

PRECALCULUS FUNCTIONS AND GRAPHS

A GRAPHING APPROACH

FOURTH EDITION



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Precalculus Functions and Graphs A Graphing Approach

Fourth Edition

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We have included examples and exercises that use real-life data as well as technology output from a variety of software. This would not have been possible without the help of many people and organizations. Our wholehearted thanks go to all their time and effort.

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Printed in the U.S.A.

Library of Congress Catalog Card Number: 2003113992

ISBN: 0-618-39476-1

123456789-DOW-08 07 06 05 04

Precalculus Functions and Graphs A Graphing Approach

Fourth Edition

A Word from the Authors

Welcome to *Precalculus Functions and Graphs: A Graphing Approach*, Fourth Edition. We are pleased to present this new edition of our textbook in which we focus on making the mathematics accessible, supporting student success, and offering instructors flexibility in how the course can be taught.

Accessible to Students

Over the years we have taken care to write this text with the student in mind. Paying careful attention to the presentation, we use precise mathematical language and a clear writing style to develop an effective learning tool. We believe that every student can learn mathematics, and we are committed to providing a text that makes the mathematics of the college algebra course accessible to all students. For the Fourth Edition, we have revised and improved many text features designed for this purpose.

Throughout the text, we present solutions to many examples from multiple perspectives—algebraic, graphic, and numeric. The side-by-side format of this pedagogical feature helps students to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.

We have found that many precalculus students grasp mathematical concepts more easily when they work with them in the context of real-life situations. Students have numerous opportunities to do this throughout the Fourth Edition, in examples and exercises, including developing models to fit current real data. To reinforce the concept of functions, we have compiled all the elementary functions as a *Library of Functions*. Each function is introduced at the first point of use in the text with a definition and description of basic characteristics; all elementary functions are also presented in a summary on the front endpapers of the text for convenient reference.

We have carefully written and designed each page to make the book more readable and accessible to students. For example, to avoid unnecessary page turning and disruptions to students' thought processes, each example and corresponding solution begins and ends on the same page.

Supports Student Success

During more than thirty years of teaching and writing, we have learned many things about the teaching and learning of mathematics. We have found that students are most successful when they know what they are expected to learn and why it is important to learn it. With that in mind, we have enhanced the thematic study thread throughout the Fourth Edition.

Each chapter begins with a list of section references and a study guide, *What You Should Learn*, which is a comprehensive overview of the chapter concepts. This study guide helps students prepare to study and learn the material in the chapter.

Using the same pedagogical theme, each section begins with a set of section learning objectives—*What You Should Learn*. These are followed by an engaging real-life application—*Why You Should Learn It*—that motivates students and illustrates an area where the mathematical concepts will be applied in an example or exercise in the section. The *Chapter Summary—What Did You Learn?*—at the end of each chapter is a section-by-section overview that ties the learning objectives from the chapter to sets of *Review Exercises* at the end of each chapter.

Throughout the text, other features further improve accessibility. *Study Tips* are provided throughout the text at point-of-use to reinforce concepts and to help students learn how to study mathematics. *Explorations* have been expanded in order to reinforce mathematical concepts. Each Example with worked-out solution is followed by a *Checkpoint*, which directs the student to work a similar exercise from the exercise set. The *Section Exercises* now begin with a *Vocabulary Check*, which gives the students an opportunity to test their understanding of the important terms in the section. *Synthesis Exercises* check students' conceptual understanding of the topics in each section and *Review Exercises* provide additional practice with the concepts in the chapter or previous chapters. *Chapter Tests*, at the end of each chapter, and periodic *Cumulative Tests* offer students frequent opportunities for self-assessment and to develop strong study- and test-taking skills.

The use of technology also supports students with different learning styles, and graphing calculators are fully integrated into the text presentation. In the Fourth Edition, a robust *Technology Support Appendix* has been added to make it easier for students to use technology. *Technology Support* notes are provided throughout the text at point-of-use. These notes guide students to the *Technology Support Appendix*, where they can learn how to use specific graphing calculator features to enhance their understanding of the concepts presented in the text. These notes also direct students to the *Graphing Technology Guide*, on the textbook website, for keystroke support that is available for numerous calculator models. *Technology Tips* are provided in the text at point-of-use to call attention to the strengths and weaknesses of graphing technology, as well as to offer alternative methods for solving or checking a problem using technology. Because students are often misled by the limitations of graphing calculators, we have, where appropriate, used color to enhance the graphing calculator displays in the textbook. This enables students to visualize the mathematical concepts clearly and accurately and avoid common misunderstandings.

Numerous additional text-specific resources are available to help students succeed in the precalculus course. These include “live” online tutoring, instructional DVDs and videos, and a variety of other resources, such as tutorial support and self-assessment, which are available on CD-ROM and the Web. In addition, the *Student Success Organizer* is a note-taking guide that helps students organize their class notes and create an effective study and review tool.

Flexible Options for Instructors

From the time we first began writing textbooks in the early 1970s, we have always considered it a critical part of our role as authors to provide instructors with flexible programs. In addition to addressing a variety of learning styles, the optional features within the text allow instructors to design their courses to meet their instructional needs and the needs of their students. For example, the

Explorations throughout the text can be used as a quick introduction to concepts or as a way to reinforce student understanding.


