Anders C. Nilsson and Bilong Liu

Vibro-Acoustics

/Volume III/

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Volume III

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PREFACE

The third volume of *Vibro-Acoustics* includes three parts plus errata for Volumes I and II. The first part of the Volume III presents the problems to each chapter. The second part of the volume lists the solutions to the problems. A few problems have been added as compared to the original texts in Volumes I and II.

A summary of some basic equations presented in Volumes I and II are given in Part 3 of Volume III.

Anders C. Nilsson
Lotorp, Sweden, March 2014

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PART 1 PROBLEMS

Chapter 1

1.1 Determine the energy dissipated over one period for a simple mass-spring system if the losses are a) viscous and b) hysteretic. Assume that the displacement of the mass is described by $x(t) = x_0 \sin(\omega t)$.

1.2 The displacement of the mass of a simple mass-spring system is given by $x(t) = x_0 \sin(\omega t)$. Determine the force required to maintain this motion if the damping force is due to i) viscous losses and ii) frictional losses. In a diagram show the force as function of displacement. Make some apropriate assumption concerning the magnitude of the properties m, k_0, c and F_d.

1.3 The mass in Fig.1-1.3-1 is excited and is thereafter left to oscillate freely. Determine the displacement as function of time if the losses are assumed to be frictional. Assume that the displacement is x_0 and the velocity zero at time $t = 0$.

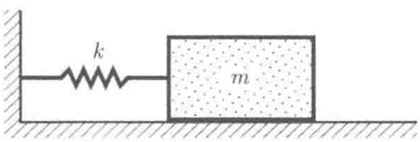

Fig. 1-1.3-1

1.4 Show that for a critically damped system the displacement can be zero for time t being finite and that this can only happen at one instance.

1.5 The mass of a simple mass-spring system is excited by an impulse I at time intervals T. Determine the response of the mass. Consider only harmonic solutions i.e. assume that the excitation process was started at $t = -\infty$. The system is lightly damped.

1.6 A mass-spring system is at rest for $t < 0$. The mass is excited by a force $F(t)$ at $t = 0$. The force is given by $F(t) = F_0$ for $0 \leqslant t \leqslant T$; $F(t) = 0$ for $t < 0$ and $t > T$.

Determine the response of the mass. In particular consider the cases for which the product $\omega_r T$ is equal to $\pi/2$, π and 2π with ω_r defined in eq. (1-14). Assume that $\beta T \ll 1$. For definitions see Section 1.2.

1.7 For the problem described in Example 1.6 determine the maximum amplitude as function of T.

1.8 A function $x(t)$ is expanded in a Fourier series as

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \omega_n t + b_n \sin \omega_n t); \quad \omega_n = 2\pi n/T; \quad n = 1, 2, \cdots$$

Show that the coefficients a_n and b_n are

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(\omega_n t)\mathrm{d}t; \quad b_n = \frac{2}{T}\int_0^T x(t)\sin(\omega_n t)\mathrm{d}t$$

1.9 A harmonic force $F(t)$ with the period T is exciting the mass of a simple 1-DOF system. Determine the displacement of the mass if

$$F(t) = F(t+T) = G_0/2 + \sum_{n=1}^{\infty} G_n \cos(\omega_n t) + \sum_{n=1}^{\infty} H_n \sin(\omega_n t); \quad \omega_n = 2\pi n/T$$

Assume the losses to be viscous.

1.10 Solve Problem 1.5 by expanding the force and response in Fourier series.

1.11 A 1-DOF system is excited by a force $F(t) = F_0 \cdot e^{i\omega t}$. Determine the time averages of kinetic and potential energies as well as the time average of the input power to the system. Assume that the equation governing the motion of the system is

$$m\ddot{x} + kx = F; \quad k = k_0(1 + i\delta)$$

According to eq. (1-81) $\delta = 2\omega m\beta/k_0$. Since $\beta = c/(2m)$ δ is written $\delta = c\omega/k_0$. Discuss the difference between viscous and structural damping.

