Jossph J. Rotman

# An Introduction to Algebraic Topology

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To my wife Marganit
and my children Ella Rose and Daniel Adam
without whom this book would have
been completed two years earlier

# Preface

There is a canard that every textbook of algebraic topology either ends with the definition of the Klein bottle or is a personal communication to J. H. C. Whitehead. Of course, this is false, as a glance at the books of Hilton and Wylie, Maunder, Munkres, and Schubert reveals. Still, the canard does reflect some truth. Too often one finds too much generality and too little attention to details.

There are two types of obstacle for the student learning algebraic topology. The first is the formidable array of new techniques (e.g., most students know very little homological algebra); the second obstacle is that the basic definitions have been so abstracted that their geometric or analytic origins have been obscured. I have tried to overcome these barriers. In the first instance, new definitions are introduced only when needed (e.g., homology with coefficients and cohomology are deferred until after the Eilenberg–Steenrod axioms have been verified for the three homology theories we treat—singular, simplicial, and cellular). Moreover, many exercises are given to help the reader assimilate material. In the second instance, important definitions are often accompanied by an informal discussion describing their origins (e.g., winding numbers are discussed before computing $\pi_1(S^1)$, Green's theorem occurs before defining homology, and differential forms appear before introducing cohomology).

We assume that the reader has had a first course in point-set topology, but we do discuss quotient spaces, path connectedness, and function spaces. We assume that the reader is familiar with groups and rings, but we do discuss free abelian groups, free groups, exact sequences, tensor products (always over **Z**), categories, and functors.

I am an algebraist with an interest in topology. The basic outline of this book corresponds to the syllabus of a first-year's course in algebraic topology designed by geometers and topologists at the University of Illinois, Urbana; other expert advice came (indirectly) from my teachers, E. H. Spanier and S. Mac Lane, and from J. F. Adams's *Algebraic Topology: A Student's Guide*. This latter book is strongly recommended to the reader who, having finished this book, wants direction for further study.

I am indebted to the many authors of books on algebraic topology, with a special bow to Spanier's now classic text. My colleagues in Urbana, especially Ph. Tondeur, H. Osborn, and R. L. Bishop, listened and explained. M.-E. Hamstrom took a particular interest in this book; she read almost the entire manuscript and made many wise comments and suggestions that have improved the text; my warmest thanks to her. Finally, I thank Mrs. Dee Wrather for a superb job of typing and Springer-Verlag for its patience.

Joseph J. Rotman

## *Addendum to Second Corrected Printing*

Though I did read the original galleys carefully, there were many errors that eluded me. I thank all who apprised me of mistakes in the first printing, especially David Carlton, Monica Nicolau, Howard Osborn, Rick Rarick, and Lewis Stiller.

*November 1992* Joseph J. Rotman

### *Addendum to Fourth Corrected Printing*

Even though many errors in the first printing were corrected in the second printing, some were unnoticed by me. I thank Bernhard J. Elsner and Martin Meier for apprising me of errors that persisted into the the second and third printings. I have corrected these errors, and the book is surely more readable because of their kind efforts.

*April, 1998* Joseph Rotman

# To the Reader

Doing exercises is an essential part of learning mathematics, and the serious reader of this book should attempt to solve all the exercises as they arise. An asterisk indicates only that an exercise is cited elsewhere in the text, sometimes in a proof (those exercises used in proofs, however, are always routine).

I have never found references of the form 1.2.1.1 convenient (after all, one decimal point suffices for the usual description of real numbers). Thus, Theorem 7.28 here means the 28th theorem in Chapter 7.

# Contents

CHAPTER 0

# Introduction

One expects algebraic topology to be a mixture of algebra and topology, and that is exactly what it is. The fundamental idea is to convert problems about topological spaces and continuous functions into problems about algebraic objects (e.g., groups, rings, vector spaces) and their homomorphisms; the method may succeed when the algebraic problem is easier than the original one. Before giving the appropriate setting, we illustrate how the method works.

## Notation

Let us first introduce notation for some standard spaces that is used throughout the book.