Our goal when developing the exercise sets was to address a wide variety of learning styles and teaching preferences. New to this edition are the *Vocabulary Check* questions, which are provided at the beginning of every exercise set to help students learn proper mathematical terminology. In each exercise set we have included a variety of exercise types, including questions requiring writing and critical thinking, as well as real-data applications. The problems are carefully graded in difficulty from mastery of basic skills to more challenging exercises. Some of the more challenging exercises include the *Synthesis Exercises* that combine skills and are used to check for conceptual understanding. *Review Exercises*, placed at the end of each exercise set, reinforce previously learned skills in preparation for the next lesson. In addition, Houghton Mifflin's Eduspace® website offers instructors the option to assign homework and tests online—and also includes the ability to grade these assignments automatically.

Several other print and media resources are also available to support instructors. The *Instructor Success Organizer* includes suggested lesson plans and is an especially useful tool for larger departments that want all sections of a course to follow the same outline. The *Instructor's Edition* of the *Student Success Organizer* can be used as a lecture outline for every section of the text and includes additional examples for classroom discussion and important definitions. This is another valuable resource for schools trying to have consistent instruction and it can be used as a resource to support less experienced instructors. When used in conjunction with the *Student Success Organizer* these resources can save instructors preparation time and help students concentrate on important concepts. For a complete list of resources available with this text, see page xvii.

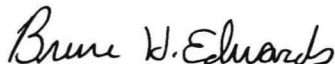
We hope you enjoy the Fourth Edition!



Ron Larson



Robert P. Hostetler



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Acknowledgments

We would like to thank the many people who have helped us prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable to us.

Fourth Edition Reviewers

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We would like to thank the staff of Larson Texts, Inc. and the staff of Meridian Creative Group, who assisted in proofreading the manuscript, preparing and proofreading the art package, and typesetting the supplements.

On a personal level, we are grateful to our wives, Deanna Gilbert Larson, Eloise Hostetler, and Consuelo Edwards for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write us. Over the past two decades we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson
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Features Highlights

Colleges and universities track enrollment figures in order to determine the financial outlook of the institution. The growth in student enrollment at a college or university can be modeled by a linear equation.



David Young/WoodfinCamp

1 Functions and Their Graphs

What You Should Learn

In this chapter, you will learn how to:

- Find and use the slope of a line to write and graph linear equations.
- Evaluate functions and find their domains.
- Analyze graphs of functions.
- Identify and graph shifts, reflections, and nonrigid transformations of functions.
- Find arithmetic combinations and compositions of functions.
- Find inverse functions graphically and algebraically.
- Use scatter plots and a graphing utility to find linear models for data.

- 1.1 Lines in the Plane
- 1.2 Functions
- 1.3 Graphs of Functions
- 1.4 Shifting, Reflecting, and Stretching Graphs
- 1.5 Combinations of Functions
- 1.6 Inverse Functions
- 1.7 Exploring Data: Linear Models and Scatter Plots

1

● “What You Should Learn” and “Why You Should Learn It”

Sections begin with *What You Should Learn*, an outline of the main concepts covered in the section, and *Why You Should Learn It*, a real-life application or mathematical reference that illustrates the relevance of the section content.

● “What You Should Learn”

Each chapter begins with *What You Should Learn*, a comprehensive overview of the chapter concepts. The photograph and caption illustrate a real-life application of a key concept. Section references help students prepare for the chapter.

1.2 Functions

Introduction to Functions

Many everyday phenomena involve pairs of quantities that are related to each other by some rule of correspondence. The mathematical term for such a rule of correspondence is a **relation**. Here are two examples.

1. The simple interest I earned on an investment of \$1000 for 1 year is related to the annual interest rate r by the formula $I = 1000r$.
2. The area A of a circle is related to its radius r by the formula $A = \pi r^2$.

Not all relations have simple mathematical formulas. For instance, people commonly match up NFL starting quarterbacks with touchdown passes, and hours of the day with temperature. In each of these cases, there is some relation that matches each item from one set with exactly one item from a different set. Such a relation is called a **function**.

Definition of a Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B . The set A is the **domain** (or set of inputs) of the function f , and the set B contains the **range** (or set of outputs).

To help understand this definition, look at the function that relates the time of day to the temperature in Figure 1.14.

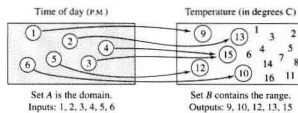


Figure 1.14

This function can be represented by the ordered pairs $\{(1, 9^{\circ}), (2, 10^{\circ}), (3, 12^{\circ}), (4, 13^{\circ}), (5, 15^{\circ}), (6, 15^{\circ})\}$. In each ordered pair, the first coordinate (x -value) is the **input** and the second coordinate (y -value) is the **output**.

Characteristics of a Function from Set A to Set B

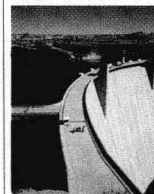
1. Each element of A must be matched with an element of B .
2. Some elements of B may not be matched with any element of A .
3. Two or more elements of A may be matched with the same element of B .
4. An element of A (the domain) cannot be matched with two different elements of B .

What you should learn

- Decide whether relations between two variables represent a function.
- Use function notation and evaluate functions.
- Find the domains of functions.
- Use functions to model and solve real-life problems.
- Evaluate difference quotients.

Why you should learn it

Many natural phenomena can be modeled by functions, such as the force of water against the face of a dam, explored in Exercise 81 on page 28.



Kurtis Ohsaki/Corbis

Application

Sequences have many applications in situations that involve a recognizable pattern. One such model is illustrated in Example 9.