1.12 A 1-DOF system is governed by the equation $m\ddot{x} + c\dot{x} + k_0 x = F(t)$. A function $h(t-\tau)$ satisfies the equation $m\ddot{h} + c\dot{h} + k_0 h = \delta(t-\tau)$ show that $x(t)$ is given by

$$x(t) = \int_{-\infty}^{t} \mathrm{d}\tau F(\tau) h(t-\tau).$$

1.13 The displacement of a 1-DOF system can be described in two different ways as

i) $m\ddot{x} + kx = F$; $k = k_0(1 + i\delta)$

ii) $m\ddot{x} + c\dot{x} + k_0 x = F$

Assume $F = F_0 \cdot e^{i\omega t}$ and $x = x_0 \cdot e^{i\omega t}$ and derive the input power to the system for both cases. Show in the first case that the input power is proportional to the potential energy of the system and in the second case to the kinetic energy.

Chapter 2

2.1 Determine the FT of the function

$$h(t) = \exp(-\beta t) \cdot \sin(\omega_n t)/(m\omega_n) \text{ for } t \geqslant 0$$

$$h(t) = 0 \text{ for } t < 0$$

where $\omega_n^2 = \omega_0^2 - \beta^2$ and $\beta = \omega_0^2\delta/(2\omega) > 0$.

2.2 A periodic signal $x(t) = x(t+T)$ is a function of time as

$$x(t) = A \text{ for } 0 \leqslant t \leqslant T/2 \text{ and } x(t) = 0 \text{ for } T/2 < t < T$$

Determine the autocorrelation function and power spectral density of the signal.

2.3 The frequency response function $H(\omega)$ of a 1-DOF system is

$$H(\omega) = \frac{1}{-m\omega^2 + k} = \frac{1}{m[(\omega_0^2 - \omega^2) + i\omega_0^2\delta]}$$

Show that for $\delta \ll 1$ the inverse FT of H is equal to

$$h(t) = \exp(-\omega_0 t\delta/2) \cdot \sin(\omega_0 t)/(m\omega_0)$$

2.4 The mass of a mass-spring system is excited by the force defined in Example 2.2. Determine the time average of the velocity squared for the mass m. The spring constant is

$$k = k_0(1 + i\delta).$$

2.5 Determine the autocorrelation functions for band-pass white noise and low-pass white noise. In the first case $G_{xx}(f) = a$ for $0 \leqslant f_0 - B/2 \leqslant f \leqslant f_0 + B/2$ and in the second case $G_{xx}(f) = a$ for $0 \leqslant f \leqslant B$.

2.6 A force $F(t)$ is applied to the mass m of a mass-spring system. The complex spring constant is given by $k = k_0(1 + i\delta)$. The force is $F(t) = A\sin(\omega_1 t) + \xi(t)$, where $\xi(t)$is a random signal with the one sided power spectral density $G_{\xi\xi} = A^2/(2\omega_0)$ where $\omega_0^2 = k_0/m$.

Determine the time average of the velocity squared of the mass.

2.7 Determine the time averages of the potential and kinetic energies of a mass-spring system for which the mass is excited by a force $F(t) = F_0 \cdot \sin(\omega_1 t)$.

2.8 Determine the time average of the velocity squared of the mass of a lightly damped mass-spring system excited by a force characterised by an exponential autocorrelation function, i.e. having a one sided power spectral density

$$G_{FF}(\omega) = \frac{4a}{a^2 + \omega^2}.$$

2.9 A mass-spring system is mounted on a foundation as shown in Fig.1-2.9-1 The point mobility of the foundation is Y_f. Determine the point mobility Y in the excitation point.

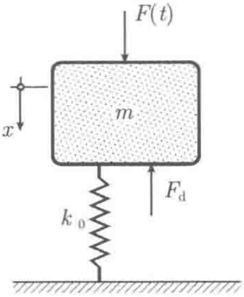

Fig. 1-2.9-1

2.10 For the system described in Example 2.9 determine the one sided power spectral density of the power transmitted to the foundation. The power spectral density of the force exciting the mass is constant and equal to G_{FF}. The point mobility of the foundation is Y_f. Determine also the time average of the power input to the foundation if Y_f is real and much smaller than unity and in addition independent of frequency.

2.11 The mass of a mass-spring system is excited by a force $F(t)$, with the one-sided power spectral density G_{FF}. The response of the mass is $z(t) = x(t) + y(t)$ where $y(t)$ is due to extraneous and random noise. The one-sided power spectral density of the random signal y is G_{yy}. The FT of the response due to the FT of the force can be written as $\hat{x} = H\hat{F}$ where H is the frequency response function for the system. Determine the coherence function between the FT of the force and the FT of the displacement z.