- $\mathbf{Z}$ = integers (positive, negative, and zero).
- $\mathbf{Q}$ = rational numbers.
- $\mathbf{C}$ = complex numbers.
- $\mathbf{I} = [0, 1]$, the (closed) unit interval.
- $\mathbf{R}$ = real numbers.
- $\mathbf{R}^n = \{(x_1, x_2, \ldots, x_n) | x_i \in \mathbf{R} \text{ for all } i\}$.

$\mathbf{R}^n$ is called **real $n$-space** or **euclidean space** (of course, $\mathbf{R}^n$ is the cartesian product of $n$ copies of $\mathbf{R}$). Also, $\mathbf{R}^2$ is homeomorphic to $\mathbf{C}$; in symbols, $\mathbf{R}^2 \approx \mathbf{C}$. If $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$, then its **norm** is defined by $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$ (when $n = 1$, then $\|x\| = |x|$, the absolute value of $x$). We regard $\mathbf{R}^n$ as the subspace of $\mathbf{R}^{n+1}$ consisting of all $(n + 1)$-tuples having last coordinate zero.

$$S^n = \{x \in \mathbf{R}^{n+1}: \|x\| = 1\}.$$

$S^n$ is called the **$n$-sphere** (of radius 1 and center the origin). Observe that $S^n \subset \mathbf{R}^{n+1}$(as the circle $S^1 \subset \mathbf{R}^2$); note also that the 0-sphere $S^0$ consists of the two points $\{1, -1\}$ and hence is a discrete two-point space. We may regard $S^n$ as the **equator** of $S^{n+1}$:

$$S^n = \mathbf{R}^{n+1} \cap S^{n+1} = \{(x_1, \ldots, x_{n+2}) \in S^{n+1}: x_{n+2} = 0\}.$$

The **north pole** is $(0, 0, \ldots, 0, 1) \in S^n$; the **south pole** is $(0, 0, \ldots, 0, -1)$. The **antipode** of $x = (x_1, \ldots, x_{n+1}) \in S^n$ is the other endpoint of the diameter having one endpoint $x$; thus the antipode of $x$ is $-x = (-x_1, \ldots, -x_{n+1})$, for the distance from $-x$ to $x$ is 2.

$$D^n = \{x \in \mathbf{R}^n: \|x\| \leq 1\}.$$

$D^n$ is called the **$n$-disk** (or **$n$-ball**). Observe that $S^{n-1} \subset D^n \subset \mathbf{R}^n$; indeed $S^{n-1}$ is the boundary of $D^n$ in $\mathbf{R}^n$.

$$\Delta^n = \{(x_1, x_2, \ldots, x_{n+1}) \in \mathbf{R}^{n+1}: \text{each } x_i \geq 0 \text{ and } \sum x_i = 1\}.$$

$\Delta^n$ is called the **standard $n$-simplex**. Observe that $\Delta^0$ is a point, $\Delta^1$ is a closed interval, $\Delta^2$ is a triangle (with interior), $\Delta^3$ is a (solid) tetrahedron, and so on. It is obvious that $\Delta^n \approx D^n$, although the reader may not want to construct[1] a homeomorphism until Exercise 2.11.

There is a standard homeomorphism from $S^n - \{\text{north pole}\}$ to $\mathbf{R}^n$, called **stereographic projection**. Denote the north pole by $N$, and define $\sigma: S^n - \{N\} \to \mathbf{R}^n$ to be the intersection of $\mathbf{R}^n$ and the line joining $x$ and $N$. Points on the latter line have the form $tx + (1 - t)N$; hence they have coordinates $(tx_1, \ldots, tx_n, tx_{n+1} + (1 - t))$. The last coordinate is zero for $t = (1 - x_{n+1})^{-1}$; hence

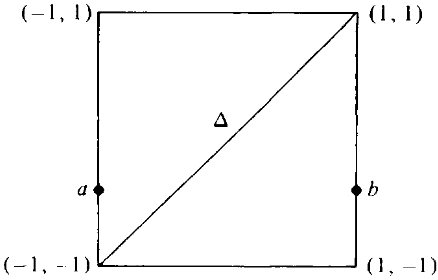
$$\sigma(x) = (tx_1, \ldots, tx_n),$$

where $t = (1 - x_{n+1})^{-1}$. It is now routine to check that $\sigma$ is indeed a homeomorphism. Note that $\sigma(x) = x$ if and only if $x$ lies on the equator $S^{n-1}$.