Example 9 Population of the United States

From 1970 to 2001, the resident population of the United States can be approximated by the model

$$a_n = 205.7 + 1.78n + 0.025n^2, \quad n = 0, 1, \dots, 31$$

where a_n is the population in millions and n represents the year, with $n = 0$ corresponding to 1970. Find the last five terms of this finite sequence. (Source: U.S. Census Bureau)

Algebraic Solution

The last five terms of this finite sequence are as follows.

$$a_{27} = 205.7 + 1.78(27) + 0.025(27)^2 \\ \approx 272.0$$

$$a_{28} = 205.7 + 1.78(28) + 0.025(28)^2 \\ \approx 275.1$$

$$a_{29} = 205.7 + 1.78(29) + 0.025(29)^2 \\ \approx 278.3$$

$$a_{30} = 205.7 + 1.78(30) + 0.025(30)^2 \\ \approx 281.6$$

$$a_{31} = 205.7 + 1.78(31) + 0.025(31)^2 \\ \approx 284.9$$

Graphical Solution

Using a graphing utility set to *dot* and *sequence* modes, enter the sequence

$$u_n = 205.7 + 1.78n + 0.025n^2.$$

Set the viewing window to $0 \leq n \leq 32$, $0 \leq x \leq 32$, and $200 \leq y \leq 300$. Then graph the sequence. Use the *value* or *trace* feature to approximate the last five terms, as shown in Figure 8.6.

$$a_{27} \approx 272.0,$$

$$a_{28} \approx 275.1,$$

$$a_{29} \approx 278.3,$$

$$a_{30} \approx 281.6,$$

$$a_{31} \approx 284.9$$

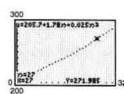


Figure 8.6

Checkpoint Now try Exercise 111.

Exploration

A $3 \times 3 \times 3$ cube is created using 27 unit cubes (a unit cube has a length, width, and height of 1 unit) and only the faces of each cube that are visible are painted blue (see Figure 8.7). Complete the table below to determine how many unit cubes of the $3 \times 3 \times 3$ cube have 0 blue faces, 1 blue face, 2 blue faces, and 3 blue faces. Do the same for a $4 \times 4 \times 4$ cube, a $5 \times 5 \times 5$ cube, and a $6 \times 6 \times 6$ cube and add your results to the table below. What type of pattern do you observe in the table? Write a formula you could use to determine the column values for an $n \times n \times n$ cube.

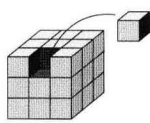


Figure 8.7

Cube	Number of blue faces	0	1	2	3
$3 \times 3 \times 3$					

Library of Functions

The *Library of Functions* feature defines each elementary function and its characteristics at first point of use.

Explorations

The *Exploration* engages students in active discovery of mathematical concepts, strengthens critical thinking skills, and helps them to develop an intuitive understanding of theoretical concepts.

Study Tips

Study Tips reinforce concepts and help students learn how to study mathematics.

Examples

Many examples present side-by-side solutions from multiple approaches—algebraic, graphical, and numerical. This format addresses a variety of learning styles and shows students that different solution methods yield the same result.

Checkpoint

The *Checkpoint* directs students to work a similar problem in the exercise set for extra practice.

Comparing the functions in Examples 2 and 3, observe that

$$F(x) = 2^{-x} = f(-x) \quad \text{and} \quad G(x) = 4^{-x} = g(-x).$$

Consequently, the graph of F is a reflection (in the y -axis) of the graph of f , as shown in Figure 3.3. The graphs of G and g have the same relationship, as shown in Figure 3.4.

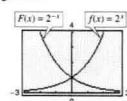


Figure 3.3

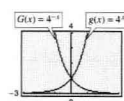


Figure 3.4

The graphs in Figures 3.1 and 3.2 are typical of the graphs of the exponential functions $f(x) = a^x$ and $f(x) = a^{-x}$. They have one y -intercept and one horizontal asymptote (the x -axis), and they are continuous.

Library of Functions: Exponential Function

The exponential function

$$f(x) = a^x, \quad a > 0, \quad a \neq 1$$

is different from all the functions you have studied so far because the variable x is an *exponent*. A distinguishing characteristic of an exponential function is its rapid increase as x increases (for $a > 1$). Many real-life phenomena with patterns of rapid growth (or decline) can be modeled by exponential functions. The basic characteristics of the exponential function are summarized below.

Graph of $f(x) = a^x$, $a > 1$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Intercept: $(0, 1)$

Increasing on $(-\infty, \infty)$

x -axis is a horizontal asymptote

($a^x \rightarrow 0$ as $x \rightarrow -\infty$)

Continuous

Graph of $f(x) = a^{-x}$, $a > 1$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

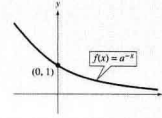
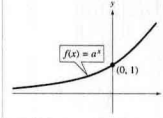
Intercept: $(0, 1)$

Decreasing on $(-\infty, \infty)$

x -axis is a horizontal asymptote

($a^{-x} \rightarrow 0$ as $x \rightarrow \infty$)

Continuous



Exploration

Use a graphing utility to graph $y = a^x$ for $a = 3, 5$, and 7 in the same viewing window. (Use a viewing window in which $-2 \leq x \leq 1$ and $0 \leq y \leq 2$.) How do the graphs compare with each other? Which graph is on the top in the interval $(-\infty, 0)$? Which is on the bottom? Which graph is on the top in the interval $(0, \infty)$? Which is on the bottom? Repeat this experiment with the graphs of $y = b^x$ for $b = \frac{1}{3}, \frac{1}{5}$, and $\frac{1}{7}$. (Use a viewing window in which $-1 \leq x \leq 2$ and $0 \leq y \leq 2$.) What can you conclude about the shape of the graph of $y = b^x$ and the value of b ?

Note in Example 6 that there are many polynomial functions with the indicated zeros. In fact, multiplying the function by any real number does not change the zeros of the function. For instance, multiply the function from part (b) by $\frac{1}{2}$ to obtain $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{2}$. Then find the zeros of the function. You will obtain the zeros $3, 2 + \sqrt{11}$, and $2 - \sqrt{11}$ as given in Example 6.