2.12 Determine the time average of the power input to 1-DOF system

$$\bar{\Pi} = -\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \cdot \omega S_{FF}(\omega) \cdot \mathrm{Im}(H)$$

when the frequency response function is defined according to eq. (2-15) as

$$H(\omega) = \frac{1}{-m\omega^2 + i\omega c + k_0} = \frac{1}{m(\omega_0^2 - \omega^2 + 2i\beta\omega)}$$

2.13 A function $x(t)$ is written $x(t) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty}(a_n \cos\omega_n t + b_n \sin\omega_n t)$ in the time interval $-T/2 \leqslant t \leqslant T/2$. Show that as $T \to \infty$ the function can be written in integral form as

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \cdot \hat{x}(\omega)e^{i\omega t} \text{ where } \hat{x}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t \cdot x(t)e^{-i\omega t}$$

2.14 Show that $E[\dot{x}^2(t)] = -\left[\dfrac{\mathrm{d}^2 R_{xx}}{\mathrm{d}\tau^2}\right]_{\tau=0}$

Chapter 3

3.1 An infinite beam is oriented along the x-axis in a coordinate system. The displacement along the x-axis is $\xi = A \cdot \sin(\omega t - k_l x)$ where k_l is the wave number for quasi-longitudinal waves. The width of the beam is b and its height h. Determine the displacement perpendicular to the x-axis of the beam. Assume that σ_y and σ_z are equal to zero in the beam.

3.2 Determine the resulting kinetic energy in the beam of Problem 3.1. Consider only the effects due to quasi L-waves.

3.3 An L-wave is propagating in an infinite and homogeneous beam oriented along the x-axis of a coordinate system. The resulting displacement is defined by $f(x - c_l t)$. Determine the kinetic and potential energies plus the energy flow due to this wave.

3.4 A semi infinite and homogeneous beam with constant cross section area S is oriented along the x-axis of a coordinate system. At $x = 0$ the beam is excited by a force $F(t)$ in the direction of the positive x-axis. Determine the displacement in the beam. Consider only L-waves. As an example let the force be given by $F(t) = F_0 \sin \omega t$.

3.5 Torsional waves are propagating in an infinite cylindrical and homogeneous shaft with radius R. Due to the wave motion the torsional angle Θ varies as $\Theta = \Theta_0 \sin(k_t x - \omega t)$. Determine the potential and kinetic energies per unit length of the shaft as well as the energy flow in the shaft which is oriented along the x-axis of a coordinate system.

3.6 Flexural waves are propagating in an infinite and homogeneous beam oriented along the x-axis of a coordinate system. The displacement of the beam is given by $w(x, t)$. Determine the potential energy per unit length of the beam based on the general expression eq. (3-17) and the definition of the strain in eq. (3-72). Neglect shear effects.

3.7 The deflection η of an infinite and homogeneous string oriented along the x-axis is at $t = 0$ equal to $\eta(x, 0) = \cos(\pi x/L)$ for $-L/2 < x < L/2$ otherwise zero.

The string is at rest at $t = 0$. Determine the displacement of the string when it is released at $t = 0$. Neglect the losses.

3.8 A thin, infinite and homogeneous beam is oriented along the x-axis in a coordinate system. The mass per unit length is m' and its bending stiffness D'. For $t < 0$ the beam is at rest having the lateral displacement $\exp[-(x/2a)^2]$. The beam is released at $t = 0$. Determine the displacement of the beam for $t > 0$. Compare the discussion in Section 3.8.

3.9 An attempt is made to measure the energy flow in a thin homogeneous beam by means of just one accelerometer. The material and geometrical parameters of the beam are known.

The bending stiffness and wavenumber are denoted D' and κ. Losses are neglected. In the first case the lateral displacement of the beam, which is oriented along the x-axis of a coordinate system, is equal to $w(x,t) = A \cdot \exp[i(\omega t - \kappa x)]$. Determine the energy flow in the beam as function of the time average of the velocity squared measured at the point $x = x_0$.

In the second case the near field can not be neglected. The displacement is $w(x,t) = A \cdot \exp(i\omega t) \cdot [\exp(-i\kappa x) - i \cdot \exp(-\kappa x)]$. Determine the ratio between the actual energy flow and the energy flow estimated by means of the velocity squared measured by means of the accelerometer at the point $x = x_0$.