## Brouwer Fixed Point Theorem

Having established notation, we now sketch a proof of the **Brouwer fixed point theorem**: if $f: D^n \to D^n$ is continuous, then there exists $x \in D^n$ with $f(x) = x$. When $n = 1$, this theorem has a simple proof. The disk $D^1$ is the closed interval $[-1, 1]$; let us look at the graph of $f$ inside the square $D^1 \times D^1$.

[1] It is an exercise that a compact convex subset of $\mathbf{R}^n$ containing an interior point is homeomorphic to $D^n$ (convexity is defined in Chapter 1); it follows that $\Delta^n$, $D^n$, and $I^n$ are homeomorphic.




**Theorem 0.1.** *Every continuous $f\colon D^1 \to D^1$ has a fixed point.*

PROOF. Let $f(-1) = a$ and $f(1) = b$. If either $f(-1) = -1$ or $f(1) = 1$, we are done. Therefore, we may assume that $f(-1) = a > -1$ and that $f(1) = b < 1$, as drawn. If $G$ is the graph of $f$ and $\Delta$ is the graph of the identity function (of course, $\Delta$ is the diagonal), then we must prove that $G \cap \Delta \neq \varnothing$. The idea is to use a connectedness argument to show that every path in $D^1 \times D^1$ from $a$ to $b$ must cross $\Delta$. Since $f$ is continuous, $G = \{(x, f(x))\colon x \in D^1\}$ is connected [$G$ is the image of the continuous map $D^1 \to D^1 \times D^1$ given by $x \mapsto (x, f(x))$]. Define $A = \{(x, f(x))\colon f(x) > x\}$ and $B = \{(x, f(x))\colon f(x) < x\}$. Note that $a \in A$ and $b \in B$, so that $A \neq \varnothing$ and $B \neq \varnothing$. If $G \cap \Delta = \varnothing$, then $G$ is the disjoint union

$$G = A \cup B.$$

Finally, it is easy to see that both $A$ and $B$ are open in $G$, and this contradicts the connectedness of $G$. □

Unfortunately, no one knows how to adapt this elementary topological argument when $n > 1$; some new idea must be introduced. There is a proof using the *simplicial approximation theorem* (see [Hirsch]). There are proofs by analysis (see [Dunford and Schwartz, pp. 467–470] or [Milnor (1978)]); the basic idea is to approximate a continuous function $f\colon D^n \to D^n$ by smooth functions $g\colon D^n \to D^n$ in such a way that $f$ has a fixed point if all the $g$ do; one can then apply analytic techniques to smooth functions.

Here is a proof of the Brouwer fixed point theorem by algebraic topology. We shall eventually prove that, for each $n \geq 0$, there is a *homology functor* $H_n$ with the following properties: for each topological space $X$ there is an abelian group $H_n(X)$, and for each continuous function $f\colon X \to Y$ there is a homomorphism $H_n(f)\colon H_n(X) \to H_n(Y)$, such that:

$$H_n(g \circ f) = H_n(g) \circ H_n(f) \tag{1}$$

whenever the composite $g \circ f$ is defined;

$$H_n(1_X) \text{ is the identity function on } H_n(X), \tag{2}$$

where $1_X$ is the identity function on $X$;

$$H_n(D^{n+1}) = 0 \quad \text{for all } n \geq 1; \tag{3}$$

$$H_n(S^n) \neq 0 \quad \text{for all } n \geq 1. \tag{4}$$

Using these $H_n$'s, we now prove the Brouwer theorem.

**Definition.** A subspace $X$ of a topological space $Y$ is a **retract** of $Y$ if there is a continuous map[2] $r: Y \to X$ with $r(x) = x$ for all $x \in X$; such a map $r$ is called a **retraction**.