Example 7 Sketching the Graph of a Polynomial Function

Sketch the graph of $f(x) = 3x^4 - 4x^3$ by hand.

Solution

1. Apply the Leading Coefficient Test. Because the leading coefficient is positive and the degree is even, you know that the graph eventually rises to the left and to the right (see Figure 2.25).

2. Find the Zeros of $f(x) = 3x^4 - 4x^3$

you can see that odd multiplicity points to your graph. Plot a few additional points, as shown to the left and

4. Draw the Graph. Figure 2.26. Be sure graph should cross shape of a portion

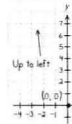


Figure 2.25

Checkpoint Now try Exercise 39.

TECHNOLOGY TIP

It is easy to make mistakes when entering functions into a graphing utility. So, it is important to have an understanding of the basic shapes of graphs and to be able to graph simple polynomials by hand. For example, suppose you had entered the function in Example 7 as $y = 3x^4 - 4x^3$. By looking at the graph, what mathematical principles would alert you to the fact that you had made a mistake?

Example 7 Ultraviolet Radiation

For a person with sensitive skin, the amount of time T (in hours) the person can be exposed to the sun with a minimal burning can be modeled by

$$T = \frac{0.37x + 23.8}{x} \quad 0 < x \leq 120$$

where x is the Sun's solar reading. The Sun's solar reading is based on the level of intensity of UVB rays. (Source: *Reading, Inc.*)

- a. Find the amount of time a person with sensitive skin can be exposed to the sun with minimal burning when $x = 10$, $x = 25$, and $x = 100$.
- b. If the model were valid for all $x > 0$, what would be the horizontal asymptote of this function, and what would it represent?

Algebraic Solution

- a. When $x = 10$, $T = \frac{0.37(10) + 23.8}{10}$

$$= 2.75 \text{ hours.}$$

$$\text{When } x = 25, T = \frac{0.37(25) + 23.8}{25}$$

$$= 1.32 \text{ hours.}$$

$$\text{When } x = 100, T = \frac{0.37(100) + 23.8}{100}$$

$$= 0.61 \text{ hour.}$$

- b. Because the degree of the numerator and denominator are the same for

$$T = \frac{0.37x + 23.8}{x}$$

the horizontal asymptote is given by the ratio of the leading coefficients of the numerator and denominator. So, the graph has the line $T = 0.37$ as a horizontal asymptote. This line represents the shortest possible exposure time with minimal burning.

Graphical Solution

- a. Use a graphing utility to graph the function

$$y_1 = \frac{0.37x + 23.8}{x}$$

using a viewing window similar to that shown in Figure 2.57. Then use the *trace* or *value* feature to approximate the value of y_1 when $x = 10$, $x = 25$, and $x = 100$. You should obtain the following values.

$$\text{When } x = 10, y_1 = 2.75 \text{ hours.}$$

$$\text{When } x = 25, y_1 = 1.32 \text{ hours.}$$

$$\text{When } x = 100, y_1 = 0.61 \text{ hour.}$$



Figure 2.57

- b. Continue to use the *trace* or *value* feature to approximate values of $f(x)$ for larger and larger values of x (see Figure 2.58). From this, you can estimate the horizontal asymptote to be $y = 0.37$. This line represents the shortest possible exposure time with minimal burning.

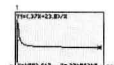


Figure 2.58

Checkpoint Now try Exercise 39.


Technology Tip

Technology Tips point out the pros and cons of technology use in certain mathematical situations. *Technology Tips* also provide alternative methods of solving or checking a problem by the use of a graphing calculator.

Technology Support


The *Technology Support* feature guides students to the *Technology Support Appendix* if they need to reference a specific calculator feature. These notes also direct students to the *Graphing Technology Guide*, on the textbook website, for keystroke support that is available for numerous calculator models.

Real-Life Applications

A wide variety of real-life applications, many using current real data, are integrated throughout the examples and exercises. The  indicates an example that involves a real-life application.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus.

 indicates an example or exercise in which the algebra of calculus is featured.

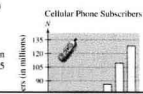
Applications

Example 7 Cellular Phone Subscribers

The number N (in millions) of cellular phone subscribers in the United States increased in a linear pattern from 1995 to 1997, as shown in Figure 1.17. Then, in 1998, the number of subscribers took a jump, and until 2001, increased in a different linear pattern. These two patterns can be approximated by the function

$$N(t) = \begin{cases} 10.75t - 20.1, & 5 \leq t \leq 7 \\ 20.11t - 92.8, & 8 \leq t \leq 11 \end{cases}$$

where t represents the year, with $t = 5$ corresponding to 1995. Use this function to approximate the number of cellular phone subscribers for each year from 1995 to 2004. (Source: Cellular Telecommunications & Internet Association)



Solution

From 1995 to 1997,

$$N(t) = 10.75t - 20.1$$

$$N(5) = 10.75(5) - 20.1 = 33.7$$

$$N(6) = 10.75(6) - 20.1 = 44.4$$

$$N(7) = 10.75(7) - 20.1 = 55.1$$

From 1998 to 2001,

$$N(t) = 20.11t - 92.8$$

$$N(8) = 20.11(8) - 92.8 = 68.1$$

$$N(9) = 20.11(9) - 92.8 = 88.2$$

$$N(10) = 20.11(10) - 92.8 = 108.3$$

$$N(11) = 20.11(11) - 92.8 = 128.4$$

Checkpoint Now try Exercise 39.