In the third case the near field but not a reflected field can be neglected. The displacement is given by $w(x,t) = A \cdot \exp(i\omega t) \cdot [\exp(-i\kappa x) + X \cdot \exp(i\kappa x)]$. Again calculate the ratio between the actual and measured energy flows at the point $x = x_0$.

3.10 Show that the bending moment per unit length induced by shear in an orthotropic plate is given by eq. (3-132) as

$$M'_{xy} = -\sqrt{D_x D_y} \cdot (1 - \sqrt{\nu_x \nu_y}) \cdot \frac{\partial^2 w}{\partial x \partial y}$$

The plate is oriented in the x-y-plane of a coordinate system.

3.11 An L-wave is propagating in an infinite beam oriented along the x-axis of a coordinate system. The displacement is $\xi(x,t) = A \cdot \exp[i(\omega t - k_l x)]$. Show that the time average of the energy flow Π is $\bar{\Pi} = c_l \bar{E}_l$ where $\bar{E}_l$ is the time average of the total energy per unit length of the beam and c_l the phase velocity of the wave.

3.12 Show that the intensity of L-waves propagating in the beam of Problem 3.11 is given by $I_x = -\sigma_x \cdot \partial\xi/\partial t$ where ξ is the displacement in the beam. Start by considering the total energy per unit volume of the beam.

Chapter 4

4.1 A T-wave is for $x < 0$ travelling in a thin semi infinite plate. The plate is oriented in the x-y-plane of a coordinate system. The wave is travelling towards a straight edge at $x = 0$. The angle of incidence is β.

The impedance of the edge is infinite. Determine the relative amplitudes of the reflected L- and T-waves at the edge.

4.2 Two semi infinite plates are oriented in the x-y-plane of a coordinate system. The junction between the plates is defined by the line $x = 0$. Plate 1 has the thickness h and plate 2 has the thickness H. An L-wave is in plate 1 travelling towards the junction. The angle of incidence is α. Determine the ratio between the incident energy flow and the energy flow transmitted to plate 2.

4.3 Use eq. (4-51) to determine the wavenumber for travelling and evanescent bending waves in a plate with thickness h. Include second order terms. Determine also the energy flow due to a plane travelling bending wave in the plate. Include second order terms in h.

4.4 Use eq. (4-49) to determine the wavenumber for quasi longitudinal waves travelling in a plate. Include only terms of the first order as the plate thickness approaches zero.

4.5 Determine the low and high frequency limits for the wavenumber describing flexural waves propagating in a sandwich plate. Geometrical and material parameters are given in Table 4-3 of Section 4.10.

4.6 A bending wave, $w(x,t)$ is propagating in a plate. Use eq. (4-56) to show that the resulting bending moment per unit width of the plate is $-D\partial^2 w/\partial x^2$ and the corresponding shear force $-D\partial^3 w/\partial x^3$. The plate is oriented in the x-y-plane of a coordinate system.

4.7 A bending wave, $w(x,t) = \eta_0 \exp[i(\omega t - \kappa x)]$ is propagating in a plate with the thickness h. Determine the intensity in the plate. Use eq. (4-56) in combination with the definition of the intensity. The plate is oriented in the x-y-plane of a coordinate system.

4.8 Determine the shear stress in a plate with thickness h as function of the distance y from the neutral plane of the plate. Use the result of eq. (4-46).

4.9 The wave number k_x for a wave propagating along a so called Timoshenko beam is in eq. (4-32) given as

$$k_x = \pm\sqrt{\frac{1}{2}\left[(k_l^2 + k_t^2/T_b) \pm \sqrt{4\kappa^4 + (k_l^2 - k_t^2/T_b)^2}\right]}$$

In the high frequency limit k_x should approach k_r, the wavenumber for Rayleigh waves. Determine the coefficient T_b for $\lim_{\omega\to\infty} k_x = k_r$.