**Remarks.** (1) Recall that a topological space $X$ contained in a topological space $Y$ is a **subspace** of $Y$ if a subset $V$ of $X$ is open in $X$ if and only if $V = X \cap U$ for some open subset $U$ of $Y$. Observe that this guarantees that the inclusion $i: X \hookrightarrow Y$ is continuous, because $i^{-1}(U) = X \cap U$ is open in $X$ whenever $U$ is open in $Y$. This parallels group theory: a group $H$ contained in a group $G$ is a **subgroup** of $G$ if and only if the inclusion $i: H \hookrightarrow G$ is a homomorphism (this says that the group operations in $H$ and in $G$ coincide).

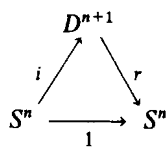
(2) One may rephrase the definition of retract in terms of functions. If $i: X \hookrightarrow Y$ is the inclusion, then a continuous map $r: Y \to X$ is a retraction if and only if

$$r \circ i = 1_X.$$

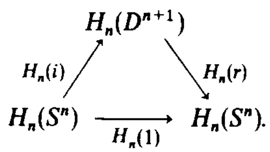
(3) For abelian groups, one can prove that a subgroup $H$ of $G$ is a retract of $G$ if and only if $H$ is a **direct summand** of $G$; that is, there is a subgroup $K$ of $G$ with $K \cap H = 0$ and $K + H = G$ (see Exercise 0.1).

**Lemma 0.2.** *If $n \geq 0$, then $S^n$ is not a retract of $D^{n+1}$.*

Proof. Suppose there were a retraction $r: D^{n+1} \to S^n$; then there would be a "commutative diagram" of topological spaces and continuous maps




(here commutative means that $r \circ i = 1$, the identity function on $S^n$). Applying $H_n$ gives a diagram of abelian groups and homomorphisms:




[2] We use the words *map* and *function* interchangeably.

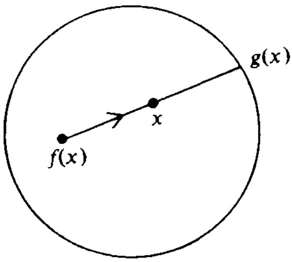
By property (1) of the homology functor $H_n$, the new diagram commutes: $H_n(r) \circ H_n(i) = H_n(1)$. Since $H_n(D^{n+1}) = 0$, by (3), it follows that $H_n(1) = 0$. But $H_n(1)$ is the identity on $H_n(S^n)$, by (2). This contradicts (4) because $H_n(S^n) \neq 0$. □

Note how homology functors $H_n$ have converted a topological problem into an algebraic one.

We mention that Lemma 0.2 has an elementary proof when $n = 0$. It is plain that a retraction $r: Y \to X$ is surjective. In particular, a retraction $r: D^1 \to S^0$ would be a continuous map from $[-1, 1]$ onto the two-point set $\{\pm 1\}$, and this contradicts the fact that a continuous image of a connected set is connected.

**Theorem 0.3** (Brouwer). *If $f: D^n \to D^n$ is continuous, then $f$ has a fixed point.*

Proof. Suppose that $f(x) \neq x$ for all $x \in D^n$; the distinct points $x$ and $f(x)$ thus determine a line. Define $g: D^n \to S^{n-1}$ (the boundary of $D^n$) as the function




assigning to $x$ that point where the ray from $f(x)$ to $x$ intersects $S^{n-1}$. Obviously, $x \in S^{n-1}$ implies $g(x) = x$. The proof that $g$ is continuous is left as an exercise in analytic geometry. We have contradicted the lemma. □

There is an extension of this theorem to infinite-dimensional spaces due to Schauder (which explains why there is a proof of the Brouwer fixed point theorem in [Dunford and Schwartz]): if $D$ is a compact convex subset of a Banach space, then every continuous $f: D \to D$ has a fixed point. The proof involves approximating $f - 1_D$ by a sequence of continuous functions each of which is defined on a finite-dimensional subspace of $D$ where Brouwer's theorem applies.