Example 8

A baseball is hit at an angle

$$f(x) = -0.03x^2 + 0.3x + 5$$

where x and y are in feet from home plate.

Algebraic Solution

The height of the ball from home plate is

$$f(x) = -0.03x^2 + 0.3x + 5$$

$$f(0) = -0.03(0)^2 + 0.3(0) + 5 = 5$$

When $x = 300$, the ball will clear a 10-ft fence.

Checkpoint Now try Exercise 39.

45. $h(x) = \frac{x^2}{x-1}$

46. $f(x) = \frac{x^3}{x^2-1}$

47. $g(x) = \frac{x^3}{2x^2-8}$

48. $f(x) = \frac{x^2-1}{x^2+4}$

49. $f(x) = \frac{x^3+2x^2+4}{2x^2+1}$

50. $f(x) = \frac{2x^2-5x+5}{x-2}$

Graphical Reasoning

In Exercises 51–54, (a) use the graph to estimate any x -intercepts of the rational function and (b) set $y = 0$ and solve the resulting equation to confirm your result in part (a).

$$51. y = \frac{x+1}{x-3}$$

$$52. y = \frac{2x}{x-3}$$

$$53. y = \frac{1}{x} - x$$

$$54. y = x - 3 + \frac{2}{x}$$

$$55. y = \frac{2x^2+x}{x+1}$$

$$56. y = \frac{x^2+5x+8}{x+3}$$

$$57. y = \frac{1+3x^2-x^3}{x^2}$$

$$58. y = \frac{12-2x-x^2}{2(4+x)}$$

Graphical Reasoning

In Exercises 59–62, (a) use a graphing utility to graph the function and determine any x -intercepts, and (b) set $y = 0$ and solve the resulting equation to confirm your result in part (a).

$$59. y = \frac{1}{x} + \frac{4}{x}$$

$$60. y = 20\left(\frac{2}{x+1} - \frac{3}{x}\right)$$

$$61. y = x - \frac{9}{x-1}$$

$$62. y = x - \frac{9}{x}$$

63. Concentration of a Mixture

A 1000-liter tank contains 50 liters of a 25% brine solution. You add x liters of a 75% brine solution to the tank.

- (a) Show that the concentration C , the proportion of brine to the total solution, of the final mixture is given by

$$C = \frac{3x + 50}{2(x + 50)}$$

- (b) Determine the domain of the function based on the physical constraints of the problem.

- (c) Use a graphing utility to graph the function. As the tank is filled, what happens to the rate at which the concentration of brine increases? What percent does the concentration of brine appear to approach?

64. Geometry

A rectangular region of length x and width y has an area of 500 square meters.

- (a) Write the width y as a function of x .

- (b) Determine the domain of the function based on the physical constraints of the problem.

- (c) Sketch a graph of the function and determine the width of the rectangle when $x = 30$ meters.

65. Page Design

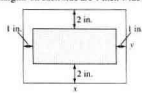
A page that is x inches wide and y inches high contains 30 square inches of print. The margins at the top and bottom are 2 inches deep and the margins on each side are 1 inch wide (see figure).

- (a) Show that the total area A of the page is given by

$$A = \frac{2x(2x + 11)}{x - 2}$$

- (b) Determine the domain of the function based on the physical constraints of the problem.

- (c) Use a graphing utility to graph the area function and approximate the page size such that the minimum amount of paper will be used. Verify your answer numerically using the *table* feature of a graphing utility.



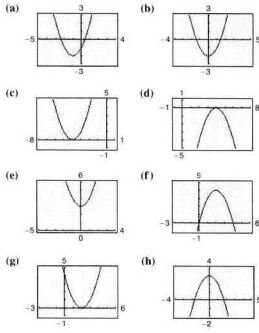
2.1 Exercises

Vocabulary Check

Fill in the blanks.

- A polynomial function of degree n and leading coefficient a_n is a function of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, $a_n \neq 0$ where n is a _____ and a_i is a _____ number.
- A _____ function is a second-degree polynomial function, and its graph is called a _____.
- The graph of a quadratic function is symmetric about its _____.
- If the graph of a quadratic function opens upward, then its leading coefficient is _____ and the vertex of the graph is a _____.
- If the graph of a quadratic function opens downward, then its leading coefficient is _____ and the vertex of the graph is a _____.

In Exercises 1–8, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]



- $f(x) = (x - 2)^2$
- $f(x) = (x + 4)^2$
- $f(x) = x^2 - 2$
- $f(x) = 3 - x^2$

- $f(x) = 4 - (x - 2)^2$
- $f(x) = x^2 + 3$
- $f(x) = (x + 1)^2 - 2$
- $f(x) = -(x - 4)^2$

In Exercises 9–12, use a graphing utility to graph each function in the same viewing window. Describe how the graph of each function is related to the graph of $y = x^2$.

- (a) $y = \frac{1}{2}x^2$ (b) $y = \frac{1}{2}x^2 - 1$
- (c) $y = \frac{1}{2}(x + 3)^2$ (d) $y = -\frac{1}{2}(x + 3)^2 - 1$
- (a) $y = \frac{1}{2}x^2$ (b) $y = \frac{1}{2}x^2 + 1$
- (c) $y = \frac{1}{2}(x - 3)^2$ (d) $y = -\frac{1}{2}(x - 3)^2 + 1$
- (a) $y = -2x^2$ (b) $y = -2x^2 - 1$
- (c) $y = -2(x - 3)^2$ (d) $y = 2(x - 3)^2 - 1$
- (a) $y = -4x^2$ (b) $y = -4x^2 + 3$
- (c) $y = -4(x + 2)^2$ (d) $y = 4(x + 2)^2 + 3$

In Exercises 13–26, sketch the graph of the quadratic function. Identify the vertex and x -intercepts. Use a graphing utility to verify your results.