4.10 According to Section 4.6 a Rayleigh wave propagating along the x-axis in a semi infinite solid can for $y \leqslant 0$ be described by the potentials

$$\phi = B_1 \exp[\alpha(y + h/2)] \exp[i(\omega t - k_r x)]/2$$
$$\Psi_z = C_2 \exp[\beta(y + h/2)] \exp[i(\omega t - k_r x)]/2$$
$$\text{where } C_2/B_1 = i \cdot \exp[h/2(\alpha - \beta)][k_r^2 - k_0^2(1 + \nu)]/(k_r \beta)$$

and k_r is the wavenumber for Rayleigh waves. The parameters α and β are

$$\beta = \sqrt{k_r^2 - k_t^2}; \quad \alpha = \sqrt{k_r^2 - k_l^2}$$

Show that $\sigma_y = 0$ and $\tau_{xy} = 0$ for $y = 0$, i.e. on the surface of the semi infinite solid. The surface of the structure is in the x-z-plane of the coordinate system. The distance from the surface is given by y.

4.11 Indicate a procedure to determine the intensity induced by a Rayleigh wave travelling in a semi-infinite solid. Use eq. (4-68).

Chapter 5

5.1 Two semi infinite beams are connected at right angels. The junction between the beams is hinged i.e. no bending moment can be transfered from one beam to the other. A longitudinal wave is incident on the junction in beam 1. Determine the transmitted and reflected energy flows as function of the incident energy flow. The two beams are identical, width b, height h, Young's modulus E, Poisson's ratio ν and density ρ.

5.2 The incident wave in Problem 5.1 is a flexural wave. Determine the transmitted and reflected energy flows.

5.3 At a junction n identical semi infinite plates are connected along a straight line. In one of the plates a plane flexural wave is incident on the junction (normal incidence). Determine the attenuation of the energy flow to any of the other plates. Neglect the translatory motion of the junction.

5.4 A longitudinal wave is propagating in Beam 1 towards the discontinuity 2 shown in Fig.1-5.4-1. Determine the ratio between the incident energy flow in Beam 1 and the transmitted energy flow to Beam 3.

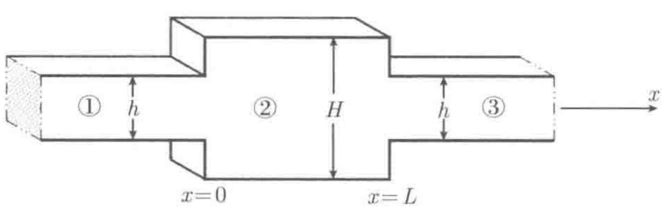

Fig. 1-5.4-1

5.5 Two semi infinite beams oriented along the same axis are connected by means of an elastic interlayer as shown in Fig.1-5.5-1. A longitudinal wave is incident on the interlayer. Determine the attenuation across the junction. Consider only longitudinal waves.

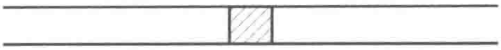

Fig. 1-5.5-1

5.6 A flexural wave is propagating in a beam towards a blocking mass as shown in Fig.1-5.6-1. A flexural wave is transmitted across the blocking mass. Determine the ratio between incident and transmitted energy flows. It is sufficient to define incident and transmitted waves and the boundary conditions necessary for solving the problem. Assume the blocking mass to be rigid. Its mass is M and its rotational mass moment of inertia J. The width of the beam is b and its height h.

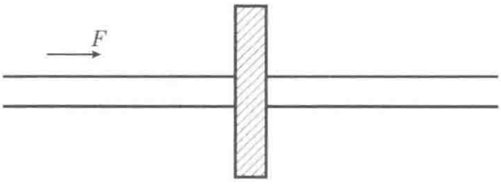

Fig. 1-5.6-1

5.7 An evanescent flexural wave on a beam is described by

$$w(x,t) = A \cdot \exp[i(\omega t + \kappa_0 \eta x/4) - \kappa_0 x]$$

where κ_0 is the real part of the wave number and η the loss factor. Determine the energy flow in the beam due to this wave.

5.8 A thin infinite plate is excited by a point force $F = F_0 \cdot \exp(i\omega t)$ perpendicular to the surface of the plate. Determine the far field displacement of the resulting flexural wave.

5.9 An infinite plate is excited by a point force. The displacement in the far field is given by the result of Example 5.8. Neglecting the losses in the plate show that the power transmitted to the far field is equal to power input at the excitation point.

5.10 Two semi-infinite plates of different thicknesses are joined together along a straight line. The joint is allowed to rotate only. A flexural plane wave, unit amplitude, is incident on the junction. The angle of incidence is α. Determine the amplitude R of the reflected wave and show that $|R| = 1$ when no propagating wave is transmitted across the junction.