Exercises

*0.1. Let $H$ be a subgroup of an abelian group $G$. If there is a homomorphism $r: G \to H$ with $r(x) = x$ for all $x \in H$, then $G = H \oplus \ker r$. (*Hint*: If $y \in G$, then $y = r(y) + (y - r(y))$.)

0.2. Give a proof of Brouwer's fixed point theorem for $n = 1$ using the proof of Theorem 0.3 and the remark preceding it.

0.3. Assume, for $n \geq 1$, that $H_i(S^n) = \mathbf{Z}$ if $i = 0, n$, and that $H_i(S^n) = 0$ otherwise. Using the technique of the proof of Lemma 0.2, prove that the equator of the $n$-sphere is not a retract.

0.4. If $X$ is a topological space homeomorphic to $D^n$, then every continuous $f: X \to X$ has a fixed point.

0.5. Let $f, g: \mathbf{I} \to \mathbf{I} \times \mathbf{I}$ be continuous; let $f(0) = (a, 0)$ and $f(1) = (b, 1)$, and let $g(0) = (0, c)$ and $g(1) = (1, d)$ for some $a, b, c, d \in \mathbf{I}$. Show that $f(s) = g(t)$ for some $s, t \in \mathbf{I}$; that is, the paths intersect. (*Hint*: Use Theorem 0.3 for a suitable map $\mathbf{I} \times \mathbf{I} \to \mathbf{I} \times \mathbf{I}$.) (There is a proof in [Maehara]; this paper also shows how to derive the Jordan curve theorem from the Brouwer theorem.)

0.6. (Perron). Let $A = [a_{ij}]$ be a real $n \times n$ matrix with $a_{ij} > 0$ for every $i, j$. Prove that $A$ has a positive eigenvalue $\lambda$; moreover, there is a corresponding eigenvector $x = (x_1, x_2, \ldots, x_n)$ (i.e., $Ax = \lambda x$) with each coordinate $x_i > 0$. (*Hint*: First define $\sigma: \mathbf{R}^n \to \mathbf{R}$ by $\sigma(x_1, x_2, \ldots, x_n) = \sum_{i=1}^n x_i$, and then define $g: \Delta^{n-1} \to \Delta^{n-1}$ by $g(x) = Ax/\sigma(Ax)$, where $x \in \Delta^{n-1} \subset \mathbf{R}^n$ is regarded as a column vector. Apply the Brouwer fixed point theorem after showing that $g$ is a well defined continuous function.)

## Categories and Functors

Having illustrated the technique, let us now give the appropriate setting for algebraic topology.

**Definition.** A **category** $\mathscr{C}$ consists of three ingredients: a class of **objects**, obj $\mathscr{C}$; sets of **morphisms** $\text{Hom}(A, B)$, one for every ordered pair $A, B \in$ obj $\mathscr{C}$; **composition** $\text{Hom}(A, B) \times \text{Hom}(B, C) \to \text{Hom}(A, C)$, denoted by $(f, g) \mapsto g \circ f$, for every $A, B, C \in$ obj $\mathscr{C}$, satisfying the following axioms:

(i) the family of $\text{Hom}(A, B)$'s is pairwise disjoint;
(ii) composition is associative when defined;
(iii) for each $A \in$ obj $\mathscr{C}$, there exists an **identity** $1_A \in \text{Hom}(A, A)$ satisfying $1_A \circ f = f$ for every $f \in \text{Hom}(B, A)$, all $B \in$ obj $\mathscr{C}$, and $g \circ 1_A = g$ for every $g \in \text{Hom}(A, C)$, all $C \in$ obj $\mathscr{C}$.

**Remarks.** (1) The associativity axiom stated more precisely is: if $f, g, h$ are morphisms with either $h \circ (g \circ f)$ or $(h \circ g) \circ f$ defined, then the other is also defined and both composites are equal.