- $f(x) = 25 - x^2$
- $f(x) = x^2 - 7$
- $f(x) = \frac{1}{2}x^2 - 4$
- $f(x) = 16 - \frac{1}{2}x^2$
- $f(x) = (x + 4)^2 - 3$
- $f(x) = (x - 6)^2 + 3$
- $h(x) = x^2 - 8x + 16$
- $g(x) = x^2 + 2x + 1$
- $f(x) = x^2 - x + \frac{1}{4}$
- $f(x) = x^2 + 3x + \frac{1}{4}$

● Synthesis and Review Exercises

Each exercise set concludes with two types of exercises.

Synthesis exercises promote further exploration of mathematical concepts, critical thinking skills, and writing about mathematics. The exercises require students to show their understanding of the relationships between many concepts in the section.

Review Exercises reinforce previously learned skills and concepts.

● Vocabulary Check

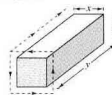
Section exercises begin with a *Vocabulary Check* that serves as a review of the important mathematical terms in each section.

● Section Exercises

The section exercise sets consist of a variety of computational, conceptual, and applied problems.

126 Chapter 2 Polynomial and Rational Functions

- Use a graphing utility and the model to create a table of estimated values for S . Compare the estimated values with the actual data.
 - Use the Remainder Theorem to evaluate the model for the year 2008. Even though the model is relatively accurate for estimating the given data, would you use this model to predict the sales from lottery tickets in the future? Explain.
81. **Geometry** A rectangular package sent by a delivery service can have a maximum combined length and girth (perimeter of a cross section) of 120 inches (see figure).



- Show that the volume of the package is given by the function $V(x) = 4x^2(30 - x)$.
 - Use a graphing utility to graph the function and approximate the dimensions of the package that yield a maximum volume.
 - Find values of x such that $V = 13,500$. Which of these values is a physical impossibility in the construction of the package? Explain.
82. **Automobile Emissions** The number of parts per million of nitric oxide emissions y from a car engine is approximated by the model $y = -5.05x^3 + 3857x - 38,411.25$, $13 \leq x \leq 18$ where x is the air-fuel ratio.
- Use a graphing utility to graph the model.
 - It is observed from the graph that two air-fuel ratios produce 2400 parts per million of nitric oxide, with one being 15. Use the graph to approximate the second air-fuel ratio.
 - Algebraically approximate the second air-fuel ratio that produces 2400 parts per million of nitric oxide. (Hint: Because you know that an air-fuel ratio of 15 produces the specified nitric oxide emission, you can use synthetic division.)

Synthesis

True or False? In Exercises 83 and 84, determine whether the statement is true or false. Justify your answer.

- If $(7x + 4)$ is a factor of some polynomial function f , then $\frac{4}{7}$ is a zero of f .
- $(2x - 1)$ is a factor of the polynomial $6x^6 + x^5 - 92x^4 + 45x^3 + 184x^2 + 4x - 48$.

Think About It In Exercises 85 and 86, perform the division by assuming that n is a positive integer.

- $\frac{x^{3n} + 9x^{2n} + 27x^n + 27}{x^n + 3}$
- $\frac{x^{3n} - 3x^{2n} + 5x^n - 6}{x^n - 2}$

87. **Writing** Complete each polynomial division. Write a brief description of the pattern that you obtain, and use your result to find a formula for the polynomial division $(x^n - 1)/(x - 1)$. Create a numerical example to test your formula.

- $\frac{x^2 - 1}{x - 1}$
- $\frac{x^3 - 1}{x - 1}$
- $\frac{x^4 - 1}{x - 1}$

88. **Writing** Write a short paragraph explaining how you can check polynomial division. Give an example.

Review

In Exercises 89–92, use any convenient method to solve the quadratic equation.

- $9x^2 - 25 = 0$
- $16x^2 - 21 = 0$
- $2x^2 + 6x + 3 = 0$
- $8x^2 - 22x + 15 = 0$

In Exercises 93–96, find a polynomial function that has the given zeros. (There are many correct answers.)

- 0, -12
- 1, -3, 8
- 0, -1, 2, 5
- $2 + \sqrt{3}$, $2 - \sqrt{3}$

Chapter Summary 443

6 Chapter Summary

What did you learn?

Section 6.1

- Use the Law of Sines to solve oblique triangles (AAS, ASA, or SSA).
- Find area of oblique triangles.
- Use the Law of Sines to model and solve real-life problems.

Section 6.2

- Use the Law of Cosines to solve oblique triangles (SSS or SAS).
- Use the Law of Cosines to model and solve real-life problems.
- Use Heron's Area Formula to find area of triangles.

Section 6.3

- Represent vectors in standard form.
- Write the components of a vector.
- Perform basic operations on vectors.
- Find the direction of a vector.
- Use vectors to solve problems.

Section 6.4

- Find the dot product of two vectors.
- Find the angle between two vectors.
- Write vector equations of lines and planes.
- Use vectors to solve problems.

Section 6.5

- Find absolute values of complex numbers.
- Write trigonometric functions in standard form.
- Multiply and divide complex numbers.
- Use De Moivre's Theorem to find powers and roots of complex numbers.
- Find nth roots of complex numbers.

