation over unobserved gamma ray polarizations.