(2) We distinguish class from set: a **set** is a class that is small enough to have a cardinal number. Thus, we may speak of the *class* of all topological spaces, but we cannot say the *set* of all topological spaces. (The set theory we accept has primitive undefined terms: class, element, and the membership relation $\in$. All the usual constructs (e.g., functions, subclasses, Boolean opera-

tions, relations) are permissible except that the statement $x \in A$ is always false whenever $x$ is a class that is not a set.)

(3) The only restriction on $\text{Hom}(A, B)$ is that it be a set. In particular, $\text{Hom}(A, B) = \varnothing$ is allowed, although axiom (iii) shows that $\text{Hom}(A, A) \neq \varnothing$ because it contains $1_A$.

(4) Instead of writing $f \in \text{Hom}(A, B)$, we usually write $f: A \to B$.

EXAMPLE 0.1. $\mathscr{C} =$ **Sets**. Here obj $\mathscr{C}$ = all sets, $\text{Hom}(A, B) = \{$all functions $A \to B\}$, and composition is the usual composition of functions.

This example needs some discussion. Our requirement, in the definition of category, that Hom sets are pairwise disjoint is a reflection of our insistence that a function $f: A \to B$ is given by its **domain** $A$, its **target** $B$, and its **graph**: $\{\text{all } (a, f(a)): a \in A\} \subset A \times B$. In particular, if $A$ is a proper subset of $B$, we distinguish the inclusion $i: A \hookrightarrow B$ from the identity $1_A$ even though both functions have the same domain and the same graph; $i \in \text{Hom}(A, B)$ and $1_A \in \text{Hom}(A, A)$, and so $i \neq 1_A$. This distinction is essential. For example, in the proof of Lemma 0.2, $H_n(i) = 0$ and $H_n(1_A) \neq 0$ when $A = S^n$ and $B = D^{n+1}$. Here are two obvious consequences of this distinction: (1) If $B \subset B'$ and $f: A \to B$ and $g: A \to B'$ are functions with the same graph (and visibly the same domain), then $g = i \circ f$, where $i: B \hookrightarrow B'$ is the inclusion. (2) One may form the composite $h \circ g$ only when target $g$ = domain $h$. Others may allow one to compose $g: A \to B$ with $h: C \to D$ when $B \subset C$; we insist that the only composite defined here is $h \circ i \circ g$, where $i: B \hookrightarrow C$ is the given inclusion.

Now that we have explained the fine points of the definition, we continue our list of examples of categories.

EXAMPLE 0.2. $\mathscr{C} =$ **Top**. Here obj $\mathscr{C}$ = all topological spaces, $\text{Hom}(A, B) = \{$all continuous functions $A \to B\}$, and composition is usual composition.

**Definition.** Let $\mathscr{C}$ and $\mathscr{A}$ be categories with obj $\mathscr{C} \subset$ obj $\mathscr{A}$. If $A, B \in$ obj $\mathscr{C}$, let us denote the two possible Hom sets by $\text{Hom}_{\mathscr{C}}(A, B)$ and $\text{Hom}_{\mathscr{A}}(A, B)$. Then $\mathscr{C}$ is a **subcategory** of $\mathscr{A}$ if $\text{Hom}_{\mathscr{C}}(A, B) \subset \text{Hom}_{\mathscr{A}}(A, B)$ for all $A, B \in$ obj $\mathscr{C}$ and if composition in $\mathscr{C}$ is the same as composition in $\mathscr{A}$; that is, the function $\text{Hom}_{\mathscr{C}}(A, B) \times \text{Hom}_{\mathscr{C}}(B, C) \to \text{Hom}_{\mathscr{C}}(A, C)$ is the restriction of the corresponding composition with subscripts $\mathscr{A}$.

EXAMPLE 0.2′. The category **Top** has many interesting subcategories. First, we may restrict objects to be subspaces of euclidean spaces, or Hausdorff spaces, or compact spaces, and so on. Second, we may restrict the maps to be differentiable or analytic (assuming that these make sense for the objects being considered).

EXAMPLE 0.3. $\mathscr{C} =$ **Groups**. Here obj $\mathscr{C}$ = all groups, $\text{Hom}(A, B) = \{$all homomorphisms $A \to B\}$, and composition is usual composition (Hom sets are so called because of this example).