Review Exercises

1–10
11–14
15–18

19–28
29–32
33–36

444 Chapter 6 Additional Topics in Trigonometry

6 Review Exercises

6.1 In Exercises 1–10, use the Law of Sines to solve the triangle. If two solutions exist, find both.

- $A = 21^\circ$, $B = 42^\circ$, $a = 6$
- $B = 110^\circ$, $C = 30^\circ$, $c = 11$
- $A = 75^\circ$, $a = 2.5$, $b = 16.5$
- $A = 130^\circ$, $a = 60$, $b = 48$
- $B = 115^\circ$, $a = 9$, $b = 14.5$
- $C = 50^\circ$, $a = 25$, $c = 22$
- $A = 15^\circ$, $a = 5$, $b = 10$
- $B = 150^\circ$, $a = 64$, $b = 10$
- $B = 25^\circ$, $a = 6.2$, $b = 4$
- $A = 74^\circ$, $b = 12.8$, $a = 12.5$

In Exercises 11–14, find the area of the triangle having the indicated angle and sides.

- $A = 27^\circ$, $b = 5$, $c = 8$
- $B = 80^\circ$, $a = 4$, $c = 8$
- $C = 122^\circ$, $b = 18$, $a = 29$
- $C = 100^\circ$, $a = 120$, $b = 74$

15. **Height** From a distance of 50 meters, the angle of elevation to the top of a building is 17° . Approximate the height of the building.

16. **Distance** A family is traveling due west on a road that passes a famous landmark. At a given time the bearing to the landmark is $N 62^\circ W$, and after the family travels 5 miles further, the bearing is $N 38^\circ W$. What is the closest the family will come to the landmark while on the road?

17. **Height** A tree stands on a hillside of slope 28° from the horizontal. From a point 75 feet down the hill, the angle of elevation to the top of the tree is 45° (see figure). Find the height of the tree.

18. **Width** A surveyor finds that a tree on the opposite bank of a river has a bearing of $N 22^\circ 30' E$ from a certain point and a bearing of $N 15^\circ W$ from a point 400 feet downstream. Find the width of the river.

6.2 In Exercises 19–28, use the Law of Cosines to solve the triangle.

- $a = 9$, $b = 12$, $c = 20$
- $a = 7$, $b = 15$, $c = 19$
- $C = 45^\circ$, $a = 6$, $b = 9$
- $B = 90^\circ$, $a = 5$, $c = 12$
- $B = 110^\circ$, $a = 4$, $c = 4$
- $B = 12^\circ$, $a = 32$, $c = 36$
- $B = 150^\circ$, $a = 10$, $c = 20$
- $a = 42$, $b = 25$, $c = 58$
- $a = 8.9$, $b = 6.1$, $c = 10.5$
- $a = 7.5$, $b = 9.8$, $c = 4.5$

29. **Geometry** The lengths of the diagonals of a parallelogram are 10 feet and 16 feet. Find the lengths of the sides of the parallelogram if the diagonals intersect at an angle of 28° .

30. **Geometry** The lengths of the diagonals of a parallelogram are 30 meters and 40 meters. Find the lengths of the sides of the parallelogram if the diagonals intersect at an angle of 34° .

31. **Navigation** Two planes leave Washington, D.C.'s Dulles International Airport at approximately the same time. One is flying at 425 miles per hour at a bearing of 35° , and the other is flying at 530 miles per hour at a bearing of 67° (see figure). Determine the distance between the planes after they have flown for 2 hours.

Chapter Tests and Cumulative Tests

Chapter Tests, at the end of each chapter, and periodic *Cumulative Tests* offer students frequent opportunities for self-assessment and to develop strong study- and test-taking skills.

Chapter Summary

The *Chapter Summary*, “What Did You Learn?”, is a section-by-section overview that ties the learning objectives from the chapter to sets of Review Exercises for extra practice.

Review Exercises

The chapter *Review Exercises* provide additional practice with the concepts in the chapter.

448 Chapter 6 Additional Topics in Trigonometry

6 Chapter Test

Take this test as you would take a test in class. After you are finished, check your work against the answers given in the back of the book.

In Exercises 1–6, use the given information to solve the triangle. If two solutions exist, find both.

- $A = 36^\circ$, $B = 98^\circ$, $c = 18$
- $a = 4$, $b = 7$, $c = 9$
- $A = 35^\circ$, $b = 8$, $c = 11$
- $A = 25^\circ$, $b = 28$, $a = 15$
- $B = 130^\circ$, $c = 10.1$, $b = 5.2$
- $A = 150^\circ$, $b = 4.8$, $a = 9.4$

7. Find the length of the pond shown in the right.

8. A triangular plot of land has boundaries of lengths 55 meters, 88 meters, and 100 meters. Find the area.

9. Find the coordinates of the point $(-8, -12)$ in polar form.

In Exercises 10–14, find a unit vector in the same direction as the given vector.

- $\mathbf{u} = (0, -4)$
- $\mathbf{u} = (-2, -)$
- $\mathbf{u} = 1 - \mathbf{j}$
- $\mathbf{u} = 21 + 3\mathbf{j}$
- Find a unit vector in the same direction as $\mathbf{u} = (3, -)$

15. Forces with magnitudes of 10 and 15 newtons act on an object at angles of 30° and 120° to the horizontal, respectively. Find the magnitude and direction of the resultant force.

16. Find the angle between the vectors $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$.

17. Find the angle between the vectors $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$.

18. Find the angle between the vectors $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$.

19. Are the vectors $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ orthogonal?

20. Find the projection of $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ onto $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$.

21. Write the vector equation of the line passing through $(1, 2)$ and $(3, 4)$.

22. Write the vector equation of the line passing through $(1, 2)$ and $(3, 4)$.

In Exercises 23–26, find the complex number z such that $z^2 = w$.

- $w = 3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$
- $w = 3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$
- $w = 3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$
- $w = 3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$

25. Find the four square roots of 16 .

26. Find all solutions of the equation $\tan^2 \theta = 3$ in the interval $[0, 2\pi)$.

4–6 Cumulative Test

Take this test to review the material from earlier chapters. After you are finished, check your work against the answers given in the back of the book.

In Exercises 1–6, use the given information to solve the triangle. If two solutions exist, find both.

- Consider the angle $\theta = -120^\circ$.
 - Sketch the angle in standard position.
 - Determine a coterminal angle in the interval $(0^\circ, 360^\circ)$.
 - Convert the angle to radian measure.
 - Find the reference angle θ' .
- Find the exact values of the six trigonometric functions of θ .
- Convert the angle $\theta = 2.35$ radians to degrees. Round your answer to one decimal place.
- Find $\cos \theta$ if $\tan \theta = -\frac{4}{3}$ and $\sin \theta > 0$.

In Exercises 7–9, sketch the graph of the function by hand. (Include two full periods.) Use a graphing utility to verify your graph.

- $f(x) = 3 - 2 \sin \pi x$
- $f(x) = \tan 3x$
- $f(x) = \frac{1}{2} \sec(x + \pi)$

7. Find a , b , and c such that the graph of the function $h(x) = a \cos(bx + c)$ matches the graph in the figure at the right.

Figure for 7

In Exercises 8 and 9, find the exact value of the expression without using a calculator.

- $\tan(\arcsin \frac{1}{2})$
- $\tan(\arcsin \frac{1}{2})$

10. Write an algebraic expression equivalent to $\sin(\arcsin 2x)$.

11. Subtract and simplify: $\frac{\sin \theta - 1}{\cos \theta} - \frac{\cos \theta}{\sin \theta - 1}$

In Exercises 12–14, verify the identity.

- $\cot^2 \alpha \sec^2 \alpha - 1 = 1$
- $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$
- $\sin^2 x \cos^2 x = \frac{1}{4}(1 - \cos 4x)$

In Exercises 15 and 16, solve the equation.

- $\sin^2 x + 2 \sin x + 1 = 0$
- $3 \tan \theta - \cot \theta = 0$

17. Approximate the solutions to the equation $\cos^2 x - 5 \cos x - 1 = 0$ in the interval $[0, 2\pi)$.

In Exercises 18 and 19, use a graphing utility to graph the function and approximate its zeros in the interval $[0, 2\pi)$. If possible, find the exact values of the zeros algebraically.

- $y = \frac{1}{2} + \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} - 4$
- $y = \tan^2 x - \tan^2 x + 3 \tan x - 3$

Supplements

Resources

Text Website (college.hmco.com)

Many text-specific resources for students and instructors can be found at the Houghton Mifflin website. They include, but are not limited to, the following features for the student and instructor.

Student Website

- Student Success Organizer
- Digital Lessons
- Graphing Technology Guide
- Graphing Calculator Programs
- Chapter Projects
- Historical Notes
- Ace Quizzes

Instructor Website

- Instructor Success Organizer
- Digital Art and Tables
- Digital Lessons
- Graphing Technology Guide
- Graphing Calculator Programs
- Chapter Projects
- Answers to Chapter Projects
- Link to Student website

Additional Resources for the Student

Study and Solutions Guide by Bruce H. Edwards (University of Florida)

HM mathSpace® Tutorial CD-ROM: This new tutorial CD-ROM allows students to practice skills and review concepts as many times as necessary by using algorithmically generated exercises and step-by-step solutions for practice. The CD-ROM contains a variety of other student resources as well.

Instructional Videotapes by Dana Mosely

Instructional Videotapes for Graphing Calculators by Dana Mosely

SMARTTHINKING™ Live, On-Line Tutoring: Houghton Mifflin has partnered with SMARTTHINKING™ to provide an easy-to-use, effective, on-line tutorial service. Through state-of-the-art tools and a two-way whiteboard, students communicate in real-time with qualified e-structors who can help the students understand difficult concepts and guide them through the problem solving process while studying or completing homework. Live online tutoring support, Question submission, Pre-scheduled tutoring time, and Reviews of past online sessions are four levels of service offered to the students.

Eduspace®: Eduspace® is a text-specific online learning environment that combines algorithmic tutorials with homework capabilities. Text-specific content is available to help you understand the mathematics covered in this textbook.

Eduspace® with eSolutions: Eduspace® with eSolutions combines all the features of Eduspace® with an electronic version of the textbook exercises and the complete solutions to the odd-numbered exercises. The result is a convenient and comprehensive way to do homework and view your course materials.

Additional Resources for the Instructor

Instructor's Annotated Edition (IAE)

Instructor's Solutions Guide and Test Item File by Bruce H. Edwards
(University of Florida)

HM ClassPrep with HM Testing CD-ROM: This CD-ROM is a combination of two course management tools.

- HM Testing 6.0 computerized testing software provides instructors with an array of algorithmic test items, allowing for the creation of an unlimited number of tests for each chapter, including cumulative tests and final exams. HM Testing also offers online testing via a Local Area Network (LAN) or the Internet, as well as a grade book function.
- HM ClassPrep features supplements and text-specific resources.

Eduspace®: Eduspace® is a text-specific online learning environment that combines algorithmic tutorials with homework capabilities and classroom management functions. Electronic grading and Course Management are two levels of service provided for instructors. Please contact your Houghton Mifflin sales representative for detailed information about the course content available for this text.

Eduspace® with eSolutions: Eduspace® with eSolutions combines all the features of Eduspace® with an electronic version of the textbook exercises and the complete solutions to the odd-numbered exercises, providing students with a convenient and comprehensive way to do homework and view course materials.

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